Microeconomics Final Exam. uc3m, May 27, 2024. Test

Name: Group:

You have 45 minutes. Mark your answer with an "x." You get 2 points for each correct answer, -0.66 for each incorrect answer, and zero points for each question you do not answer.

1. Which of these axioms is not satisfied by the Pareto preferences?

☒ Completeness ($A.1$) ☐ Monotonicity ($A.3$)
☐ Transitivity ($A.2$) ☐ Continuity ($A.4$)

2. The optimal consumption bundle of a consumer that lexicographically prefers y to x when his income is $I = 12$ and the prices are $p_x = 1$ and $p_y = 2$ is

☐ $(6,3)$ ☐ $(12,0)$ ☒ $(0,6)$ ☐ $(4,4)$.

3. The preferences of an individual are represented by the utility function $u(x,y) = x^2y$. At the current prices she consumes the bundle $(1,4)$. Hence the compensated variation of a 1-euro tax per unit of good x is

☐ zero ☒ greater than zero, but less than 1 euro ☐ 1 euro ☐ greater than 1 euro.

4. If x is a normal good, then the signs of the substitution effect (SE), income effect (IE) and total effect (TE) of an increase of its price p_x are:

☐ $SE > 0$, $IE > 0$, $TE > 0$ ☐ $SE < 0$, $IE > 0$, TE indeterminate
☒ $SE < 0$, $IE < 0$, $TE < 0$ ☐ $SE > 0$, $IE < 0$, TE indeterminate.

5. If the prices were $(p_x, p_y) = (2,1)$ at the base period and they are $(p'_x, p'_y) = (2,2)$ at the current period, then the Laspeyres CPI for an individual whose consumption in the base period was $(x,y) = (3,6)$ is:

☐ $\frac{2}{3}$ ☐ $\frac{4}{3}$ ☒ $\frac{3}{2}$ ☐ 2.

6. If the preferences of the consumer in question 1.5 are represented by the utility function $u(x,y) = \min\{2x, y\}$, then her true CPI is:

☐ $\frac{2}{3}$ ☐ $\frac{4}{3}$ ☒ $\frac{3}{2}$ ☐ 2.

7. The expected utility and risk premium of a lottery l that pays $x = (2,8)$ with probabilities $p = (2/3, 1/3)$ for an individual whose preferences are represented by the Bernoulli utility function $u(x) = \sqrt{2x}$ is

☐ $Eu(l) = 8/9$, $RP(l) = 5/3$ ☐ $Eu(l) = 8/3$, $RP(l) = 5/3$
☐ $Eu(l) = 8/9$, $RP(l) = 4/9$ ☒ $Eu(l) = 8/3$, $RP(l) = 4/9$.

8. A worker with preferences represented by the Bernoulli utility function $u(x) = x$ receives a job offer paying the wages $x_A = 15$, $x_B = 9$ and $x_C = 6$, respectively, depending on whether the economy enters a boom (A), maintains its actual course (B) or enters a recession (C). The probabilities of A, B and C are $p_A = p_B = p_C = 1/3$. The worker's wage is his current job is 9 euros. Hence, the value of perfect information to him is

☐ 0 ☒ 1 ☐ greater than 1, but less than 2 ☐ 2.

9. A firm that produces a good according to the production function $F(L, K) = 2\sqrt{L} + K$ has

☒ decreasing returns to scale ☐ constant returns to scale
☐ economies of scale ☐ a concave total cost function.

10, 11 and 12. The conditional demand of labor $L(Q)$ for $Q > 0$ of a firm whose production function is $F(L, K) = \sqrt{2\,(L-1)\,K}$ when the input prices are $(w, r) = (1, 2)$ is

☐ $2Q^2$ ☐ $2\sqrt{Q}$ ☒ $Q+1$ ☐ $2Q+1$,

the firm's cost function is

☐ $2Q^2+1$ ☐ $2\sqrt{Q}-1$ ☒ $2Q+1$ ☐ $(2Q+1)^2$.

and therefore, the firm produces the good with

☒ economies of scale ☐ constant returns to scale
☐ diseconomies of scale ☐ decreasing returns to scale.

13. The short run supply at the price $p = 9$ of a competitive firm that produces the good with cost $C(q) = 25 + q^2$ is

☐ $S(9) = 9$ ☐ $S(9) = 18$ ☐ $S(9) = 0$ ☒ $S(9) = 9/2$.

14 and 15. In a market in which the demand is $D(p) = \max\{22 - p, 0\}$ there are 6 firms that produce the good with cost $C(q) = q^2 + 6q + 1$ for $q \geq 0$. Hence in the short run competitive equilibrium the price and output of each firm are

☐ $p^* = 12$, $q_i^* = 5/3$ ☒ $p^* = 10$, $q_i^* = 2$ ☐ $p^* = 6$, $q_i^* = 3$ ☐ $p^* = 9$, $q_i^* = 2$,

while in the long run competitive equilibrium the price and the number of firms are

☐ $p_L = 10$, $n_L = 12$ ☐ $p_L = 6$, $n_L = 8$ ☐ $p_L = 9$, $n_L = 13$ ☒ $p_L = 8$, $n_L = 14$.

Microeconomics, Final Exam: Exercises (uc3m, May 27, 2024)

Name: Group:

Exercise 1. The preferences of a consumer over food (x) and clothes (y) are represented by the utility function $u(x, y) = 2x + \ln y$.

A. (10 points) Calculate her ordinary demand functions, $x(p_x, p_y, I)$ and $y(p_x, p_y, I)$.

B. (15 points) Assuming that the consumer's income is $I = 4$ and prices are $(p_x, p_y) = (4, 1)$, calculate the income, substitution and total effects over the demand of clothes of a sale tax on clothes of 1 euro per unit.

C. (10 points) Calculate the compensated variation of the tax in part (B) and, using this result calculate the consumer's *true* price index taking $(4, 1)$ as the prices in the base period and $(4, 2)$ as the prices in the current period. Calculate as well the Laspeyres CPI and explain why these indices differ.

Exercise 2. Consider a market for a good demanded by both households and firms. The households' demand is $D_H(p) = \max\{50 - p/2, 0\}$ and the firms' demand is $D_E(p) = \max\{100 - p, 0\}$. The market is monopolized by a firm that produces the good with total cost $C(q) = 2000 + q^2/3$.

A. (20 points) Calculate the monopoly equilibrium without price discrimination, including the price, the quantities served to households and firms, the surpluses of households and firms, and the Lerner index.

B (10 points) Determine the market equilibrium resulting if the government introduces the price cap $\bar{p} = 60$, and verify how this regulation would affect the surplus of households, firms and the monopoly, as well as the deadweight loss.

C. (5 points) Calculate the monopoly equilibrium with third degree price discrimination, and explain how and why it differs or does not differ from that of part A.

Grading Rules:

Exercise 1

A. MRS: 2 points. Solution to the system of equations: 5 points. Corner solutions: 3 points.

B. System of equations to identify the substitution effect: 6 points. Solution: 4 points. Income effect: 3 points. Total effect: 2 points.

C. Compensated Variation: 5 points. CPI^*: 1 points. $LCPI$: 2 points.

Exercise 2

A. Price, quantity: 8 points. Surpluses: 2+2+2 points. Deadweight loss: 3 points. Lerner Index: 3 points.

B. Equilibrium price and quantity: 8 points. Surpluses: 2 points.

C. System of equations: 2 points. Solution: 2 points. Comment: 1 point.

Exercise 1. A. Since $RMS(x,y)=2y$*, an interior solution is identified by the system of equations*

$$2y=\frac{p_x}{p_y},\ xp_x+yp_y=I,$$

whose solution is

$$x=\frac{I}{p_x}-\frac{1}{2},\ y=\frac{p_x}{2p_y}>0.$$

Thus, for $x\geq 0$ *to hold we must have* $I\geq p_x/2$*. If this inequality does not hold, the optimal bundle is* $(x,y)=(0,I/p_y)$*. Hence, the ordinary demands are*

$$x(p_x,p_y,I)=\begin{cases}\frac{I}{p_x}-\frac{1}{2} & \text{if } I\geq\frac{p_x}{2}\\ 0 & \text{if } I<\frac{p_x}{2},\end{cases},\ y(p_x,p_y,I)=\begin{cases}\frac{p_x}{2p_y} & \text{if } I\geq\frac{p_x}{2}\\ \frac{I}{p_y} & \text{if } I<\frac{p_x}{2}.\end{cases}$$

B. *At prices* $(p_x,p_y)=(4,1)$ *and income* $I=4$ *the optimal bundle is* $(x^*,y^*)=(1/2,2)$*, and the consumer's utility is* $u_0=1+\ln 2$*. The cheapest bundle that allows the consumer to reach the utility* u_0 *at prices* $(4,2)$ *is identified by the system of equations*

$$\begin{aligned}2\hat{y}&=2\\ 2\hat{x}+\ln\hat{y}&=1+\ln 2,\end{aligned}$$

whose solution is

$$(\hat{x},\hat{y})=(\frac{1}{2}\left(1+\ln 2\right),1).$$

Therefore, the substitution, total and income effects are

$$SE=1-2=-1,\ TE=1-2=-1,\ IE=TE-SE=0.$$

C. The compensated variation is the difference between the cost of the bundle $(\hat{x},\hat{y})$ *at prices* $(4,2)$ *and the consumer's income* $I=4$,

$$VC=4\left(\frac{1}{2}\left(1+\ln 2\right)\right)+2\left(1\right)-4\approx 1.386.$$

The consumer's true CPI is

$$CPI^*=\frac{4\left(\frac{1}{2}\left(1+\ln 2\right)\right)+2\left(1\right)}{4}=1.346,$$

and the Laspeyres CPI is

$$LCPI=\frac{0.5\left(4\right)+2\left(2\right)}{4}=1.5.$$

The inequality $IPCL>IPC^*$ *is due to the index LCPI to ignoring the substitution effect caused by the change of prices, which results in an upward bias of the increase of the cost of living.*

Exercise 2. A. The inverse demand is

$$P(q) = \max\{100 - \frac{2}{3}q, 0\}.$$

Thus, for $q \leq 150$, the monopoly's revenue function is

$$R(q) = P(q)q = \left(100 - \frac{2}{3}q\right)q, \ R'(q) = 100 - \frac{4}{3}q.$$

The first order condition for profit maximization is

$$R'(q) = C'(q) \Leftrightarrow 100 - \frac{4}{3}q = \frac{2}{3}q,$$

The solution to this equation identifies the output, which in turns identifies the price, at the monopoly equilibrium

$$q_M = 50, \ p^M = P(q_M) = 100 - \frac{2}{3}(50) = \frac{200}{3}.$$

Households and firms demand

$$q_H = 50 - \frac{200}{6} = \frac{50}{3}, \ q_F = 100 - \frac{200}{3} = \frac{100}{3}.$$

and their surpluses are

$$CS_H = \frac{1}{2}\left(100 - \frac{200}{3}\right)\frac{50}{3} = \left(\frac{50}{3}\right)^2, \ CS_F = \frac{1}{2}\left(100 - \frac{200}{3}\right)\frac{100}{3} = \frac{1}{2}\left(\frac{100}{3}\right)^2.$$

The monopoly's surplus is

$$PS_M = R(q_M) - \frac{q_M^2}{3} = \frac{200}{3}(50) - \frac{50^2}{3} = 2500.$$

In order to calculate the deadweight loss we identify the output at the "competitive equilibrium, which is the solution to the equation

$$P(q) = C'(q) \Leftrightarrow 100 - \frac{2}{3}q = \frac{2}{3}q \Leftrightarrow q_C = 75.$$

Hence, the deadweight loss is

$$DWL = \frac{1}{2}(p_M - C'(q_M))(q_C - q_M) = \frac{1}{2}\left(\frac{200}{3} - \frac{2}{3}(50)\right)(75 - 50) = \frac{1250}{3}.$$

The Lerner index is

$$L = \frac{p^M - C'(q^M)}{p^M} = \frac{\frac{200}{3} - \frac{2(50)}{3}}{\frac{200}{3}} = \frac{1}{2}.$$

B. With a price-cap $\bar{p} = 60$ the monopoly serves the demand at $\bar{p}$, i.e.,

$$\bar{p} = 60 = 100 - \frac{2}{3}q \Leftrightarrow \bar{q} = 60.$$

Hence, consumer surpluses are

$$\overline{CS}_H = \frac{1}{2}(100 - 60)\,20 = 20^2,\ \overline{CS}_F = \frac{1}{2}(100 - 60)\,40 = \frac{40^2}{2}.$$

and the producer surplus is

$$\overline{PS}_M = I(\bar{q}) - \frac{\bar{q}^2}{3} = 60\,(60) - \frac{60^2}{3} = 2400.$$

The deadweight loss decreases to

$$\overline{DWL} = \frac{1}{2}\left(\bar{p} - C'(\bar{q})\right)(q_C - \bar{q}) = \frac{1}{2}\left(60 - \frac{2}{3}(60)\right)(75 - 60) = 150.$$

C. With third degree price discrimination the monopoly serves to households and firm the quantities that solve the system of equations

$$\begin{aligned} I'_H(q_H) &= C'(q_H + q_F) \Leftrightarrow 100 - 4q_H = \frac{2}{3}(q_H + q_F) \\ I'_E(q_H) &= C'(q_H + q_F) \Leftrightarrow 100 - 2q_F = \frac{2}{3}(q_H + q_F), \end{aligned}$$

whose solution are the quantities (q_H, q_F) found in part A. Therefore, in this setting the possibility of exercising third degree price discrimination does not modify the monopoly decisions.