Consumer Theory – Exercises

Universidad Carlos III, Department of Economics

2025

1 Preferences and Utility Functions

Exercise 1

Suppose there are two Cercanías lines that stop in Getafe:

- C4a with stops: Parla - Las Margaritas - Atocha - Sol - Colmenar Viejo.
- C4b with stops: Parla - Las Margaritas - Atocha - Sol - San Sebastián de los Reyes.

The number of trains per hour on these lines are x for C4a and y for C4b. The possible bundles are $(x, y) \in \mathbb{R}^2_+$.

(a) Cristina studies Economics at UC3M and lives near Atocha. What could be reasonable preferences for her? Based on the above information and your answer, draw a possible indifference map for her.

(b) Agustina studies Business Administration at UC3M and lives in San Sebastián de los Reyes. What could be reasonable preferences for her? Based on the above information and your answer, draw a possible indifference map for her.

Now suppose that instead of line C4a, there is the following Cercanías line:

- C7 with stops: Alcalá de Henares - Atocha - Recoletos - Las Rozas - Principe Pio.

The number of trains per hour on this line is z. Hence, the possible bundles are now $(x, z) \in \mathbb{R}^2_+$.

(c) José studies Law at UC3M and lives in Las Rozas. What could be reasonable preferences for him? Based on your answer and the above information, draw a possible indifference map for José.

Exercise 2

Graph indifference maps that are consistent with the preferences described below.

(a) My wellbeing increases with higher income (x) and decreases with higher pollution (y).

(b) 1000 milligrams of Tylenol (x) give me the same pain relief as 500 milligrams of Aspirin (y).

(c) I only drink my martinis with exactly one part of vermouth (x) and exactly five parts of gin (y).

(d) I am always willing to trade two hamburgers (x) for one beer (y).

(e) I always drink one soft drink (x) with one hamburger (y).

(f) I enjoy drinking beer (x) but am allergic to peanuts (y).

Exercise 3

Identify the Axioms of Consumer Behavior That Imply the following Properties of Indifference Maps.

(a) Indifference sets are curves (i.e., they are not "thick").

(b) Every bundle belongs to one indifference curve.

(c) Indifference curves do not intersect.

(d) Indifference curves slope downward.

Exercise 4

(a) Two objects weigh 50 kg and 55 kg. Since $55/50 = 1.1$, we say the second object is 10% heavier than the first. Does this remain true if the weights are measured in pounds?

(b) The temperatures of two objects are measured as 50°F and 55°F. We state, "The second object is 10% hotter than the first." Does this hold true if the temperatures are measured in Celsius? The temperature of a third object is 65°F. If we claim, "The difference in temperature between the third and second objects is twice that between the second and the first," does this statement hold irrespective of the measurement scale?

(c) A consumer has preferences represented by a utility function u, with values measured in utils. If bundles A and B have utilities of 50 and 55 utils respectively, we say B provides 10% more utility than A. Does this statement remain true if preferences are represented by $U = u^2$ (i.e., U is obtained by squaring u)? Does B provide 10% more utils than A in this case?

Exercise 5

For each of the below utility function, calculate and graph the indifference curves that pass through the bundles $(x, y) = (1, 1)$ and $(x, y) = (1, 2)$.

(a) $u(x, y) = \sqrt{xy}$

(b) $u(x, y) = \frac{xy}{4}$

(c) $u(x, y) = y + 2 \ln x$

(d) $u(x, y) = 4(x + 2y)^2$

(e) $u(x, y) = \min\{x^2, 2y\}$

Multiple Choice Questions

MC 1.1: Roberto's preferences over consumption bundles $(x, y) \in \mathbb{R}^2_+$ are complete and transitive (axioms A.1 and A.2). If he views good x as detrimental to his welfare and good y as beneficial, his indifference curves:

(i)	☐ cross	(ii)	☐ are increasing
(iii)	☐ are concave	(iv)	☐ have area / are thick

MC 1.2: Alicia's preferences are monotonic (axiom A.3). Then her indifference curves:

(i)	☐ do not cross	(ii)	☐ are increasing
(iii)	☐ are decreasing	(iv)	☐ are thick

MC 1.3: Which axiom guarantees that indifference curves do not cross?

(i)	☐ Completeness (A.1)	(ii)	☐ Monotonicity (A.3)
(iii)	☐ Transitivity (A.2)	(iv)	☐ Convexity (A.5)

MC 1.4: The Pareto preference relation is defined as $(x, y) \succsim_P (x', y')$ if $x \geq x'$ and $y \geq y'$. Which of the following statements about this preference relation is correct?

(i)	☐ It violates axiom A.1 (completeness)	(ii)	☐ It violates axiom A.3 (monotonicity)
(iii)	☐ It violates axiom A.2 (transitivity)	(iv)	☐ It satisfies axioms A.1-A.3

MC 1.5: A consumer's preferences $\succsim$ are known to satisfy axioms A.1, A.2, and A.3, with $A = (0, 2) \succ B = (1, 1)$. Thus, we can infer the following relationship between these bundles and a third bundle, $C = (1, 2)$:

(i)	☐ $C \succsim B$	(ii)	☐ $C \sim A$
(iii)	☐ $C \sim B$	(iv)	☐ $C \succ A$

MC 1.6: A consumer's preferences for bundles $(x, y) \in \mathbb{R}^2_+$ are complete, transitive, and monotonic (axioms A.1, A.2, and A.3). Then the preference relations between $A = (1, 1)$, $B = (2, 2)$, and $C = (1, 2)$ necessarily satisfy:

(i)	☐ $C \succsim B$	(ii)	☐ $C \succsim A$
(iii)	☐ $C \succ A$	(iv)	☐ $B \succ C$

MC 1.7: It is known that a consumer's preference relation $\succsim$ satisfies axioms A.1, A.2, and A.3, and that the consumer is indifferent between the two bundles $A = (1, 2)$ and $B = (2, 1)$. Therefore, we can infer the following relation between these bundles and a third bundle $C = (1, 3)$:

(i)	☐ $C \succ B$	(ii)	☐ $C \succsim B$
(iii)	☐ $C \sim B$	(iv)	☐ $C \succ A$

2 The Marginal Rate of Substitution (MRS)

Exercise 6

The preferences of Juan and María over leisure activities differ. Juan likes concerts, but prefers going to the football stadium. María, on the other hand, likes football, but prefers to go to concerts.

(a) Graph possible indifference maps for Juan and María.

(b) Using the concept of the marginal rate of substitution, explain the differing curvatures of their indifference curves.

Exercise 7

Nicolas' preferences can be described by the utility function $u(x, y) = x + 2\sqrt{y}$.

(a) Analyze the impact of increased consumption of clothing (x) on his MRS.

(b) Determine whether increasing the consumption of clothing (x) or food (y) will increase his MRS.

(c) Identify all the bundles where the MRS is equal to 4.

Exercise 8

Calculate the MRS for the preferences represented by the utility functions given in Exercise 5. In each case, determine whether an individual with 2 units of each good would be willing to give up 1 infinitesimal unit of x in exchange for 1.5 infinitesimal units of y. Would the individual be willing to exchange 1 unit of x for 1.5 units of y? Would they agree to either of these exchanges if they have 2 units of x and 1 unit of y?

Exercise 9

Graph the indifference curve containing the bundle $(x, y) = (3, 3)$ and calculate the MRS at this bundle and an arbitrary bundle on this curve for individuals with preferences represented by the utility functions below. What do you realize?

(a) $u(x, y) = xy$

(b) $u(x, y) = 2\sqrt{xy}$

(c) $u(x, y) = \left(4 + 3\sqrt{xy}\right)^2$

3 The Budget Set and the Consumer's Optimal Choice

Exercise 10

Assume the price of natural gas is 0.08 Euros per kilowatt-hour (€/kWh) and the price of electricity is 0.18 €/kWh. However, after purchasing 1500 kWh of electricity, the price falls to 0.15 €/kWh. A household has 300 Euros available to spend on energy.

(a) Draw the household's budget set.

(b) Now assume that the electricity provider increases the threshold at which the price per unit drops from 1500 kWh to 2000 kWh. How does the household's budget set change?

(c) Comparing the situations in (a) and (b), is the household necessarily worse off in (b)?

Exercise 11

Water tariffs sometimes have the following structure: To receive any water supply, the consumer must pay an initial connection fee $T > 0$, which covers the cost of consuming up to $x_1 > 0$ liters of water. If one consumes more than x_1, each additional liter costs $p_1 > 0$ until a total amount of $x_2 > x_1$ is reached. For each liter in excess of x_2, the water bill increases by $p_2 > p_1$.

(a) Graph the budget set for a household that consumes water (x) and money available for the purchase of other goods (y), assuming that the consumer's income I satisfies $I > T + p(x_2 - x_1)$.

(b) If someone with monotone preferences pays the connection fee T, would you expect them to ever consume an amount of water less than x_1?

(c) Assume that Axioms A1-A4 are satisfied. Is it possible for an individual to be indifferent between two bundles that differ in water consumption?

Exercise 12

A consumer's preferences over x and y are described by the utility function $u(x, y) = 2x + y$. Their monetary income is $I = 15$ euros. Calculate their optimal consumption bundle when the prices of the goods are:

(a) $(p_x, p_y) = (1, 2)$.

(b) $(p'_x, p'_y) = (3, 1)$.

(c) $(p''_x, p''_y) = (2, 1)$.

Exercise 13

A consumer's preferences for food (x) and clothes (y) are represented by the utility function $u(x, y) = x + \sqrt{y}$. The prices are $p_x = 4$ and $p_y = 1$ euros per unit, respectively, and the consumer's income is $I = 10$ euros.

(a) Represent the consumer's budget set.

(b) Calculate the optimal consumption bundle.

Exercise 14

Mario's preferences for Slices of Pizza (x) and Pinchos de Tortilla (y) are represented by the utility function $u(x, y) = \ln x + \ln y$. The prices are $p_x = 1$ and $p_y = 2$, respectively, and his income is $I = 10$ euros.

(a) Represent his budget set.

(b) Calculate his optimal consumption bundle.

Exercise 15

Sara has an income of $I = 200$, which she spends on buying water (x) and food (y), with prices $p_x = 4$ and $p_y = 2$, respectively. Her preferences over these goods are represented by the utility function $u(x, y) = \min\{x, y\}$.

(a) Graph her indifference curves, her budget constraint, and calculate her optimal consumption bundle.

(b) The government introduces a tax of $t = 1$ euro whenever $x > 10$. For example, if Sara consumes 12 units of water, the first 10 units cost $p_x = 4$ euros each, and the remaining 2 units cost $p_x + t = 5$ euros each. Repeat part (a) under this taxation scheme.

Multiple Choice Questions

MC 3.1: If the price of good y increases by 20%, the budget line will:

(i)	☐ remain in the same position	*(ii)*	☐ rotate around its intersection with the y-axis
(iii)	☐ shift parallel toward the origin	*(iv)*	☐ rotate around its intersection with the x-axis

MC 3.2: If all prices increase by 20%, the budget line will:

(i)	☐ shift parallel toward the origin	*(ii)*	☐ rotate around its intersection with the x-axis
(iii)	☐ remain unchanged	*(iv)*	☐ shift parallel away from the origin

MC 3.3: If a consumer's income increases by 10%, the price of good x increases by 5%, and the price of good y increases by 10%, the budget line will:

(i)	☐ shift parallel toward the origin	*(ii)*	☐ rotate around its intersection with the y-axis
(iii)	☐ remain in the same position	*(iv)*	☐ rotate around its intersection with the x-axis

MC 3.4: A consumer whose monetary income is $I = 4$ is considering buying the bundle $(2, 0)$. If $p_x = 2$, $p_y = 1$, and $MRS(2, 0) = 3$, the consumer should:

(i)	☐ buy more x and less y	*(ii)*	☐ buy more of both x and y
(iii)	☐ buy more y and less x	*(iv)*	☐ purchase the bundle $(2, 0)$

MC 3.5: If a consumer's marginal rate of substitution is $MRS(x, y) = 2$ (constant) and their income is $I = 8$, then at the prices $(p_x, p_y) = (1, 2)$, the optimal consumption bundle is:

(i)	☐ $(2, 3)$	*(ii)*	☐ $(8, 0)$
(iii)	☐ $(4, 2)$	*(iv)*	☐ $(2, 4)$

MC 3.6: The optimal bundle (x^*, y^*) satisfies the budget constraint $p_x x^* + p_y y^* = I$ due to:

(i)	☐ Axiom A.1 (completeness)	*(ii)*	☐ Axiom A.3 (monotonicity)
(iii)	☐ Axiom A.2 (transitivity)	*(iv)*	☐ Axiom A.5 (convexity)

MC 3.7: If a consumer's preferences are represented by the utility function $u(x, y) = (x+y)^2$, their income is $I = 4$, and the prices are $(p_x, p_y) = (1, 2)$, then the optimal consumption bundle is:

(i)	☐ $(0, 2)$	*(ii)*	☐ $(4, 0)$
(iii)	☐ $(2, 1)$	*(iv)*	☐ $(4, 1)$

4 Demand Functions, IE, and SE

Exercise 16

A consumer has preferences described by the utility function $u(x, y) = 2xy$.

(a) Compute the demand curve for good x. Is good x an inferior or normal good? Is x a Giffen good? What is the price elasticity of this good? Represent the Engel curve of x for $p_x = 2$ and $p_y = 3$.

(b) Determine and represent the optimal consumption bundle if the consumer's income is $I = 15$ and the prices of the goods are $p_x = 2$ and $p_y = 3$. Compute the income and substitution effects for good x when the price increases to $p'_x = 3$.

Exercise 17

A consumer's preferences for food (x) and clothes (y) are represented by the utility function $u(x, y) = xy^2$. The prices are p_x and p_y euros per unit, respectively, and the consumer's income is I euros.

(a) Calculate the consumer's ordinary demand functions, $x(p_x, p_y, I)$ and $y(p_x, p_y, I)$.

(b) Assume that the consumer's income is $I = 12$ and prices are $(p_x, p_y) = (1, 2)$. To raise revenue for public expenditures, the government levies a 1 euro/unit sales tax on good x. Calculate the substitution and income effects of this tax on the demand for good x. If the government replaced this tax with an income tax that raises the same amount of revenue, would the consumer be better or worse off compared to the sales tax on good x?

Exercise 18

A consumer's preferences over clothing (x) and food (y) are represented by the utility function $u(x, y) = x + 2\sqrt{y}$. The price of clothing is $p_x = 1$ euro per unit, the price of food is $p_y = p$ euros per unit, and the consumer's income is I euros.

(a) Calculate the consumer's demand for food, $y(p, I)$, for $I \geq \frac{1}{p^2}$ and for $I < \frac{1}{p^2}$.

(b) Represent graphically the consumer's budget set and calculate her optimal consumption bundle and utility level for $p = \frac{1}{2}$ and $I = 1$. Compute the income and substitution effects on the demand for food resulting from a unit tax of 50 cents on the price of food. Calculate the tax revenue.

Exercise 19

A consumer's preferences for goods x and y are represented by the utility function $u(x, y) = y + 2 \ln x$.

(a) Calculate the consumer's ordinary demand functions. Use the notation p_x, p_y, and I for prices and income, respectively.

(b) Assume that the prices are $p_x = p_y = 1$ and the consumer's monetary income is $I = 30$. A sales tax of one euro per unit of x is introduced. Show that the revenue obtained from this sales tax is less than the revenue that could be obtained with a direct tax on monetary income, R, which has an identical effect on the consumer's welfare as the sales tax.

Exercise 20

A consumer's preferences over clothing (x) and food (y) are represented by the utility function $u(x, y) = 2 \ln x + \ln y$.

(a) Calculate the consumer's ordinary demand functions for food and clothing. Represent the consumer's budget set graphically and calculate their optimal consumption bundle and utility level for $(p_x, p_y) = (2, 1)$ and income $I = 12$.

(b) Compute the substitution and income effects on the demand for food (y) resulting from a unit sales tax of one euro.

Multiple Choice Questions

MC 4.1: If a consumer's preferences are represented by the utility function $u(x, y) = \min\{x, y\}$, then a decrease in the price of x causes:

(i)	☐ a decrease in the demand for x	(ii)	☐ a negative income effect
(iii)	☐ an ambiguous effect on the demand for x	(iv)	☐ a substitution effect equal to zero

MC 4.2: A consumer's preferences are represented by the utility function $u(x, y) = 2x + y$, and market prices are $p_x = p_y = 2$. A decrease in the price of good x to $p'_x = 1$ causes:

(i)	☐ an increase in the demand for y	(ii)	☐ a substitution effect equal to 0
(iii)	☐ a decrease in the demand for x	(iv)	☐ an income effect equal to 0

MC 4.3: If x is an inferior good, then a decrease of it's price p_x:

(i)	☐ increases the demand for x	(ii)	☐ increases the demand for y
(iii)	☐ decreases the demand for x	(iv)	☐ has an ambiguous effect on the demand for x

MC 4.4: If x is a Giffen good, then the substitution (SE), income (IE), and total (TE) effects of an increase in its price p_x are:

(i)	☐ $SE < 0$, $IE > 0$, $TE < 0$	(ii)	☐ $SE < 0$, $IE > 0$, $TE > 0$
(iii)	☐ $SE < 0$, $IE < 0$, $TE < 0$	(iv)	☐ $SE > 0$, $IE > 0$, $TE > 0$

MC 4.5: An individual's preferences are represented by the utility function $u(x, y) = x + 2y$, and the prices are $p_x = p_y = 2$. A decrease in the price of y to $p'_y = 1$ causes:

(i)	☐ an increase in the demand for good x	(ii)	☐ a substitution effect equal to zero
(iii)	☐ a decrease in the demand for good y	(iv)	☐ an income effect equal to zero

5 Labor Supply and the Consumption-Leisure Model

Exercise 21

Esther's parents grant her a monthly budget of M euros. In addition, her aunt offers her the possibility of taking care of her cousins, Elena and Sara, during the weekend, paying her a daily wage of w euros. Esther's preferences for leisure during the weekend (h, measured in days) and consumption (c, measured in euros) are represented by the utility function $u(h, c) = \sqrt[3]{h^2 c}$. Esther is endowed with $H = 9$ weekend days. Describe Esther's problem and calculate her demand for consumption and leisure, and her "labor" supply (the number of days she will offer to take care of her cousins) as a function of M and w.

(a) Represent Esther's budget set and calculate her optimal consumption-leisure bundle for $M = 120$ and $w = 40$.

(b) If $M = 120$, what is the lowest wage w for which Esther's labor supply is positive?

(c) If $w = 40$, what is the lowest monthly budget M for which Esther's labor supply is zero?

Exercise 22

A consumer-worker, who receives a non-wage rent of 240 euros every day has the following preferences over consumption (c) and leisure (h), represented by the utility function $U(h, c) = c^2 h$. Assume $H = 16$.

(a) Determine the lowest wage per hour for which the consumer-worker is willing to work a positive amount of time.

(b) How many hours will the consumer-worker work at a wage of 4 euros per hour?

(c) How much time will the consumer-worker work at a wage of 9 euros per hour? And at 11.25 euros per hour?

(d) Determine how the income and substitution effects influence the demand for leisure when the hourly wage increases from 9 to 11.25 euros.

Exercise 23

The preferences of a worker for leisure (h, measured in hours) and consumption (c, measured in euros) are represented by the utility function $u(h, c) = h + 2\sqrt{c}$. The worker has $H = 16$ hours to use as labor or leisure, and has no other sources of income than his labor income. Calculate and represent his labor supply. (Note that $p_c = 1$ since we are measuring consumption in euros). Assume that the wage is $w = 4$, and that there is an unemployment subsidy of S euros (which is received only by those workers who do not work at all).

(a) Calculate the leisure-consumption bundle of the worker assuming that $S = 5$.

(b) What is the smallest value of S for which the worker would choose not to work?

Exercise 24

María has a daily endowment of 12 hours (to work or use for leisure activities) and a monetary (non-labor) income of M euros. Her preferences are represented by the utility function $u(h,c) = 2\ln h + \ln c$, where h denotes the number of hours of leisure and c denotes her consumption. Set the price of consumption to $p_c = 1$, and denote by w the hourly wage.

(a) Describe María's problem, and calculate her demands for leisure and consumption, as well as her labor supply as functions of M and w.

(b) Using your results in (a), represent María's budget set and calculate her optimal consumption-leisure bundle for $M = 6$ and $w = 4$. Calculate the income and substitution effects over the demand for leisure caused by a 25% tax on labor income.

Exercise 25

Ana is a student whose welfare depends on her average grade $m \in \mathbb{R}_+$ and her consumption $c \in \mathbb{R}_+$. (Assume that consumption is measured in euros, so that $p_c = 1$.) Her preferences are represented by the utility function $u(m,c) = \ln m + \ln c$. Ana has $H = 15$ hours that she can allocate to study and/or supply as labor. Ana's average grade m is determined by the number of hours she studies e according to the formula $m = \frac{2}{3}e$. The hourly wage is $w \geq 0$ euros, and Ana has no other income (besides her labor income).

(a) Describe Ana's budget constraint and graph her budget set in the plane (m,c). Calculate the number of hours she studies and works as a function of w. Assuming that $w = 4$, calculate Ana's optimal average grade and consumption (m^*, c^*) bundle, and represent it in the graph.

(b) Assume now that a program is introduced that rewards students with an average grade $\bar{m} = 7$ or above with $M = 10$ euros. Assuming that $w = 4$, graph Ana's new budget set and determine the impact of this change on her average grade and consumption.

Exercise 26

Consider an individual about to retire with preferences for leisure (h, measured in hours) and consumption (c, measured in euros) represented by the utility function $u(h,c) = hc$, whose salary is 15 euros/hour, and has a total of 140 hours per month available to use as leisure or supply as labor. He is entitled to a monthly pension of 1200 euros, but if he continues working, this pension would be reduced by $t(15l)$, where l is the number of hours he works and $t \in [0, 1/2]$.

(a) Write down the individual's budget constraint, draw his budget set, and calculate his labor supply $l(t)$.

(b) For which values of t will he choose to retire altogether?

Exercise 27

There are 10 consumer-workers with preferences over leisure (h) and consumption (c) described by the utility function $u(h, c) = h + \ln c$. Each individual has 1 unit of time (one day, for example) for leisure or labor activities, and a monetary income of M euros. The price of the consumption good is $p = 1$ and the wage is w.

(a) Derive the labor supply of each individual as a function of w and M.

(b) Compute the aggregate labor supply, assuming $M = 3$ for 5 individuals and $M = 0$ for the remaining 5 individuals. Calculate the equilibrium wage and the employment level, assuming that the labor demand is $L^D(w) = \frac{20}{w}$. Graph the labor market demand and supply curves and identify the equilibrium.

6 Compensated Variation, Equivalent Variation, and Price Indices

Exercise 28

Assume that x and y represent housing services, measured in squared meters per year, and the rest of goods, respectively. A representative consumer has preferences for those goods represented by the utility function $u(x, y) = xy^2$. Prices are $p_x = 3$ and $p_y = 1$. The government proposes a subsidy of 1 euro per square meter consumed. The opposition complains, arguing that the value of the subsidy to the individual is inferior to the cost incurred by the State. What would you recommend? Why?

Exercise 29

Consider the following situation of a pensioner who consumes two goods, food (x) and clothes (y). When he got retired in 2019, the Social Security awarded a pension of 15,000 euros to him. In that year, the prices of food and clothes were 8 euros and 50 euros, respectively. Suppose that the utility function of the pensioner is $u(x, y) = x\sqrt{y}$.

(a) Determine and represent the pensioner's choice under those conditions.

(b) Suppose that in 2024 the prices of food and clothes are 10 euros and 75 euros, respectively. Determine and represent the choice of the pensioner in case that his pension is not updated.

(c) Which pension should we give the pensioner for him to recover his initial utility level with the minimum cost for the social security?

Exercise 30

The preferences of a consumer between two goods, x and y, are described by the utility function $u(x, y) = \ln x + \ln y$. The prices of these goods are $p_x = 1$ and $p_y = \frac{1}{2}$.

(a) Determine the solution to the consumer's problem at those prices for any income I.

(b) Because of an ecological disaster, the supply of good x decreases and its price doubles. As a consequence, the welfare of the consumer decreases. Trying to mitigate the disaster, the local authority is willing to subsidize the consumer. Compute the monetary quantity S that must be given to the consumer to keep the same utility level he had before the disaster.

Exercise 31

János only consumes two goods: paprika (x) and brandy (y). The preferences of János are represented by the utility function $u(x, y) = y + \ln x$. The price of paprika is $p_x = p$ euros, and the price of brandy is $p_y = 1$ euro. János' income is I euros.

(a) Calculate János' demand for paprika and brandy as a function of (p, I) for $I > 1$.

(b) Represent János' budget set for $(p, I) = \left(\frac{1}{2}, 10\right)$ and calculate his optimal bundle and utility level.

(c) Calculate the income and substitution effects on the demand for paprika of an increase in its price from $p = \frac{1}{2}$ to $p' = 1$.

(d) How much is János willing to pay to avoid the price of paprika increasing from $p = \frac{1}{2}$ to $p' = 1$?

Exercise 32

Let's classify goods into two groups: clothes and shoes (x) and food (y). The preferences of Manuel, who retired with a pension $I_0 = 750$ euros, are represented by a utility function $u(x, y) = x^{2/5}y^{3/5}$. The prices in the year he retired, which will be taken as the base year, were $p_0 = (1, 1)$. The current prices are $p_1 = (2, \frac{3}{2})$.

(a) Calculate the pension I_1 that would guarantee Manuel the same welfare level as he had in his retirement year.

(b) Manuel's true consumer price index is $CPI^* = \frac{I_1}{I_0}$. Verify that the Laspeyres price index CPI_L is larger than CPI^*.

Exercise 33

A consumer's preferences for food (x) and clothes (y) are represented by the utility function $u(x, y) = x + \sqrt{y}$. The prices are p_x and p_y euros per unit, respectively, and the consumer's income is I euros.

(a) Calculate the consumer's ordinary demands, $x(p_x, p_y, I)$ and $y(p_x, p_y, I)$. Represent the consumer's budget set and calculate her optimal consumption bundle if her income is $I = 8$ and prices are $(p_x, p_y) = (4, 1)$.

(b) Assuming that $I = 8$ and $(p_x, p_y) = (4, 1)$, calculate the equivalent variation of a tax of 1 euro per unit of good y, and compare it to the tax revenue. How do you explain the difference?

Multiple Choice Questions

MC 6.1: Assume that in 2007 prices were $(p_x, p_y) = (3, 2)$, and in 2008 they were $(p'_x, p'_y) = (2, 4)$. If the bundle of the representative consumer is $(2, 2)$, and one measures the CPI (consumer price index) using a Laspeyres Index, then the percentage change in prices is:

(i)	☐ 25%	(ii)	☐ 20%
(iii)	☐ 30%	(iv)	☐ 50%

MC 6.2: Assume that in 2007 prices were $(p_x, p_y) = (2, 3)$, and in 2008 they were $(p'_x, p'_y) = (3, 4)$. If the bundle of a consumer in 2007 was $(2, 2)$, then her true CPI index is:

(i)	☐ less than 40%	(ii)	☐ exactly 40%
(iii)	☐ greater than 40%	(iv)	☐ undetermined

MC 6.3: Between 2009 and 2010, the prices of both goods, x and y, increased by 10%. If the consumer's income increased by a rate equal to the consumer's true price index, then her budget line:

(i) ☐ shifted parallel towards the origin
(ii) ☐ rotated around its intersection with the x-axis
(iii) ☐ shifted parallel away from the origin
(iv) ☐ maintained its position

MC 6.4: The prices of the goods x and y in the base period were $(p_x, p_y) = (2, 2)$, and the optimal bundle of the representative agent was $(x^*, y^*) = (2, 1)$. If the current prices are $(p'_x, p'_y) = (1, 4)$, then the Laspeyres CPI is:

(i)	☐ $CPI_L = 1$	(ii)	☐ $CPI_L = 1.66$
(iii)	☐ $CPI_L = 1.5$	(iv)	☐ $CPI_L = 0.8$

MC 6.5: If a consumer's income is multiplied by the Laspeyres consumer price index, then:

(i) ☐ the consumer receives exactly the compensated variation
(ii) ☐ the consumer's budget set remains unchanged to be that of the base period
(iii) ☐ the consumer's welfare increases relative to that of the base period
(iv) ☐ the consumer's welfare remains unchanged to be that of the base period

MC 6.6: A consumer's preferences are described by the utility function $u(x, y) = xy$. The prices in the base period were $(p_x, p_y) = (1, 1)$ and her optimal consumption bundle was $(x^*, y^*) = (1, 1)$. If the current prices are $(p'_x, p'_y) = (1, 4)$, then the difference between her Laspeyres consumer price index (CPI) and her true CPI is:

(i)	☐ 0.1	(ii)	☐ 0.2
(iii)	☐ 0.5	(iv)	☐ 1

MC 6.7: If a consumer's preferences are represented by the utility function $u(x, y) = 2x + y$, her income is $I = 4$, and the prices are $p_x = p_y = 1$, then the equivalent variation of a sales tax on 1 euro per unit of good x is:

(i)	☐ 0	(ii)	☐ 1
(iii)	☐ 2	(iv)	☐ 4

7 Choice Under Uncertainty

Exercise 34

Oscar has just graduated. He has received an inheritance of 4 million euros and is considering whether to invest 2 million euros in a start-up business. If the business is successful, he expects a gross profit of 6 million euros, but if it fails, he will lose the investment. The probability of success is $p = \frac{1}{2}$. Determine whether Oscar will make the investment if his preferences are represented by the Bernoulli utility function:

$$(i)\ u(x) = x; \quad (ii)\ u(x) = x^2; \quad (iii)\ u(x) = \sqrt{x}.$$

Exercise 35

Pedro Banderas has a wealth of 100 thousand euros and is considering whether to produce a blockbuster movie whose budget is 250 thousand euros. A film company is willing to finance the movie but wants Pedro to share some of the risk (and profits); specifically, it is willing to finance 80% of the budget. Assuming that the distributors like the movie, Pedro expects the movie to generate box office revenue of 250 thousand euros if the reviews are bad, and as much as 1.5 million euros if the reviews are good. It is known that distributors like 8 out of 10 movies that are produced, and that 1 out of 10 movies that are distributed get good reviews. Pedro's preferences are represented by the Bernoulli utility function $u(x) = \sqrt{x}$.

(a) Represent the decision problem and determine whether or not Pedro should produce the movie.

(b) Determine whether Pedro may be willing to finance 40% (instead of 20%) of the movie's budget.

Exercise 36

The oil company Tibitrol has bought some deserted land in Monegros. The company's geologist estimates that the probability that they will find oil in this land is $p = 0.2$. The drilling of the land to check whether or not it really has oil costs 100 million euros. If they find oil, the company will make revenues of 300 million euros. If they do not find oil, the drilling will be completely useless.

(a) If the company is risk-neutral, what will it decide to do to maximize its expected utility?

(b) What will the company do if it is risk-averse?

Exercise 37

An individual must decide whether to finance their house with a mortgage at a fixed interest rate (FM), or at a variable interest rate (VM). FM involves an annual payment of P thousand euros, while VM involves an annual payment of 17 thousand euros with probability $\frac{1}{2}$, 20 thousand euros with probability $\frac{1}{3}$, and 30 thousand euros with probability $\frac{1}{6}$. The individual's annual income is 50 thousand euros, and their welfare depends on their net income x, measured in thousands of euros, which is equal to their income minus the mortgage payment. The individual's preferences over lotteries are represented by the Bernoulli utility function $u(x) = 2\sqrt{x}$. For which values of P would they prefer to finance their house with the FM mortgage?

Exercise 38

A professor is preparing a multiple-choice exam in which there are four possible answers to each question, of which only one is correct. A correct answer is worth 1 point. Students' preferences satisfy the usual assumptions and are described by a Bernoulli utility function $u(x)$, where x is the number of points obtained in the exam. Assume that when a student does not know the answer to a question, they believe that all answers are equally likely to be correct.

(a) Assuming that there is no penalty for an incorrect answer—that is, an incorrect answer is worth zero points—describe the alternative decisions (lotteries) of a student who does not know the answer to a question. If a student is risk-neutral, is it optimal to answer the question randomly? What if they are risk-averse?

(b) Assuming that the student is risk-neutral, calculate the certainty equivalent of the lottery of responding to a question for which the student does not know the answer. If it is known that students are either risk-averse or risk-neutral, what is the minimum penalty for each incorrect answer that would induce students not to respond to questions whose answers they do not know?

Exercise 39

An entrepreneur has an income of 150 thousand euros and is considering introducing a new touristic product that requires an investment of 200 thousand euros. An investment fund offers them the possibility of financing 50% of the investment in exchange for 60% of the revenues it generates. The entrepreneur believes that if the weather is favorable during the tourist season, then the new product would generate a revenue of 200 thousand euros or 500 thousand euros with equal probability, while if the weather is unfavorable, the product would be a fiasco (that is, it would generate no revenue). Historically, the region enjoys favorable weather with probability 0.6. The preferences of the entrepreneur are represented by the Bernoulli utility function $u(x) = x^2$, where x is the net disposable income in thousands of euros.

(a) Determine whether the entrepreneur should introduce the new product (I), accepting the offer of the investment fund, or rather abstain from introducing the product (NI), maintaining their current income.

(b) Would the entrepreneur be willing to pay 25 thousand euros to know before making the decision whether the weather will be favorable during the tourist season? (Your answers must be supported by appropriate calculations.)

Exercise 40

In the market of car insurance, there are two kinds of drivers: good drivers (who have one accident per year with probability 0.1 and no accident with probability 0.9) and bad drivers (who have one accident per year with probability 0.1, two accidents with probability 0.05, and no accident with probability 0.85). Repairing a car costs, on average, 2,000 euros. The proportion of good and bad drivers in the population is 2 to 1.

(a) Assume that the insurance companies are risk-neutral and cannot distinguish between good and bad drivers. What is the minimum price that these companies would be willing to offer in order to cover the risk of an accident?

(b) Imagine that the preferences of the drivers are represented by the utility function $u(x) = \sqrt{x}$, and that their initial wealth is 5,000 euros. Which type of drivers (good and/or bad) will subscribe to an insurance policy with the minimum price determined in part (a)?

Exercise 41

A risk-neutral person needs to put a mortgage on one of their buildings in order to get 200,000 euros. They have to pay back this amount in 2 annual payments of 100,000 euros each, plus the corresponding interest rate. The mortgage credit options they can choose from are:

(1) Fixed interest rate: 10% per year.

(2) Interest rate: 9% in the first year, which may increase to 14%, decrease to 8%, or remain the same in the second year.

(3) Interest rate: 7% in the first year, which may increase to 20%, decrease to 6%, or remain the same in the second year.

(a) Determine the decision which maximizes the expected profits, knowing that the interest rate increases with probability 0.6 and decreases with probability 0.2.

(b) How much is this person willing to pay in order to learn whether the interest rate will increase, decrease, or remain the same?

Exercise 42

A consumer must choose between buying an apartment in Madrid or a house in the suburbs. Both choices would cost 120,000 euros. The consumer is indifferent between the two options, except for their expectation regarding revaluation. If housing prices keep increasing (event E_1), the price of the apartment will reach 140,000 euros, while the price of the house will reach 340,000 euros. The probability of this happening is 30%. Otherwise, if housing prices decrease (event E_2), the price of the apartment will drop to 70,000 euros, and the price of the house to 20,000 euros. The consumer's preferences are represented by the utility function $u(x) = \sqrt{x}$, where x is their wealth in euros. The consumer's initial wealth is 140,000 euros.

(a) Represent the decision problem and determine whether the consumer should buy the house or the apartment.

(b) Should the consumer pay 20,000 euros to learn whether housing prices will decrease or increase?

Exercise 43

The introduction of a new product in the market includes three stages: Design, Experimentation, and Production. 7 out of 10 products do not pass the design stage. From those that do pass, only 10% pass the experimentation stage and proceed to production. Only 1 out of 5 products produced are successful in the market. For each new product, the costs of each stage are 100,000, 20,000, and 200,000 euros, respectively. The expected profits from a product that successfully passes all three stages are 60 million euros.

(a) What is the expected value of constructing a new product?

(b) For 15,000 euros, a consultant can predetermine (without any uncertainty) whether a product that has already passed the design stage will pass the experimentation stage. What is the value of the consultant's services, assuming the entrepreneur is risk-neutral?

Exercise 44

The marketing chief of a large computer producer has to decide whether to launch a new campaign before (d_1) or after the month of May (d_2). If the campaign is launched before May, sales will amount to 100 million euros. If launched after May, there is a risk that the competitor launches its own campaign before (C), which occurs with probability 0.4. Additionally, sales depend on the state of the economy, which can be good (A) with probability 0.5, stable (E) with probability 0.3, or bad (R). If the economy is good and the competitor has not launched its campaign, sales will reach 150 million euros, whereas if the competitor has launched, sales will be 120 million euros. If the economy is stable, sales will reach 90 million euros if the competitor launches its campaign and 110 million euros if it does not. If the economy is bad, sales will be 70 million euros if the competitor launches its campaign and 80 million euros otherwise. Assuming the producer is risk-neutral:

(a) What is the best decision?

(b) How much would the marketing chief be willing to pay to know with certainty all the uncertain variables of the problem?

(c) How much would the marketing chief be willing to pay to an industrial spy who could tell with certainty whether the competitor will launch its campaign?

Exercise 45

An individual has an initial wealth of 500,000 euros and an annual income of 250,000 euros. The income tax rate is 50%. He is considering whether to declare his full income, only half of it, or nothing at all. The probability that he is inspected by Hacienda is 0.1. If an inspection detects misdeclared income, he must pay the missing taxes plus an identical amount as a fine. His preferences are represented by the Bernoulli utility function $u(x) = 2\sqrt{x}$, where x is wealth plus his income net of taxes and/or fines.

(a) Describe the decision problem and identify the optimal decision.

(b) Now suppose the individual decides not to file a tax form but is now worried about the possibility of being inspected by Hacienda. The individual consults with an expert lawyer who suggests that the probability of an inspection is higher than expected and equals 50%. She suggests as a solution filing a voluntary tax return and paying a fine of m thousand euros. For which values of m will the individual be willing to adopt this solution?

(c) Alternatively, before he makes the decision wether to declare, someone offers to inform him whether or not he is on Hacienda's inspection list. Derive the equation identifying the value of this information. Would the individual pay 20,000 euros for this information?

Multiple Choice Questions

MC 7.1: An individual's risk premium for the lottery $l = (x, p)$, that pays $x = (0, 4, 16)$ with probabilities $p = \left(\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right)$, is $RP(l) = 2$. Hence his certainty equivalent is:

(i)	$\square$ $CE(l) = 2$	(ii)	$\square$ $CE(l) = 1$
(iii)	$\square$ $CE(l) = 3$	(iv)	$\square$ $CE(l) = 4$

MC 7.2: If the certainty equivalent of a lottery l, which pays 20 thousand euros or 10 thousand euros with the same probability, is 14 thousand euros, then the individual:

(i)	$\square$ is risk loving	(ii)	$\square$ is risk neutral
(iii)	$\square$ is risk averse	(iv)	$\square$ has an indeterminate risk attitude

MC 7.3: If an individual's certainty equivalent of lottery $l = \left((0, 2, 10), (\frac{3}{10}, \frac{1}{2}, \frac{1}{5})\right)$ is $CE(l) = 2$, then his risk premium is:

(i)	$\square$ $RP(l) = -1$	(ii)	$\square$ $RP(l) = 1$
(iii)	$\square$ $RP(l) = 2$	(iv)	$\square$ $RP(l) = 0$

MC 7.4: The risk premium of the lottery $l = \left((0, 8), (\frac{1}{4}, \frac{3}{4})\right)$ for an individual A whose preferences are represented by the Bernoulli utility function $u_A(x)$ is $RP_A(l) = 2$. If the preferences of an individual B are represented by the Bernoulli utility function $u_B(x) = \frac{1}{3}u_A(x)$, then her certainty equivalent of the lottery l is:

(i)	$\square$ $CE(l) = 2$	(ii)	$\square$ $CE(l) = 6$
(iii)	$\square$ $CE(l) = 4$	(iv)	$\square$ $CE(l) = 0$

MC 7.5: Identify the certainty equivalent and risk premium of the lottery $l = (x, p)$ that pays $x = (0, 2, 4)$ with probabilities $p = \left(\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right)$ for an individual with preferences represented by the Bernoulli utility function $u(x) = x^2$:

(i)	$\square CE(l) = 2, RP(l) = \sqrt{6} - 2$	(ii)	$\square CE(l) = 2, RP(l) = 2 - \sqrt{6}$
(iii)	$\square CE(l) = \sqrt{6}, RP(l) = 2 - \sqrt{6}$	(iv)	$\square CE(l) = \sqrt{6}, RP(l) = \sqrt{6} - 2$

MC 7.6: A consumer's preferences over lotteries are represented by the Bernoulli utility function $u(x) = \sqrt{x}$. Identify the expected utility and certainty equivalent of the lottery $l = (x, p)$ that pays $x = (0, 4, 9)$ with probabilities $p = \left(\frac{1}{6}, \frac{1}{2}, \frac{1}{3}\right)$:

(i)	$\square Eu(l) = 1, CE(l) = 1$	(ii)	$\square Eu(l) = 4, CE(l) = 2$
(iii)	$\square Eu(l) = 2, CE(l) = 2$	(iv)	$\square Eu(l) = 2, CE(l) = 4$