Static Games of Incomplete Information

A *static game of incomplete information* (or *static Bayesian game*) is a collection (N, T, A, u, p), where

- $N = \{1, ..., n\}$ is the set of players,
- $T = T_1 \times .... \times T_n$ is the set of profiles of types of players, that is, T_i is the set of types player $i \in N$ can be;
- $A = A_1 \times .... \times A_n$ is the set of profiles of actions, that is, A_i is the set of actions player $i \in N$ can take;
- $u = (u_1, ..., u_n)$ is the profile of payoff functions, where for $i \in N$, $u_i : T \times A \to \mathbb{R}$;
- $p \in \triangle T$ is the probability distribution according to which players types are selected by nature.

In a static Bayesian game, a pure strategy for player i is a mapping

$$s_i : T_i \to A.$$

Assume that the sets T and A are finite, and given profiles of types $t \in T$, actions $a \in A$, and strategies $s = (s_1, ..., s_n))$, denote by

$$t_{-i} \in T_{-i} := \prod_{j \in N \setminus \{i\}} T_j,$$

$$a_{-i} \in A_{-i} := \prod_{j \in N \setminus \{i\}} A_j,$$

and

$$s_{-i} = (s_j)_{j \in N \setminus \{i\}},$$

respectively, the profile of types, actions, and strategies obtained by deleting the ith coordinate of t, a, and s.

The payoff of player $i \in N$ if the other players' strategies are s_{-i}, his type is $\bar{t}_i \in T_i$ and his action $\bar{a}_i \in A_i$ is

$$U_i(\bar{a}_i, s_{-i} \mid \bar{t}_i) = \sum_{t_{-i} \in T_{-i}} u_i(\bar{a}_i, s_{-i}(t_{-i}), \bar{t}_i, t_{-i}) p(t_{-i} \mid \bar{t}_i),$$

where $s_{-i}(t_{-i}) = ((s_j(t_j))_{j \in N \setminus \{i\}}$ is the profile of actions taken by players other than i when their type profile is t_{-i}, and

$$p(t_{-i} \mid \bar{t}_i) = \frac{p(t_{-i}, \bar{t}_i)}{\sum_{t'_{-i} \in T_{-i}} p(t'_{-i}, \bar{t}_i)}$$

is the probability that the profile of types for players other than i is $t_{-i} \in T_{-i}$.

A solution concept appropriate for static Bayesian games is *Bayesian Nash equilibrium* (BNE).

A profile of pure strategies s^* is a BNE if for all $i \in N$, all $t_i \in T_i$ and all $a_i \in A_i$:

$$U_i(s_i(t_i), s_{-i} \mid t_i) \geq U_i(a_i, s_{-i} \mid t_i).$$

In a BNE every type of every player behaves optimally, that is, maximizes its expected payoff calculated consistently with Bayes' Rule.