EXAM 2

Convocatoria Extraordinaria

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Question 1 (SOLUCIÓN): Smoking one more cigarette per day implies a variation of income of $100\beta_1\%$, remaining the rest of explanatory variables fixed. An increase of 1% in income implies s varition in cigarettes consumption of $\beta_1/100$ units, remaining the rest of explanatory variables constant. If we sustitute *cigs* in equation (1) into equation (2), we get

$$\begin{aligned} cigs &= \frac{\gamma_0+\beta_0}{1-\beta_1\gamma_1}+\frac{\gamma_2+\gamma_1\beta_2}{1-\beta_1\gamma_1}educ+\frac{\gamma_3+\gamma_1\beta_3}{1-\beta_1\gamma_1}age+\frac{\gamma_4+\gamma_1\beta_4}{1-\beta_1\gamma_1}age^2 \qquad (1)\\ &+\frac{\gamma_5}{1-\beta_1\gamma_1}\log(cigpric)+\frac{\gamma_6}{1-\beta_1\gamma_1}restaurn+\left(U_2+\frac{\gamma_1 U_1}{1-\beta_1\gamma_1}\right). \end{aligned}$$

Therefore, $Cov\,(cigs, U_1) = \gamma_1 Var(U)/\,(1-\beta_1\gamma_1) \neq 0$ whenever $\gamma_1 \neq 0$, which shows that *cigs* is endogenous in equation (1). The OLS estimators of $\beta' s$ coefficients suffer from simultaneity bias.

Question 2 (SOLUCIÓN): The reduced equation (1) can be written more compactly as

$$cigs = \pi_0 + \pi_1 educ + \pi_2 age + \pi_3 age^2 + \pi_4 \log(cigpric) + \pi_5 restaurn + V,$$

with $\pi_4 = \gamma_5/\,(1-\beta_1\gamma_1)$ and $\pi_5 = \gamma_6/\,(1-\beta_1\gamma_1)$. Therefore, $\pi_4 = \pi_5 = 0$ if and only if (iff) $\gamma_5 = \gamma_6 = 0$, i.e. the model is not identified iff $\gamma_5 = \gamma_6 = 0$. We only require that $\beta_1\gamma_1 \neq 1$, otherwise the reduce equation doesn't exist, i.e. the system doesn't have a solution. We can test $H_0 : \pi_4 = \pi_5 = 0$ vs $H_1 : \pi_4 \neq 0$ or/and $\pi_5 \neq 0$ using a F test assuming homoskedasticity (the question doesn't say anything about this). Using GRETL we get $F = 3.13265$, which is smaller than 10 and we conclude that the instruments are not strong.

Question 3 (SOLUCIÓN): We get $\hat{\beta}_3 = 0.0938181$ and $\hat{\beta}_4 = -0.00105083$. Therefore, the critical age such that the income elasticity with repect to age changes sign is

$$\widehat{age}^* = -\frac{\hat{\beta}_3}{2\hat{\beta}_4} = -\frac{0.0938181}{-0.00105083 \cdot 2} = 44.6 \text{ years.}$$

Now, the elasticity is a function of age, i.e.

$$\xi_{income,age}(age) = \frac{d \; \log(income)}{d \; age} \cdot age = \hat{\beta}_3 \cdot age + 2 \cdot age^2 \cdot \hat{\beta}_4,$$

Thus, the estimated elasticity for a 20 years old person is

$$\begin{aligned} \hat{\xi}_{income,age}(20) &= 20 \cdot \hat{\beta}_3 + 2 \cdot 20^2 \cdot \hat{\beta}_4 \\ &= 20 \cdot 0.0938181 + 2 \cdot 20^2 \cdot (-0.00105083) \\ &= 1.0357. \end{aligned}$$

We calculate the standard error

$$SE\left(\hat{\xi}_{income,age}(20)\right) = \sqrt{20^2 \cdot \widehat{Var}\left(\hat{\beta}_3\right) + 2^2 \cdot 20^4 \cdot \widehat{Var}\left(\hat{\beta}_4\right) + 2 \cdot 2 \cdot 20^3 \cdot \widehat{Cov}\left(\hat{\beta}_3, \hat{\beta}_4\right)}$$

$$= \sqrt{20^2 \cdot 5.68982 \cdot 10^{-4} + 2^2 \cdot 20^4 \cdot 7.52574 \cdot 10^{-8} + 2 \cdot 2 \cdot 20^3 \cdot (-6.52285 \cdot 10^{-6})}$$

$$= 0.25889$$

and the 95% confidence interval is

$$\begin{aligned} \hat{\xi}_{income,age}(20) \pm 1.96 \cdot SE\left(\hat{\xi}_{income,age}(20)\right) &= 1.0357 \pm 1.96 \cdot 0.25889 \\ &= (0.528\,28, 1.5431). \end{aligned}$$

QUESTION 4 (SOLUCIÓN): We compute the 2SLS residuals

$$\hat{U}_{1i} = \log(income_i) - \hat{\beta}_0 - \hat{\beta}_1 cigs_i - \hat{\beta}_2 educ_i - \hat{\beta}_3 age_i - \hat{\beta}_4 age_i^2.$$

Then, compute the regression

$$\hat{U}_{1i} = \gamma_0 + \gamma_1 educ_i + \gamma_2 age_i + \gamma_3 age_i^2 + \gamma_4 \log(cigpric_i) + \gamma_5 restaurn_i + error,$$

then we test

$$H_0 : \gamma_4 = \gamma_5 = 0 \text{ vs } H_1 : \gamma_4 \neq 0 \text{ or/and } \gamma_5 \neq 0.$$

If we reject the hypothesis, we conclude that some of the instruments is not exogenous. Then, compute the F statistic and use the test statistic

$$J = 2F.$$

Under H_0, J is distributed as a $\chi^2_{(2-1)}/(2-1) = \chi^2_1$. Reject at the 5% of significance when $J > \chi^2_{1,0.05} = 3.84146$. Using GRETL we get $J = 1.35589$ with $p - value = 0.244252$, and we conclude that there is no sample evidence to reject the hypothesis that the two instruments are exogenous.