Theory of the Firm – Exercises

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1 Production

Exercise 1

Answer the questions of parts (a)-(e) for firms whose production functions are: (i.) $F(L,K) = \sqrt{LK}$, (ii.) $F(L,K) = L + 4K$, and (iii.) $F(L,K) = 2\min\{2L, K\}$.

(a) Compute and graph the isoquants for $Q_0 = 2$ and $Q_0 = 10$.

(b) Calculate the $MRST(L,K)$ and evaluate it at $(L,K) = (4,1)$ and $(L,K) = (10,10)$.

(c) Determine and graph the short run production functions $f(L) = F(\bar{K}, L)$ for $\bar{K} = 4$ and for $\bar{K} = 10$.

(d) Determine and graph the marginal productivity of labor $F_L(L,K) = \partial F(L,K)/\partial L$ for $\bar{K} = 4$ and for $\bar{K} = 10$.

(e) What are the firms' returns to scale in the long run?

Exercise 2

How are the answers to the questions in Exercise 1 changed if the functions in (i.), (ii.) and (iii.) are transformed as $\hat{F}(L,K) = (F(L,K))^2$?

Exercise 3

A famous bakery's production capacity of bread loaves, given its capital equipment, depends on the number of workers as described by the following table:

Number of Workers	1	2	3	4	5	6	7
Bread Loaves (thousands)	1	1.8	2.4	2.8	3	2.8	2.5

(a) Calculate the marginal and average productivity of each worker.

(b) Determine the types of returns to scale the bakery has.

(c) Discuss the reasons why the marginal productivity of a worker may be negative. What are the implications of this fact over the slope of the isoquants?

Multiple Choice Questions

MC 1.1: If a firm's production technology is $F(L,K) = (4L+K)^2$, the firm's Marginal Rate of Technical Substitution (MRTS) is:

☐ $MRTS = \frac{2}{3}$	☐ $MRTS = \frac{4L}{K}$
☐ $MRTS = 4$	☐ none of those.

MC 1.2: *GMW S.A.* (Getafe Motor Works) has the production function $F(L,K) = \sqrt{L}K$. Determine the isoquants of GMW.

☐ $K = \frac{Q_0}{L}$	☐ $K = \left(\frac{Q_0}{L}\right)^2$
☐ $K = 4Q_0L$	☐ none of those.

MC 1.3: Lolita, the competitive cow in all markets of Holstein, produces milk (L) using oats (A) and hay (H) according to the production function $L = A^2 + H$. Hence, as a milk producer Lolita's returns to scale are

☐ increasing	☐ constant
☐ decreasing	☐ undetermined.

MC 1.4: If a firm's production function is $F(L,K) = \min\{2L, \sqrt{K}\}$, then the firm's returns to scale are

☐ increasing	☐ constant
☐ decreasing	☐ undetermined.

2 Cost Minimization and Conditional Input Demands

Exercise 4

A certain firm produces a good using two factors of production: energy and "other" inputs.

(a) Assume that the energy price, controlled by an international cartel, rises by 100%. Determine how the long-run expansion path of the firm varies, and how its long-run total, average and marginal-cost curves are affected.

(b) Assume now that the government imposes an upper bound to the amount of energy firms can import. Determine the effects of this additional restriction on the expansion path and the cost curves.

Exercise 5

The production function of a firm is $F(K, L) = \sqrt{LK}$.

(a) Derive the conditional demands for the factors of production.

(b) Derive the long-run total-cost function for $w = r = 1$.

(c) Obtain the long-run marginal-cost and average-cost functions for $w = r = 1$.

(d) Obtain the average-cost, marginal-cost and variable-average-cost curves in the short run for $K = \bar{K} = 25$.

Exercise 6

A firm produces a good with labor L and capital K. Input prices are w and r, respectively. For each of the production functions given below, calculate the conditional input demands. Calculate the total, average and marginal cost functions when the input prices are $w = r = 1$.

(a) $F(L, K) = \sqrt{L + 2K}$.

(b) $F(L, K) = (L - 1)^{\frac{1}{4}} K^{\frac{1}{4}}$.

(c) $F(L, K) = 2\left(\min\{2L, K\}\right)^2$.

Exercise 7

The production function of a firm is $F(K, L) = \min\{2L, K\}$.

(a) Derive the conditional demands for the factors of production.

(b) How does the conditional labor demand change when the interest rate r increases? Do you think you would obtain the same result with the production function from Exercise 5? Explain.

(c) Suppose that $w = 2$ and $r = 1$. Determine the long run cost function.

Multiple Choice Questions

MC 2.1: If a firm has constant returns to scale, then

☐ its total cost function is strictly concave
☐ its total cost function is strictly convex
☐ its marginal cost is less than its average cost
☐ its average cost is constant.

MC 2.2: Lolita, the competitive cow in all markets of Holstein, produces milk (L) using oats (A) and hay (H) according to the production function $L = \sqrt{A + 2H}$. Hence, as a milk producer Lolita has

☐ diseconomies of scale	☐ decreasing marginal cost
☐ increasing returns to scale	☐ constant average cost.

3 Costs

Exercise 8

The production function of a firm is $F(K, L) = 4KL^{0.5}$, where K is the amount of capital used in the production process and L is the amount of labor. The wage is $w = 2$ and the price of capital is $r = 4$. Calculate the firm's total-cost, average-cost and marginal-cost functions, and determine whether the firm has economies or diseconomies of scale.

Exercise 9

The production function of a firm is $F(L, K) = L^\alpha K^\beta$, where $\alpha \in (0, 1)$ and $\beta \in (0, 1)$. The wage is $w = 1$ and the price of capital is $r = 1$.

(a) Calculate the firm's conditional input demands.

(b) Calculate the firm's long run cost function.

(c) When does the firm have economies of scale? When does it have diseconomies of scale?

(d) Determine the firm's short run expansion path if $\bar{K} = 100$ and it's long run expansion path for parameter values of your choice.

Exercise 10

A firm has the production function $F(L, K) = \sqrt{\min\{4L, K\}}$. Input prices are w and r, respectively.

(a) Calculate the firm's conditional input demands.

(b) Calculate and draw the firm's long run cost function.

(c) Calculate and draw the firm's marginal and average cost function.

(d) Does the firm have economies of scale?

(e) Determine the firm's long run expansion path.

Exercise 11

Graph the total cost functions (a) $C(Q) = 100 + Q^2$, and (b) $C(Q) = Q^3 - 4Q^2 + \frac{37}{3}Q$. In each case, calculate and graph the average and marginal cost functions. For which output levels, if any, has the firm economies or diseconomies of scale?

Multiple Choice Questions

MC 3.1: If a firm has economies of scale, then its average cost is

☐ decreasing and smaller than its marginal cost
☐ decreasing and larger than its marginal cost
☐ increasing and larger than its marginal cost
☐ increasing and smaller than its marginal cost.

MC 3.2: A firm whose total cost function is $C(Q) = \frac{Q^2}{2} + Q$ has

☐ economies of scale ☐ decreasing returns to scale
☐ diseconomies of scale ☐ increasing returns to scale.

MC 3.3: A firm whose total cost function is $C(Q) = 5Q + 7$ has

☐ economies of scale ☐ constant returns to scale
☐ diseconomies of scale ☐ increasing returns to scale.

MC 3.4: If a firm has diseconomies of scale, then its

☐ marginal cost is decreasing ☐ average cost is less than its marginal cost
☐ average cost is decreasing ☐ total cost function is concave.

MC 3.5: Lolita is a competitive cow that produces milk using oats (O) and hay (H) according to the production function $F(O, H) = \min\{2O^2, H^2\}$. Therefore, as a milk producer Lolita has

☐ diseconomies of scale ☐ constant returns to scale
☐ economies of scale ☐ a convex total cost function.

4 Competitive Markets

Exercise 12

In a market, there are 5 competitive firms. Each of these firms has the same cost function $C(q) = \frac{q^2}{5} + q + 250$. The market demand is $Q^D(p) = 250 - p$.

(a) Determine the supply function of each firm.

(b) Calculate the aggregate supply function of the 5 firms.

(c) Find the (short-run) market equilibrium (p^E, Q^E). How much profit does each firm make? What is the consumer surplus?

(d) Now suppose the fixed cost increases from $FC = 250$ to $FC' = 500$. What is each firm's profit now? Explain why your solution from part (c) nevertheless remains the market equilibrium.

Exercise 13

The aggregate demand for a good is $D(p) = \max\{150 - 2p, 0\}$. The market is supplied by 4 price-taking firms whose average cost is given by $AC(q) = \frac{100}{q} - 5 + q$.

(a) Determine the short-run equilibrium in this industry, specifying the aggregate quantity traded in the market and the profits for each firm.

(b) The total cost of a foreign firm is $C(Q) = 8Q$. Assuming that the industry opens to foreign trade and that the foreign firm operates as a price-taker, determine the total quantity sold, the amount of imports, the market price, and the short-run equilibrium profits (or losses) for both types of firms.

(c) Assume the competition authority sets a price equal to the minimum average cost of the domestic firms, guaranteeing that they break even. Determine the resulting equilibrium. What is the change in consumer surplus?

Exercise 14

The inverse demand function for a good in a European Union country, where all agents are price-takers, is given by $P_{EU}(q) = \max\{500 - 4q, 0\}$, whereas the national inverse supply function is $P^S_{EU}(q) = 5(1 + q)$.

(a) Determine the market equilibrium in the absence of international trade.

(b) Suppose now that the country opens to international trade and that the euro/dollar exchange rate is equal to one. The inverse supply function of the rest of the world (measured in the national currency) is given by $p(Q) = 2 + 20Q$. Determine the new equilibrium after aggregating both supply curves. Who gains and who loses in this situation? Do you think the country should open up to international trade?

(c) Assume that the euro depreciates so that one euro now only buys 0.8 dollars. Determine how the supply curve from the rest of the world changes and find the new equilibrium. Who gains and who loses in this situation?

Exercise 15

Assume that there are only two kinds of suppliers of ethanol in the US market: the Brazilian producers and the national ones.

(a) Suppose that the ethanol market is perfectly competitive and, for whatever reason, the Brazilian producers are considerably more efficient. Represent in three different graphs the Brazilian supply curve, the national supply curve and the aggregate supply curve in the US.

(b) Represent an equilibrium situation in which all the traded quantity corresponds to Brazilian producers.

(c) Suppose that a tariff is imposed on the ethanol imports from Brazil. Show a new equilibrium situation in which the traded quantity still corresponds to the Brazilian producers.

(d) Represent a third equilibrium in which, due to a subsidy, the national producers are now able to sell a positive quantity of the total product.

Exercise 16

Consider a country whose demand and supply curves for a product in a competitive market are, respectively,

$$P_N^d(q) = \max\{400 - 10q, 0\}, \quad \text{and} \quad P_N^s(q) = 10 + 20q.$$

(a) Determine the market equilibrium.

(b) Assume that the country opens up to international trade and that the supply curve of the rest of the world is infinitely elastic at the price of 190. Determine the total quantity demanded, the quantity supplied by national firms, and the amount of imports in the new equilibrium.

(c) Compute the consumer and the producer surplus. Is opening up to international trade beneficial for the country as a whole?

(d) Suppose that the government imposes a tariff with the purpose of reducing the amount of imports by 50%. What is the price, the quantity demanded, and the quantity supplied by national producers in the new equilibrium?

(e) Calculate the consumer surplus, the producer surplus, and the government revenue obtained through the introduction of the tariff. Is introducing the tariff beneficial for the country as a whole?

Exercise 17

In a market, there are 3 price-taking firms. The cost functions for each of them are:

$$C_1(Q) = Q^2 + 2Q + 36, \quad C_2(Q) = 2Q^2 + 2Q + 10, \quad C_3(Q) = Q^2 + 6Q + 6.$$

At the current equilibrium, the first firm produces a positive amount but makes zero profits.

(a) Calculate the market equilibrium price, and each firm's production level and profits.

(b) Calculate the market equilibrium price in the long run with free entry and exit. Which firm(s) survive?

Exercise 18

The supply and demand schedules for residential housing in Getafe are $S(p) = \frac{p}{2}$ and $D(p) = \max\{600 - 2p, 0\}$, respectively, where p thousand euros/unit.

(a) Calculate the equilibrium price and quantity traded (built).

(b) Determine the effect 50 thousand euros subsidy on the number of houses built. Calculate the change in total surplus (make sure to account for the government expenditure).

Exercise 19

Once upon a time, in the small and prosperous town Getrid, there was a thriving market for widgets. The townsfolk relied heavily on widgets for their everyday tasks, and demand for these devices grew steadily year after year. As the market matured, two innovative firms—AltoTech and BravoMakers—emerged, each championing a distinct production technology. AltoTech had developed an elegant, high-efficiency assembly line, leading to a cost function of

$$C_{AT}(q_{AT}) = 10 + 2q_{AT} + 0.5q_{AT}^2,$$

where q_{AT} represented the output of the firm.BravoMakers, on the other hand, opted for a more labor-intensive approach, which required less initial capital but incurred higher ongoing costs. BravoMakers' cost function was

$$C_{BM}(q_{BM}) = 20 + 3q_{BM} + 0.25q_{BM}^2,$$

The town's demand for widgets was described by the equation:

$$Q^D(p) = 100 - p,$$

where p represented the price per widget and $Q^D(p)$ the quantity demanded.

(a) Assuming both firms are price takers, what are their respective supply functions?

Now assume that only two technologies described above are feasible. Moreover, the technologies are no longer protected by patents or other regulations, and any firm can just acquire each of the two technologies and enter the market.

(b) Which of the two technologies will survive in the long run with free entry and exit?

(c) What will be the market price in this equilibrium, and how many firms will be active in the market?

Multiple Choice Questions

MC 4.1: If a competitive firm produces a positive output, then the market price is

☐ equal to its marginal cost, and greater than or equal to its average cost
☐ equal to its marginal cost, and greater than or equal to its average variable cost
☐ equal to its average cost, and less than or equal to its marginal cost
☐ equal to its average variable cost, and greater than or equal to its marginal cost.

MC 4.2: If the total cost function of a competitive firm is $C(Q) = 2Q^3 - 12Q^2 + 38Q$, and the long run equilibrium price is $P = 20$, then the firm

☐ obtains losses ☐ produces $Q^* = 0$ units
☐ obtains profits ☐ produces $Q^* = 3$ units.

MC 4.3: There are two technologies for the production of a certain good which generate the cost functions $C_A(Q) = Q^2 + 3Q + 1$, and $C_B(Q) = 2Q^2 + Q + 6$. The good is traded in a competitive market where demand is $D(P) = \max\{10 - P, 0\}$. If there is free entry and neither of these technologies is protected by patents, which of these technologies will survive in a the long run competitive equilibrium?

☐ Technology A ☐ Both technologies
☐ Technology B ☐ Neither technology.

MC 4.4: In a short run equilibrium of a competitive industry each firm's

☐ average cost is less than or equal to the market price ☐ profits are zero
☐ surplus is greater than or equal to its profits ☐ profits are positive.

MC 4.5: If a competitive firm produces a positive output in the short run, then its

☐ profit is greater or equal to zero
☐ average cost is decreasing
☐ total average cost is less than the market price
☐ marginal cost is greater or equal to its average variable cost.

MC 4.6: If a competitive firm produces a good with total cost $C(q) = q^3 - 6q^2 + 10q$, then its supply at the price $p = 10$ is

☐ $S(10) = 6$ ☐ $S(10) = 4$
☐ $S(10) = 0$ ☐ $S(10) = 2$.

5 Monopolistic Firms and Markets

Exercise 20

The demand for a good produced by a monopolist is $D(p) = \max\{20 - p/2, 0\}$.

(a) Calculate and graphically represent the inverse demand function, the total revenue curve and the marginal revenue curve.

(b) Can the marginal revenue become negative when the price is positive? Explain.

Exercise 21

A monopolist produces at constant marginal costs, $c = 10$, a good with a market-demand of $D(p) = \max\{210 - p, 0\}/4$.

(a) Compute the monopoly equilibrium. Verify that if the marginal cost increases to $c = 20$ then in equilibrium the monopoly's total revenue decreases.

(b) Calculate the competitive equilibrium assuming that there are several firms producing the good with the given cost. Verify that if the firms' marginal cost increases to $c = 20$ then in equilibrium the industry's total revenue increases.

Exercise 22

A firm producing a good with total cost $C(q) = 1200 + q^2/2$ monopolizes a market in which the demand is $D(p) = \max\{300 - p, 0\}$.

(a) Calculate the monopolist's optimal price and quantity.

(b) Calculate the monopolist's profit, the consumer surplus, and the deadweight loss.

(c) Determine the Lerner index at the monopolist's optimum.

Exercise 23

Outel owns the monopoly in a country's market for microprocessors with an inverse demand of $D(p) = \max\{9,000,000 - p, 0\}$. Nothing is known about the production costs of Outel, except that the second order condition is satisfied. Do you think that Outel would sell 7 million microprocessors in such a country? (Assume that Outel is a profit-maximizing monopolist).

Exercise 24

Connected Airlines monopolizes the route Chicago-Nebraska. The monthly market demand on this route is given by $D(p) = \max\{a_t - bp, 0\}$, where $a_t = a_w$ is in winter and $a_t = a_s < a_w$ in summer. The marginal costs of Connected are the same in both seasons and independent on the amount of tickets sold. When would Connected flights be cheaper, in the summer or in the winter?

Exercise 25

A firm producing a good with marginal cost $MC(q) = q/2$ monopolizes a market in which the demand is $D(p) = \max\{100 - p, 0\}/2$.

(a) Compute the monopoly equilibrium.

(b) Compute the equilibrium assuming that the monopolist is a price-taker.

(c) Compute the efficiency loss caused by the monopoly.

Exercise 26

A firm producing a good with the cost function $C(q) = 32 + 3q + q^2/2$ monopolizes a market in which demand is $D(p) = \max\{21 - p, 0\}$.

(a) Determine the monopoly equilibrium and compute the firm's profit, the consumer surplus, and the efficiency loss.

(b) The government decides to regulate the price in order to maximize the aggregate surplus in the market, subject to the constraint of not imposing losses on the firm. What is the optimal regulated price? Calculate also the firm's profit, the consumer surplus, and the efficiency loss.

Exercise 27

A firm producing a good with a cost function $C(q) = q^2 + 100q$ monopolizes a market in which demand is $D(p) = \max\{700 - p, 0\}$.

(a) Calculate the monopoly equilibrium.

(b) Suppose the government imposes a 30% profit tax. How would this tax affect the monopoly equilibrium and the firm's profit?

(c) Alternatively, suppose a sales tax of 20 euros per unit is imposed. How would this tax affect the monopoly equilibrium and the firm's profit?

(d) Which of these taxes is preferred by consumers? Explain.

Exercise 28

An electricity-generating firm produces energy according to the cost function $C(q) = 30q$, where q is the production in megawatts per hour (MW/h). The electricity demand of households is $D_H(p) = \max\{50 - p, 0\}$, while the electricity demand of firms is $D_F(p) = \max\{400 - 10p, 0\}$.

(a) Determine graphically and analytically the monopoly equilibrium with and without third-degree price discrimination.

(b) Are households better off when price discrimination is not allowed?

Exercise 29

A firm producing a good with costs function is $C(q) = 4q + FC$ monopolizes a market in which the demand of type 1 consumers is $D_1(p) = \max\{20 - p, 0\}$, and that of type 2 consumers is $D_2(p) = \max\{60 - 2p, 0\}$.

(a) Represent both demand curves and the corresponding marginal-revenue curves graphically.

(b) We are informed that the monopolist is charging a price of 18 euros to both types of consumers. Compute the consumption level for both types, the monopoly profits and the surplus of both types of consumers.

(c) Suppose now that the monopolist can price-discriminate. Determine the choices he would make, the profits he would reach and the consumer surplus for both groups of consumers.

Multiple Choice Questions

MC 5.1: A monopolist's Lerner Index is

☐ inversely proportional to the absolute value of demand elasticity
☐ directly proportional to the absolute value of demand elasticity
☐ larger the larger is the monopolist total cost
☐ larger the smaller is the monopolist profit.

MC 5.2: If a market is monopolized by a firm that produces the good at zero cost, then in the market equilibrium the

☐ price is equal to zero ☐ Lerner index is equal to one
☐ consumer surplus is equal to zero ☐ producer surplus is equal to zero.

MC 5.3: Assume a monopolist with marginal costs of zero. In comparison with the monopoly equilibrium without price discrimination, third degree price discrimination induces

☐ a decrease of the level of output
☐ an increase of the producer surplus
☐ an increase of the price to all consumers
☐ a decrease of the surplus of every consumer.

MC 5.4: If a monopoly produces the good with total cost $C(q) = 2q$ and the market demand is $D(p) = \max\{10 - p, 0\}$, then the monopoly's Lerner index is

☐ $L = \frac{1}{3}$ ☐ $L = \frac{1}{2}$ ☐ $L = \frac{2}{3}$ ☐ $L = 1$.

MC 5.5: If a monopoly produces the good with zero cost and the market demand is $D(p) = \max\{10 - p, 0\}$, then under first degree price discrimination

☐ its output is $q^M = 5$ ☐ the deadweight loss is $25/2$
☐ the total surplus is $25/2$ ☐ the producer surplus is 50.