ECONOMETRICS FINAL EXAM: B
EXTRAORDINARY CALL 2022-23

Universidad Carlos III de Madrid
Answer the three questions in 90 minutes

1. (30%) Christensen and Greene (1976) estimated a generalized Cobb-Douglas cost function for electricity generation of the form

$$\ln(C) = \alpha + \beta \ln Q + \gamma \left[\frac{1}{2}(\ln Q)^2\right] + \delta_k \ln P_k + \delta_l \ln P_l + \delta_f \ln P_f + u,$$

where P_k, P_l and P_f indicate the unit prices of capital, labour and fuel, respectively, q is output and C are the total costs. The purpose of this generalization was to produce a U-shaped average total cost curve. To conform to the underlying theory of production, the cost function should be homogeneous of degree one in the prices, $\delta_k + \delta_l + \delta_f = 1$. (In the questions about tests below, the null and alternative hypotheses, the test statistic, its distribution under the null, the decision rule and the assumptions under which the test is valid must be provided.)

   (a) (7%) Explain in detail the relationship between the error u and the explanatory variables so that δ_k, δ_l and δ_f are the expected elasticities of C with respect to P_k, P_l and P_f. (5%) If this relationship holds, what is the expected elasticity of costs with respect to output for a level $Q = \bar{Q}$? (2%).

   (b) (8%) Provide a test for the hypothesis $\delta_k + \delta_l + \delta_f = 1$ against the alternative $\delta_k + \delta_l + \delta_f \neq 1$, at the $100\alpha\%$ significance level by an F test (4%). How would you test that $\delta_k + \delta_l + \delta_f = 1$ against the alternative $\delta_k + \delta_l + \delta_f < 1$ using a significance test on an individual coefficient? (4%)

   (c) (15%) An estimator of this generalized Cobb-Douglas cost function that satisfies the degree 1 homogeneity condition based on data from 158 firms is

$$\widehat{\ln\left(\frac{C}{P_f}\right)} = \underset{(0.252)}{-6.818} + \underset{(0.031)}{0.403} \ln Q + \underset{(0.004)}{0.061} \left[\frac{1}{2}(\ln Q)^2\right] + \underset{(0.040)}{0.162} \ln\left(\frac{P_k}{P_f}\right) + \underset{(0.047)}{0.152} \ln\left(\frac{P_l}{P_f}\right),$$

   The standard errors are in parenthesis. Compute the restricted estimate of δ_f based on these results (5%). Based on the restricted estimator, provide a 95% confidence interval for δ_f taking into account that the estimated covariance between the OLS estimators of δ_k and δ_l is 0.0018. (10%).

2. (20%) Consider the model

$$Y_i = \beta_0 + \beta_1 X_i + u_i, \;\; i = 1, ..., n,$$

where the explanatory variable can be correlated with the error term. Suppose we have an exogenous variable Z_i that is correlated with X_i.

(a) (6.5%) Derive algebraically an expression for β_1 in terms of the covariances of the explained and explanatory variables with respect to the exogenous variable (4.5%). From this expression obtain the instrumental variables estimator of β_1 (2%).

(b) (6.5%) Derive an expression for β_1 in terms of the coefficients of the slopes of the reduced forms of the two endogenous variables (4.5%). From this expression, obtain the estimator of instrumental variables in terms of the OLS estimators of the slopes of the two reduced forms (2%).

(c) (7%) Show that the two estimates in (a) and (b) are identical.

3. (20%) Consider the model

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 W_i + u_i, \;\; i = 1, ..., n,$$

where W_i is a control variable.

(a) (10%) Show that β_1 has a causal interpretation when $\mathbb{E}\left(u_i | X_i, W_i\right) = \mathbb{E}\left(u_i | W_i\right)$.

(b) (10%) Show that the OLS estimate of β_1 is consistent (4.5%), but that the estimate of β_2 might not (5.5%), if $\mathbb{E}\left(u_i | X_i, W_i\right) = \delta_0 + \delta_1 W_i$.