Screening

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The Agency Problem with Adverse Selection

A risk neutral principal wants to offer a menu of contracts to be offered to an agent randomly drawn from a heterogenous population of agents.

In the population of agents, a fraction $q \in (0, 1)$ is of type H, and the remaining fraction, $1 - q$, is of type L. Agents of type $\tau \in \{H, L\}$ are characterized by:

- A von N-M utility function $u : \mathbb{R} \to \mathbb{R}$ such that $u(0) = 0$, $u' > 0$ and $u'' \leq 0$, representing his preferences.
- A real number $\underline{u} \geq 0$ specifying his reservation utility, and
- a function $k_\tau v(e)$ describing his cost of effort, where $k_\tau > 0$ and $v : \mathbb{R}_+ \to \mathbb{R}$ satisfies $v(0) = 0$, $v' > 0$, and $v'' > 0$.

Thus, agents only differ in the value of the parameter k_τ. Without loss of generality, let us assume that $k_L = 1$, and $k_H = k > 1$.

The Principal's Problem

The menu of contracts is designed in order to maximize expected profits and assure that every agent will accept one of the contracts offered. Hence the principal's problem is:

$$\max_{\{(e_H,w_H),(e_L,w_L)\}\in\mathbb{R}^4_+} q\left(\mathbb{E}X(e_H)-w_H\right)+(1-q)\left(\mathbb{E}X(e_L)-w_L\right)$$

subject to:

$$\begin{array}{ll}
(PC_H;\,\lambda_H) & u(w_H)\geq kv(e_H)+\underline{u} \\
(PC_L;\,\lambda_L) & u(w_L)\geq v(e_L)+\underline{u} \\
(IC_H;\,\mu_H) & u(w_H)-kv(e_H)\geq u(w_L)-kv(e_L) \\
(IC_L;\,\mu_L) & u(w_L)-v(e_L)\geq u(w_H)-v(e_H).
\end{array}$$

The Optimal Menu of Contracts

This problem may be simplified by noticing that in an interior solution

$$\begin{aligned} u(w_L) - v(e_L) &\geq u(w_H) - v(e_H) \text{ (by } IC_L) \\ &> u(w_H) - kv(e_H) \text{ (because } k > 1 \text{ and } e_H > 0) \\ &\geq \underline{u} \text{ (by } PC_H), \end{aligned}$$

that is, the constraint PC_L is not binding, and hence it can be ignored.

The Principal's Problem

Suppressing the inequality PC_L we may write the Lagrangian as:

$$\begin{aligned}\mathcal{L}(\cdot) &= q\left(\mathbb{E}X(e_H) - w_H\right) + (1-q)\left(\mathbb{E}X(e_L) - w_L\right) \\ &+\lambda_H\left(u(w_H) - kv(e_H) - \underline{u}\right) \\ &+\mu_H\left(u(w_H) - kv(e_H) - u(w_L) + kv(e_L)\right) \\ &+\mu_L\left(u(w_L) - v(e_L) - u(w_H) + v(e_H)\right).\end{aligned}$$

The Optimal Menu of Contracts

The first order conditions identifying an interior solution of the problem are

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial e_H} &= q\left(\mathbb{E}X(e_H)\right)' - \lambda_H k v'(e_H) - \mu_H k v'(e_H) + \mu_L v'(e_H) = 0 \\
\frac{\partial \mathcal{L}}{\partial w_H} &= -q + \lambda_H u'(w_H) + \mu_H u'(w_H) - \mu_L u'(w_H) = 0 \\
\frac{\partial \mathcal{L}}{\partial e_L} &= (1-q)\left(\mathbb{E}X(e_L)\right)' + \mu_H k v'(e_L) - \mu_L v'(e_L) = 0 \\
\frac{\partial \mathcal{L}}{\partial w_L} &= -(1-q) - \mu_H u'(w_L) + \mu_L u'(w_L) = 0,
\end{aligned}$$

The Optimal Menu of Contracts

This system may be rewritten as

$$q\frac{(\mathbb{E}X(e_H))'}{v'(e_H)} = \lambda_H k - (\mu_L - \mu_H k) \tag{1}$$

$$\frac{q}{u'(w_H)} = \lambda_H - (\mu_L - \mu_H) \tag{2}$$

$$(1-q)\frac{(\mathbb{E}X(e_L))'}{v'(e_L)} = \mu_L - k\mu_H \tag{3}$$

$$\frac{1-q}{u'(w_L)} = \mu_L - \mu_H. \tag{4}$$

In addition, the slackness conditions

$$\begin{aligned} \lambda_H \left(u(w_H) - kv(e_H) - \underline{u}\right) &= 0 & (5)\\ \mu_H \left(u(w_H) - kv(e_H) - u(w_L) + kv(e_L)\right) &= 0 & (6)\\ \mu_L \left(u(w_L) - v(e_L) - u(w_H) + v(e_H)\right) &= 0. & (7) \end{aligned}$$

must hold.

Since $\mu_L > 0$ by equation (4), then equation (7) implies that the constrain IC_L is binding.

Likewise, since $\lambda_H > 0$ by equations (2) and (4), then equation (5) implies that the constrain PC_H is binding.

The Optimal Menu of Contracts

Let us show that in a solution to this system the incentive constraint of the low type, IC_H, is not active, i.e., that

$$u(w_H) - kv(e_H) > u(w_L) + kv(e_L),$$

and therefore the slackness condition (7) implies $\mu_H = 0$.

The Optimal Menu of Contracts

To prove this we first show that in a solution $e_L \geq e_H$.

Since

$$\begin{aligned} v(e_L) - v(e_H) &\leq u(w_L) - u(w_H) \text{ (by } IC_L) \\ &\leq k\,(v(e_L) - v(e_H)) \text{ (by } IC_H), \end{aligned}$$

then

$$(1-k)\,(v(e_L) - v(e_H)) \leq 0.$$

This implies

$$v(e_L) \leq v(e_H),$$

and hence

$$e_L \geq e_H.$$

The Optimal Menu of Contracts

Next we show that $e_L \neq e_H$.

Suppose by way of contradiction that $e_L = e_H$. This implies that $w_L = w_H$ for otherwise both types will choose the contract involving the largest wage (for identical effort).
Formally, the inequalities IC_H and IC_L imply

$$0 = v(e_L) - v(e_H) \leq u(w_L) - u(w_H) \leq k\,(v(e_L) - v(e_H)) = 0.$$

Hence

$$u(w_L) - u(w_H) = 0,$$

and therefore

$$w_L = w_H.$$

The Optimal Menu of Contracts

But if $e_L = e_H$ and $w_L = w_H$, then we can suppress the arguments in the system of first order conditions, and write it as:

$$q\frac{(\mathbb{E}X)'}{v'} = \lambda_H k - (\mu_L - \mu_H k) \quad (1)$$

$$\frac{q}{u'} = \lambda_H - (\mu_L - \mu_H) \quad (2)$$

$$(1-q)\frac{(\mathbb{E}X)'}{v'} = \mu_L - k\mu_H \quad (3)$$

$$\frac{1-q}{u'} = \mu_L - \mu_H. \quad (4)$$

The Optimal Menu of Contracts

Substituting $\lambda_H = 1/u'$ from (2) and (4) into equation (2) we get $(1-q)\lambda_H = \mu_L - \mu_H$

$$\mu_L = (1-q)\lambda_H + \mu_H.$$

Substituting $k\lambda_H = (\mathbb{E}X)' / v'$ from (1) and (3) into equation (1) we get

$$\mu_L = k[(1-q)\lambda_L + \mu_H].$$

Since $k > 1$ and $(1-q)\lambda_H + \mu_H > 0$ these two equations cannot hold. Hence

$$e_L \neq e_H,$$

and since $e_L \geq e_H$, we have

$$e_L > e_H.$$

The Optimal Menu of Contracts

Finally we show that IC_H holds with strict inequality, and therefore $\mu_H = 0$.

Since $\mu_L > 0$, then IC_L and $e_L > e_H$ imply

$$u(w_L) - u(w_H) = v(e_L) - v(e_H) > 0.$$

Then $k > 1$ implies

$$k(v(e_L) - v(e_H)) > v(e_L) - v(e_H) = u(w_L) - u(w_H),$$

that is

$$u(w_H) - kv(e_H)) > u(w_L) - kv(e_L).$$

Thus, IC_H holds with strict inequality, and hence $\mu_H = 0$ by the complementary slackness condition (6).

The Optimal Menu of Contracts

Substituting $\mu_H = 0$ into the system of first order conditions, we get

$$q\frac{(\mathbb{E}X(e_H))'}{v'(e_H)} = \lambda_H k - \mu_L \qquad (1)$$

$$\frac{q}{u'(w_H)} = \lambda_H - \mu_L \qquad (2)$$

$$(1-q)\frac{(\mathbb{E}X(e_L))'}{v'(e_L)} = \mu_L \qquad (3)$$

$$\frac{1-q}{u'(w_L)} = \mu_L. \qquad (4)$$

The Optimal Menu of Contracts

This system may be rewritten as

$$
\begin{aligned}
(\mathbb{E}X(e_H))' &= \frac{kv'(e_H)}{u'(w_H)} + \frac{1-q}{q}(k-1)\frac{v'(e_H)}{u'(w_L)} \qquad (1,2) \\
(\mathbb{E}X(e_L))' &= \frac{v'(e_L)}{u'(w_L)} \qquad (3,4).
\end{aligned}
$$

These two equations together with the two binding constrains

$$
\begin{aligned}
u(w_H) &= kv(e_H) - \underline{u} \qquad (5) \\
u(w_L) - v(e_L) &= u(w_H) - v(e_H) \qquad (7)
\end{aligned}
$$

identify the optimal contract.

The Optimal Menu of Contracts

Properties of the optimal menu:

- The contract offered to the low cost type is *optimal:* by equation $(3, 4)$, the Principal selects a contract on her demand of effort from the low cost type.
- The contract offered to the high cost type distorts the demand of effort downward: the contract satisfying equation (1,2) is below the Principal's demand of effort from the high cost type. This distortion makes the contract for the high types less attractive to the low type, which relaxes the incentive constraint for this type.
- As observed earlier, the low cost type captures a positive surplus – which we can refer to as *information rents*.

The Optimal Menu of Contracts

Exercise. $\mathbb{E}X(e) = 2e$, and $u(x) = x$, $v(e) = e^2$, $\underline{u} = 0$, $k = 2$, $q = 1/2$.

Optimal contracts with complete information:

Effort Supplies: $w_H = 2e_H^2$; $w_L = e_L^2$.
Effort Demands: $2 = 4e_H$, i.e., $e_H = 1/2$; $2 = 2e_L$, i.e., $e_L = 1$.

Thus, the optimal contracts are

$$(e_L^*, w_L^*) = (1, 1), (e_H^*, w_H^*) = (1/2, 1/2).$$

And the principal's expected profit is

$$\mathbb{E}\pi^* = \frac{1}{2}(2(1) - 1) + \frac{1}{2}\left(2\left(\frac{1}{2}\right) - 2\left(\frac{1}{2}\right)^2\right) = \frac{3}{4}.$$

The Optimal Menu of Contracts

Exercise. $\mathbb{E}X(e) = 2e$, and $u(x) = x$, $v(e) = e^2$, $\underline{u} = 0$, $k = 2$, $q = 1/2$.

With adverse selection the optimal menu of contracts solves,

$$\begin{aligned} 2 &= 2e_L \\ 2 &= 2\frac{2e_H}{1} + \frac{1-\frac{1}{2}}{\frac{1}{2}}(2-1)\frac{2e_H}{1} \\ w_H &= 2e_H^2 \\ w_L - e_L^2 &= w_H - e_H^2 \end{aligned}$$

Solving the system we get

$$(\tilde{e}_L, \tilde{w}_L) = (1, 10/9), (\tilde{e}_H, \tilde{w}_H) = (1/3, 2/9).$$

The principal's expected profit is

$$\mathbb{E}\tilde{\pi} = \frac{1}{2}\left(2(1) - \frac{10}{9}\right) + \frac{1}{2}\left(2\left(\frac{1}{3}\right) - \frac{2}{9}\right) = \frac{2}{3}.$$

Surplus is

The Optimal Menu of Contracts

Exercise. $\mathbb{E}X(e) = 2e$, and $u(x) = x$, $v(e) = e^2$, $\underline{u} = 0$, $k = 2$, $q = 1/2$.

With complete information the principal captures the entire surplus. Hence the social surplus is

$$\mathbb{E}\pi^* = \frac{3}{4}.$$

With adverse selection agents of type H capture some surplus. In this example, each L agent captures $1/16$, and there is a fraction $q = 1/2$ of L agents in the population. Hence the social surplus is

$$\mathbb{E}\tilde{\pi} + \frac{1}{2}(\frac{1}{16}) = \frac{22}{32} < \frac{3}{4}.$$

Adverse selection reduces the social surplus!