Universidad Carlos III de Madrid May 2019

Microeconomics

Name: Group:

1	2	3	4	5	Grade

You have 2 hours and 45 minutes to answer all the questions.

1. Multiple Choice Questions. (Mark your choice with an "x." You get 2 points if your answer is correct, -0.66 points if it is incorrect, and zero points if you do not answer.)

1.1. The Pareto preferences

☒ do not satisfy axiom $A.1$ (completeness) ☐ do not satisfy axiom $A.3$ (monotonicity)
☐ do not satisfy axiom $A.2$ (transitivity) ☐ do not satisfy axiom $A.4$ (continuity).

Questions 1.2 and 1.3 refer to a consumer with preferences over x and y represented by the utility function $u(x, y) = 2x + y$, and a monetary income $I = 12$.

1.2. At prices $(p_x, p_y) = (3, 1)$, his optimal consumption bundle is

☒ $(0, 12)$ ☐ $(4, 0)$
☐ $(2, 6)$ ☐ $(3, 3)$.

1.3. The substitution and income effects, SE and IE, of a increase of the price of y to $p'_y = 2$ over the demand of y are

☒ $SE = -12$, $IE = 0$ ☐ $SE = 0$, $IE = -6$
☐ $SE = -6$, $IE = -6$ ☐ $SE = 0$, $IE = -12$.

Questions 1.4 and 1.5 refer to a consumer with preferences over food (x) and clothing (y) represented by the utility function $u(x, y) = xy$, and whose monetary income in 2018 was $I = 2$. In 2018 prices were $(p_x^{2018}, p_y^{2018}) = (1, 1)$, and in 2019 they are $(p_x^{2019}, p_y^{2019}) = (4, 1)$.

1.4. The true consumer price index of this individual is

☐ 1 ☐ 1.5 ☒ 2 ☐ 2.5.

1.5. The consumer price index of this individual calculated as a Laspeyres index is

☐ 1 ☐ 1.5 ☐ 2 ☒ 2.5.

Questions 1.6 and 1.7 refer to a consumer with preferences over lotteries represented by the Bernoulli utility function $u(x) = \sqrt{4x}$, who receives two job offers, X and Y, that pay wages that depend on whether the economy enters a boom (A), maintains its actual growth (B) or enters a recession (C). Specifically, X pays $(x_A, x_B, x_C) = (64, 16, 0)$ and Y pays $(y_A, y_B, y_C) = (36, 16, 16)$. The probabilities of scenarios A, B and C are $p_A = 1/4$, $p_B = 1/2$ and $p_C = 1/4$, respectively.

1.6. Identify the expected utilities of X and Y for this individual.

☐ $Eu(X) = 9$, $Eu(Y) = 10$ ☐ $Eu(X) = 9$, $Eu(Y) = 8$
☒ $Eu(X) = 8$, $Eu(Y) = 9$ ☐ $Eu(X) = 8$, $Eu(Y) = 10$.

1.7. Identify the certainty equivalents of X and Y for this individual.

☐ $CE(X) = 25$, $CE(Y) = 16$ ☐ $CE(X) = 25$, $CE(Y) = 20.25$
☒ $CE(X) = 16$, $CE(Y) = 20.25$ ☐ $CE(X) = 16$, $CE(Y) = 25$.

Questions 1.8 and 1.9 refer to Lolita, a competitive cow that produces milk Q using oats O and barley B according to the production function $Q = \sqrt{O(B-2)}$.

1.8. Lolita has

☒ increasing returns to scale ☐ decreasing returns to scale
☐ constant returns to scale ☐ undetermined returns to scale.

1.9. The prices of oats and barley are $p_O = 4$ and $p_B = 6$, respectively. In the short run Lolita cannot change the among of barley she uses, $\bar{B} = 6$. Then, depending on her milk production, in the short run Lolita has

☐ diseconomies of scale for $Q < 6$ ☒ economies of scale for $Q < 6$
☐ diseconomies of scale for $Q > 4$ ☐ economies of scale for $Q > 4$.

1.10. A firm that produces a good with constant marginal cost monopolizes two markets, A and B. If the demand elasticity in A is larger than in B, then relative to the monopoly equilibrium without price discrimination, third degree price discrimination

☒ results in an increase of the consumer surplus in market A
☐ results in an increase of the consumer surplus in market B
☐ results in an decrease of the consumer surplus in markets A and B
☐ results in an increase of the total surplus.

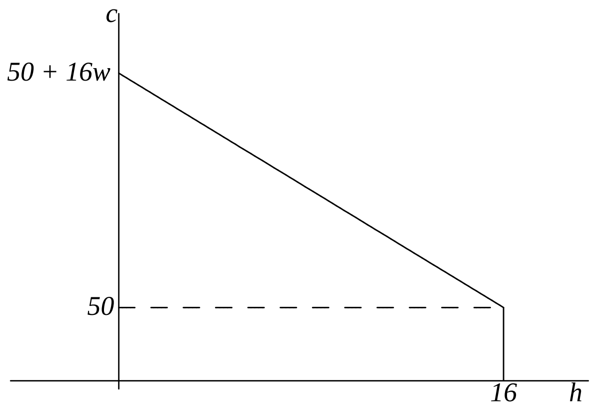
2. Elisa's preferences for leisure and consumption are represented by the utility function $u(h, c) = c + 64 \ln h$, where h is the number of hours of leisure and c is consumption measured in euros. Elisa has 16 hours a day for leisure and labor activities, and has a daily non-labor income of 50 euros.

(a) (15 points) Specify Elisa's budget constraint, and graph her budget set. Then calculate her demand of leisure and consumption, as well as her labor supply, as functions of the hourly wage w. (Verify the possible existence of corner solutions.)

Elisa's budget constraints are

$$c + wh \leq 50 + 16w, \; 0 \leq h \leq 16, \; c \geq 0.$$

The graph shows her budget set.



Since $RMS(h, c) = 64/h$, *an interior solution to Elisa's problem solves the system*

$$\begin{aligned} \frac{64}{h} &= w \\ c + wh &= 50 + 16w. \end{aligned}$$

Since $h \leq 16$ *must hold, the solution to Elisa's problem is*

$$h(w) = \begin{cases} 16 & \text{if } w < 4 \\ \dfrac{64}{w} & \text{if } w \geq 4, \end{cases}$$

$$c(w) = \begin{cases} 50 & \text{if } w < 4 \\ 50 + \left(16 - \dfrac{64}{w}\right) w & \text{if } w \geq 4, \end{cases}$$

and her labor supply is

$$l(w) = \begin{cases} 0 & \text{if } w < 4 \\ 16 - \dfrac{64}{w} & \text{if } w \geq 4. \end{cases}$$

(b) (10 points) If the wage is $w = 10$, what are the income and substitution effects over the demand of leisure of a 20% tax on labor income?

For $w = 10$ *Elisa's leisure demand is* $h(10) = 6.4$, *and her demand of consumption is* $c(10) = 50 + (9.6)\,10 = 146$. *With the 20%* ($t = 0.2$ *in per euro terms), the effective wage is* $\bar{w} = 10\,(1 - 0.2) = 8$, *and the leisure demand is* $h(8) = 8$. *Hence the total effect over the demand of leisure is*

$$TE = h(8) - h(10) = 8 - 6.4 = 1.6.$$

In order to calculate the substitution effect we solve the system

$$\begin{aligned} c + 64 \ln h &= 146 + 64 \ln(6.4) \\ \frac{64}{h} &= 8. \end{aligned}$$

Since a solution to this system involves $h = 8$ *hours of leisure, the substitution effect is*

$$SE = 8 - h(10),$$

which is qual to the total effect. Therefore the income effect is $IE = 0$.

(c) (10 points) Calculate the equivalent variation and the tax revenue of the 20% tax on labor income for $w = 10$.

The tax revenue is

$$R = h(8)tw = 8(0.2)10 = 16.$$

Since $h(8) = 8$ *and* $c(8) = 50 + (16 - 8)8 = 114$, *in order to calculate the equivalent variation* EV *we solve the system*

$$\begin{aligned} \frac{64}{h} &= 10 \\ c + 64 \ln h &= 114 + 64 \ln 8, \end{aligned}$$

whose solution is

$$\tilde{h} = 6.4,\ \ \tilde{c} = 114 + 64(\ln 8 - \ln 6.4) \simeq 128.28.$$

As the consumer's total income equals her consumption, and $c(10) = 146$ *euros, the equivalent variation is*

$$EV = 146 - 128.28 = 17.72 > 16 = R.$$

3. In a competitive market in which the demand is $D(p) = \max\{130 - 5p, 0\}$, there are firms producing the good with the technologies A and B, at a total cost given by functions

$$C^A(q) = \begin{cases} 0 & \text{if } q = 0 \\ 2q^2 + 2 & \text{if } q > 0, \end{cases} \qquad C^B(q) = \begin{cases} 0 & \text{if } q = 0 \\ q^2 + q + 4 & \text{if } q > 0, \end{cases}$$

respectively. Currently, the number of firms of each type are $n_A = n_B = 20$.

(a) (10 points) Calculate the supply of the firms of type A and B, and the market supply.

Taking derivatives of the average cost functions for each type of firm, A and B, for $q > 0$, $AC^A(q) = C^A(q)/q = 2q + 2/q$ and $AC^B(q) = C^B(q)/q = q + 1 + 4/q$, and solving the equations

$$\frac{dAC^A(q)}{dq} = 2q - \frac{2}{q^2} = 0, \quad \frac{dAC^B(q)}{dq} = 1 - \frac{4}{q^2} = 0,$$

we get the outputs that minimize average cost in each case: $q_A^* = 1$ and $q_B^* = 2$. For these outputs the average costs are $AC^A(q_A^*) = 4$ and $AC^B(q_B^*) = 5$, respectively.

Therefore the supply of firms of type A is 0 for $p < 4$, 0 or 1 for $p = 4$, and for $p > 4$ the supply q solves the equation

$$\frac{dC^A(q)}{dq} = p \Leftrightarrow 4q = p;$$

that is,

$$S^A(p) = \begin{cases} 0 & \text{if } p < 4 \\ \{0, 1\} & \text{if } p = 4 \\ \frac{p}{4} & \text{if } p > 4. \end{cases}$$

The supply of firms of type B is 0 for $p < 5$, 0 or 2 for $p = 5$, and for $p > 5$ the supply q solves the equation

$$\frac{dC^B(q)}{dq} = p \Leftrightarrow 2q + 1 = p;$$

that is,

$$S^B(p) = \begin{cases} 0 & \text{if } p < 5 \\ \{0, 2\} & \text{if } p = 5 \\ \frac{p-1}{2} & \text{if } p > 5. \end{cases}$$

The market supply is $S(p) = n_A S^A(p) + n_B S^B(p)$, that is,

$$S(p) = \begin{cases} 0 & \text{if } p < 4 \\ \{0, 1, ..., 20\} & \text{if } p = 4 \\ 5p & \text{if } 4 < p < 5 \\ \{5p, 5p + 2, ..., 5p + 40\} & \text{if } p = 5 \\ 15p - 10 & \text{if } p > 5. \end{cases}$$

(b) (10 points) Graph the market supply and demand and identify the price, the output and the consumer surplus in the short run competitive equilibrium.

The competitive equilibrium price p^* *solves the equation* $D(p) = S(p)$. *Since for* $p = 5$ *we have*

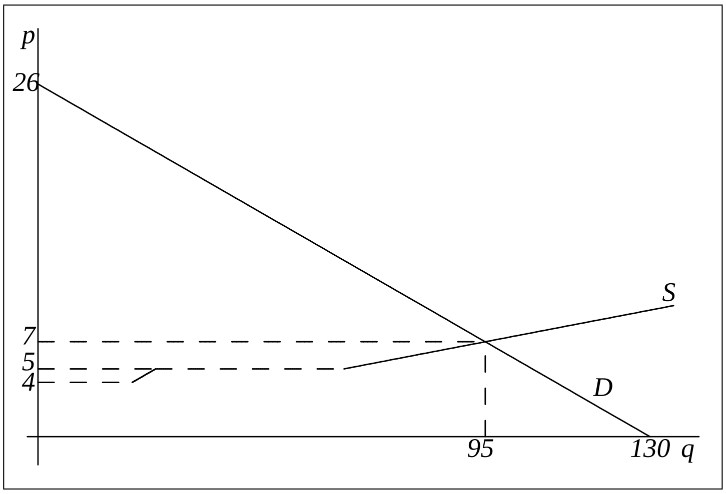
$$D(5) = 130 - 5(5) = 105 > 5(5) + 40 \geq S(5),$$

then $p^* > 5$. *Therefore* p^* *is the solution to the equation*

$$130 - 5p = 15p - 10;$$

that is, $p^* = 7$. *The equilibrium output is* $q^* = 95$. *In equilibrium, the consumer surplus is*

$$CS^* = \frac{1}{2}(26 - 7)\,95 = 902.5.$$



(c) (5 points) Calculate the long run competitive equilibrium price and output. What are the outputs of the firms of type A and B? How many firms of each type are there?

In the long run competitive equilibrium the price is

$$p_L^* = \min\{AC^A(q_A^*),\ AC^B(q_B^*)\} = AC^A(q_A^*) = 4.$$

Hence only the firms using technology A *survive – that is,* $n_B^* = 0$.

The demand at this price is

$$D(p_L^*) = 130 - 5(4) = 110.$$

Since minimizing average cost implies that each firm produces $q_A^* = 1$, *in order to serve this demand* $n_A^* =$ *110 firms of type* A *are required.*

4. A pharmaceutical firm is considering developing a new drug to treat an illness. The demand for such drug, in **thousands** of units, is $D(p) = \max\{600 - 6p, 0\}$. The investment in $R\&D$ required to develop the drug is $\bar{I} = 5$ million euros. The drug would be produced with a constant average cost equal to 40 euros. If the firm makes the investment, it would maintain the formula a secret, monopolizing the market. If no investment is made, then the firm incurs in not cost.

(a) (10 points) Determine whether the firm will develop the drug, and if it does so, calculate the monopoly equilibrium price and output, and well as the consumer surplus, the firm's profit and the Lerner index.

If the firm develops the drug, its profits would be equal to the revenue minus the investment in R&D, $\bar{I}$, minus the cost of production. Let q be the firm's output, in thousand of units. The inverse demand is

$$P(q) = \max\{100 - \frac{q}{6}, 0\}.$$

For $q < 600$ the monopolist's marginal revenue is

$$I'(q) = P(q) + P'(q)q = 100 - \frac{q}{6} + \left(\frac{1}{6}\right) q = 100 - \frac{q}{3}.$$

The marginal cost is $MC(q) = 40$. Hence, the monopoly output q solves the equation

$$I'(q) = MC(q) \Leftrightarrow 100 - \frac{q}{3} = 40,$$

whose solution is $q_M = 180$ thousand units. The equilibrium price is

$$p_M = P(q_M) = 100 - \frac{180}{6} = 70 \text{ euros/unit.}$$

The monopoly's profit is

$$\pi_M = p_M q_M - \bar{I} - 40q_M = 70\,(180) - 5000 - 40\,(180) = 400 \text{ thousand euros}.$$

Since profit is positive, the firm will develop the drug.

In the monopoly equilibrium the consumer surplus is

$$CS_M = \frac{1}{2}\,(100 - 70)\,180 = 2700 \text{ thousand euros,}$$

and the Lerner index is

$$L = \frac{p_M - CM_a(q_M)}{p_M} = \frac{70 - 40}{70} = \frac{3}{7}.$$

(b) (10 points) A non-profit organization (NPO) has decided to make an offer for the drug's patent. Obviously, for the offer to be acceptable it must compensate the firm for the investment made ($\bar{I}$), and the profits that the firm will obtain if it exploits the patent. The NPO only has 600 thousand euros, and therefore it must rise the revenue to cover the offer for the patent by setting an appropriate price. If the objective of the NPO is to maximize the consumer surplus, at what price will it sell the drug? By how much would the consumer surplus increase with respect to the situation in (a)? What is the social benefit of this program? (Recall that the solution to a second degree equation, $ax^2 + bx + c = 0$, is $\left(-b \pm \sqrt{b^2 - 4ac}\right)/2a$.)

(b') If you do not know how to tackle question (b), for 5 points you may alternatively calculate the price, output and consumer surplus that would result if the government regulates this market by setting the price in order to rise enough revenue to pay for the investment $\bar{I}$ and the cost of production.

(b) The NPO would need to make the minimal acceptable offer for the patent, which is the sum of the investment in R&D needed, and the monopoly profit,

$$\bar{I} + \pi_M = 5.4 \textit{ million euros.}$$

Since the NPO only has 600 thousand euros, it will set a price that allows to rise the 4,8 million euros needed to pay for the patent, and also to cover the production cost. Hence it has to set the price p to solve

$$pD(p) = 40D(p) + 4800 \Leftrightarrow (p - 40)(600 - 6p) = 4800$$

This equation has two solutions, 80 *and* 60. *Obviously, the most favorable to the consumers interests is* $p^* = 60$. *At this price the NPO serves* $D(60) = 240$ *thousand units of the drug. Hence, the consumer surplus is*

$$CS^* = \frac{1}{2}(100 - p^*)D(p^*) = \frac{1}{2}(100 - 60)240 = 4800 \textit{ thousand euros.}$$

Therefore the consumer surplus increases by $4.8 - 2.7 = 2.1$ *million euros. Since the firm collects monopoly profits, el social benefit of this program is* $2.1 - 0.6 = 1.5$ *million euros.*

(b') If the government mandates the firm to develop the drug and regulates the market by setting a price that allows to rise the revenues to compensate the firm for all the costs, then p must solve the equation

$$pD(p) = 40D(p) + 5000 \Leftrightarrow (p - 40)(600 - 6p) = 5000$$

The smallest, more favorable to consumers, solutions to this equation is $\bar{p} \simeq 61.83$. *The demand, and output, and this price is*

$$D(\bar{p}) = 600 - 6\bar{p} = 600 - 6(61.83) = 229.02 \textit{ thousand units,}$$

and the consumer surplus is

$$\overline{CS} = \frac{1}{2}(100 - \bar{p})D(\bar{p}) = 4370.8 \textit{ thousand units.}$$