Exercise List 1: Bayesian Games

Exercise 1. Two firms compete *à la Cournot* in a market in which the inverse demand is $P(Q) = \max\{A - Q, 0\}$. Both firms produce the good with the same technology and maximize expected profits.

I. Assume that the technology exhibits constant returns to scale and $A = 12$.

(a) The firms choose their output with uncertainty about their marginal cost, which can be either 1 or 3 with the same probability. Represent the situation firms face as a Bayesian game and calculate the Bayesian Nash equilibrium in pure strategies and the equilibrium profits.

(b) Assume now that firm 1 observes privately the marginal cost before it chooses its output, whereas firm 2 does not observe it (but knows that firm 1 has observed it). Describe the new Bayesian game and calculate the Bayesian Nash equilibrium in pure strategies and the equilibrium profits. Does firm 1 has incentives to acquire this information? Would firm 1 be willing to share its information with firm 2 if it could do it credibly? Does the information advantage of firm 1 lead to greater profits? Do you think this result is general or it is peculiar to the example?

II. Assume now that the technology exhibits decreasing returns to scale and that the firms' cost function is $C(q) = q^2$. Also, A is uncertain and equal to 6 or 12 con with the same probability. Reconsider the questions (a) and (b) of part I with these data.

Exercise 2. Two firms, each of which produces a good with constant returns to scale and zero marginal cost, compete *á la Bertrand*. The goods they produce are imperfect substitutes, and their demands are $q_1(p_1, p_2) = \max\{10 - p_1 - bp_2, 0\}$ and $q_2(p_1, p_2) = \max\{10 - p_2, 0\}$. Only firm 1 knows the value of b, which can be either 0 or 1 with probabilities 2/3 and 1/3, respectively. Describe Bayesian game the firms face, and calculate the Bayesian Nash equilibrium in pure strategies and the equilibrium profits.

Exercise 3. Firm 1 monopolizes a market and must decide whether to increase its capacity taking into account that Firm 2 is considering entering the market. The firms make these decisions simultaneously. The profits resulting are described by the table:

	Enter	No Enter
Increase Capacity	$2 - c, -1$	$4 - c, 0$
Do not Increase Capacity	$2, 1$	$3, 0$

The constant c is the cost to firm 1 to increase capacity, which can be either high ($c = 2$) or low ($c = 1/2$) with probabilities $1/3$ and $2/3$, respectively. Firm 1 observes privately this cost before deciding whether to increase its capacity. Describe the Bayesian game the firms face, and calculate its Bayesian Nash equilibria.

Exercise 4. An object is sold using a sealed bid first-price auction (Bidders submit their bids in a sealed envelope, and the object is allocated to the bidder who bids the largest amount at a price equal to her bid – if there is a tie the object is allocated to each largest bidder with the same probability.) There are two bidders. Bidder 1's value of object is 5 euros, whereas the Bidder 2's value is either 7 or 4 euros with the same probability. Find at least two Bayesian equilibria in pure strategies and show that at least one bidder is using a weakly dominated strategy.

Exercise 5. Two candidates (1 and 2) compete for public office in a majority vote election – in case of a tie both candidates are elected with the same probability. Who is the best candidate for office depends of the state of the economy: candidate 1 is the best if the state is A and candidate 2 is the best if the state is B. Only two individuals have the right to vote (or abstain) and their payoffs are identical and equal to 1 if the best candidate is elected and 0 otherwise. Voter 1 observes the state of the economy before voting, while voter two only knows that the state is A with probability $9/10$ and B with probability $1/10$. Describe this Bayesian game, and calculate its Bayesian Nash equilibria.

Suggested exercises (not to be discussed in recitation): exercises 1 to 5 in Gibbons, Chapter 4.