Microeconomics: Exercises Final Exam (June 8, 2020)

General Rules for Grading:

Exercises on Consumer Theory:

A. (15 points) Setting the consumer problem properly (budget constraints): 3 points. Calculating the MRS: 2 points. Solving the system of equations: 6 points. Dealing properly with corners: 4 points.

B. (5 points) Labor Supply: 3 points. Corner: 1 point. Graph: 1 point.

C. (10 points) Calculating the substitution effect: 6 points. Calculating the Income effect: 2 points. Understanding how this affects labor supply: 2 points.

Exercises on the Theory of the Firm:

A. (10 points) 1.5 points for each item and 1 point for presentation

B. (10 points) 5 points for the correct calculation of equilibrium prices and quantities, 1.5 point for each consumer surplus, 1 point for the profit, 1 point for the presentation.

C. (8 points) Calculating the total demand: 3 points. Calculating the price-quantity equilibrium: 3 points. Calculating the consumer surplus and profit: 1+1 points.

CT1.A worker has daily a non-labor income of M euros and has 12 hours for leisure and labor activities. Her preferences for leisure-consumption are represented by the utility function $u(h,c) = h^2c$, where h denotes the number of hours of leisure she enjoys and c her consumption measured in euros. The hourly wage is w euros.

A. (15 points) Describe the worker's problem, including its budget constraints, and calculate her demand of leisure and consumption, $h(M,w)$ and $c(M,w)$.

B. (5 points) Calculate and graph her labor supply for $M = 6$.

C. (10 points) For $w = 4$ and $M = 6$, determine the substitution and income effects over her labor supply of a 25% tax on labor income.

Solution

A. *The worker's problem is:*

$$\begin{aligned} \max_{h,c}\ u(h,c) &= h^2c \\ c + wh &\leq M + 12w \\ 0 &\leq h \leq 12,\ c \geq 0 \end{aligned}$$

The worker's marginal rate of substitution is

$$RMS(h,c) = \frac{2c}{h},$$

A solution to the worker's problem solves the system

$$\begin{aligned} \frac{2c}{h} &= w \\ c + wh &= M + 12w \end{aligned}$$

Solving the system we get

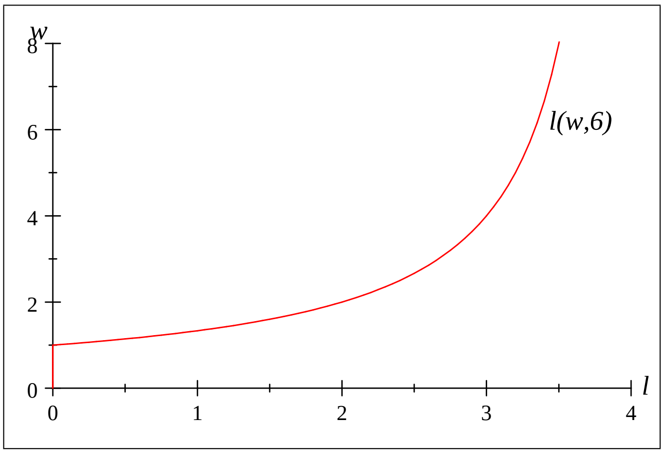
$$h^* = 8 + \frac{2M}{3w};\; c^* = \frac{M}{3} + 4w.$$

For $0 \leq h^* \leq 12$ *to hold we must have* $2M/3w \leq 4$; that is, $w \geq M/6$. *If* $w < M/6$, *the solution to the problem is* $h^* = 12$, $c^* = M$. *Therefore, the worker's demand functions are*

$$h^* = h(w,M) = \begin{cases} 8 + \frac{2M}{3w} & \textit{if } w \geq \frac{M}{6} \\ 12 & \textit{if } w < \frac{M}{6} \end{cases};\; c^* = c(w,M) = \begin{cases} \frac{M}{3} + 4w & \textit{if } w \geq \frac{M}{6} \\ M & \textit{if } w < \frac{M}{6} \end{cases}.$$

B. *The worker's labor supply is*

$$l(w,6) = 12 - h(w,6) = \begin{cases} 4 - \dfrac{4}{w} & \textit{if } w \geq 1 \\ 0 & \textit{if } w < 1. \end{cases}$$



C. *The tax on labor income amounts to wage reduction from* $w = 4$ *to* $w = (1 - 0.25)\,4 = 3$ *euros/hour. For* $w = 4$, *worker's optimal bundle is* $(h(4,6), c(6,6)) = (9,18)$ *and her utility is* $u(9,18) = 9^2\,(18)$. *To calculate the substitution effect over the labor supply we need to identify leisure and consumption that allow the worker to preserve its welfare at the smallest income. That is, the solution to the system:*

$$\begin{aligned} 9^2\,(18) &= h^2 c \\ \frac{2c}{h} &= 3, \end{aligned}$$

Solving this system we get $\tilde{h} = \sqrt[3]{972} \simeq 9.9$. *Hence the substitution effect over leisure is*

$$SE_h = 9.9 - 9 = 0.9.$$

That is, leisure increases by 0.9 *hours, and hence labor supply but that amount. Therefore, the substitution effect on labor supply is* $SE = -0.9$. *The total effect on labor supply is*

$$TE = l(3,6) - l(4,6) = \left(4 - \frac{4}{3}\right) - \left(4 - \frac{4}{4}\right) = -\frac{1}{3}.$$

Hence the income effect is

$$IE = TE - SE = -\frac{1}{3} - (-0.9) \simeq 0.56.$$

CT2.A worker has daily a non-labor income of M euros and has 18 hours for leisure and labor activities. Her preferences for leisure-consumption are represented by the utility function $u(h,c) = h^2 c$, where h denotes the number of hours of leisure she enjoys and c her consumption measured in euros. The hourly wage is w euros.

A. (15 points) Describe the worker's problem, including its budget constraints, and calculate her demand of leisure and consumption, $h(M,w)$ and $c(M,w)$.

B. (5 points) Calculate and graph her labor supply for $M = 18$.

C. (10 points) For $w = 6$ and $M = 18$, determine the substitution and income effects over her labor supply of a 33% tax on labor income.

Solution

A. The worker's problem is:

$$\begin{aligned} \max_{h,c}\ u(h,c) &= h^2 c \\ c + wh &\leq M + 18w \\ 0 &\leq h \leq 18,\ c \geq 0 \end{aligned}$$

The worker's marginal rate of substitution is

$$RMS(h,c) = \frac{2c}{h},$$

A solution to the worker's problem solves the system

$$\begin{aligned} \frac{2c}{h} &= w \\ c + wh &= M + 18w \end{aligned}$$

Solving the system we get

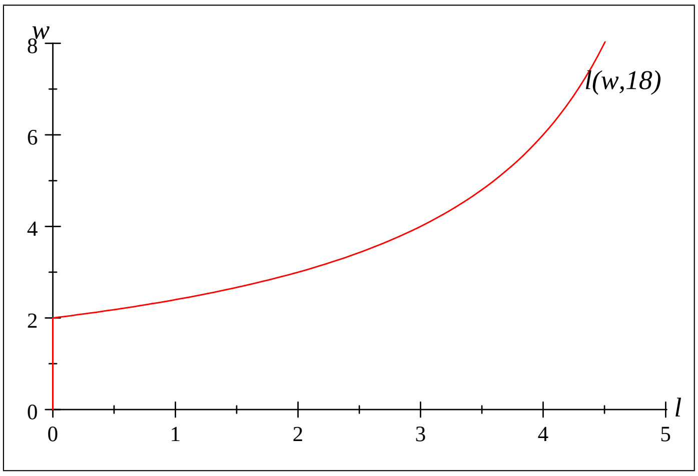
$$h^* = 12 + \frac{2M}{3w}; \; c^* = \frac{M}{3} + 6w.$$

For $0 \leq h^* \leq 18$ *to hold we must have* $2M/3w \leq 6$; that is, $w \geq M/9$. *If* $w < M/9$, *the solution to the problem is* $h^* = 18$, $c^* = M$. *Therefore, the worker's domed functions are*

$$h^* = h(w, M) = \begin{cases} 12 + \frac{2M}{3w} & \text{if } w \geq \frac{M}{9} \\ 18 & \text{if } w < \frac{M}{9} \end{cases} ; \; c^* = c(w, M) = \begin{cases} \frac{M}{3} + 6w & \text{if } w \geq \frac{M}{9} \\ M & \text{if } w < \frac{M}{9} \end{cases} .$$

B. *The worker's labor supply is*

$$l(w, 18) = 18 - h(w, 18) = \begin{cases} 6 - \dfrac{12}{w} & \text{if } w \geq 2 \\ 0 & \text{if } w < 2. \end{cases}$$



C. *The tax on labor income amounts to wage reduction from* $w = 6$ *to* $w = (1 - 0.33)\,6 = 4$ *euros/hour. For* $w = 6$, *worker's optimal bundle is* $(h(6, 18), c(6, 18)) = (14, 42)$ *and her utility is* $u(14, 42) = 14^2\,(42)$. *To calculate the substitution effect over the labor supply we need to identify leisure and consumption that allow the worker to preserve its welfare at the smallest income. That is, the solution to the system:*

$$\begin{aligned} (14)^2\,(42) &= h^2 c \\ \frac{2c}{h} &= 4 \end{aligned}$$

Solving this system we get $\tilde{h} = \sqrt[3]{4116} \simeq 16$. *Hence the substitution effect over leisure is*

$$16 - 14 = 2.$$

That is, leisure increases by $0,9$ *hours, and hence labor supply but that amount. Therefore, the substitution effect on labor supply is* $SE = -0,9$. *The total effect on labor supply is*

$$TE = l(4, 18) - l(6, 18) = 3 - 4 = -1.$$

Hence the income effect is

$$IE = TE - SE = -1 - (-2) = 1.$$

CT3.A worker has daily a non-labor income of M euros and has 18 hours for leisure and labor activities. Her preferences for leisure-consumption are represented by the utility function $u(h, c) = hc^2$, where h denotes the number of hours of leisure she enjoys and c her consumption measured in euros. The hourly wage is w euros.

A. (15 points) Describe the worker's problem, including its budget constraints, and calculate her demand of leisure and consumption, $h(M, w)$ and $c(M, w)$.

B. (5 points) Calculate and graph her labor supply for $M = 72$.

C. (10 points) For $w = 6$ and $M = 72$, determine the substitution and income effects over her labor supply of a 33% tax on labor income.

Solution

A. The worker's problem is:

$$\begin{aligned} \max_{h,c}\ u(h,c) &= hc^2 \\ c + wh &\leq M + 18w \\ 0 &\leq h \leq 18,\ c \geq 0 \end{aligned}$$

The worker's marginal rate of substitution is

$$RMS(h, c) = \frac{2c}{h},$$

A solution to the worker's problem solves the system

$$\begin{aligned} \frac{2c}{h} &= w \\ c + wh &= M + 18w \end{aligned}$$

Solving the system we get

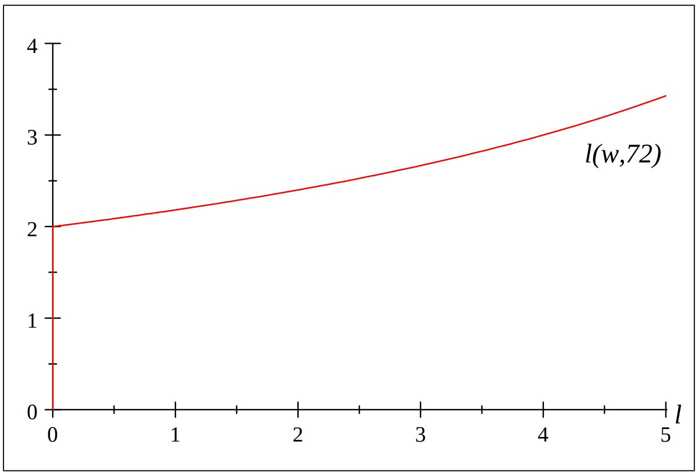
$$h^* = 6 + \frac{M}{3w};\ c^* = \frac{2M}{3} + 12w.$$

For $0 \leq h^* \leq 18$ *to hold we must have* $M/3w \leq 12$; that is, $w \geq M/36$. *If* $w < M/36$, *the solution to the problem is* $h^* = 18$, $c^* = M$. *Therefore, the worker's demand functions are*

$$h^* = h(w, M) = \begin{cases} 6 + \frac{M}{3w} & \textit{if } w \geq \frac{M}{36} \\ 18 & \textit{if } w < \frac{M}{36} \end{cases};\ c^* = c(w, M) = \begin{cases} \frac{2M}{3} + 12w & \textit{if } w \geq \frac{M}{36} \\ M & \textit{if } w < \frac{M}{36} \end{cases}.$$

B. *The worker's labor supply is*

$$l(w, 72) = 18 - h(w, 72) = \begin{cases} 12 - \dfrac{24}{w} & \textit{if } w \geq 2 \\ 0 & \textit{if } w < 2. \end{cases}$$



C. *The tax on labor income amounts to wage reduction from* $w = 6$ *to* $w = (1 - 0.33)\, 6 = 4$ *euros/hour. For* $w = 6$, *worker's optimal bundle is* $(h\,(6,72)\,, c\,(6,72)) = (10,120)$ *and her utility is* $u(10,120) = 10^2\,(120)\,.$ *To calculate the substitution effect over the labor supply we need to identify leisure and consumption that allow the worker to preserve its welfare at the smallest income. That is, the solution to the system:*

$$\begin{aligned} 10\,(120)^2 &= hc^2 \\ \frac{c}{2h} &= 4 \end{aligned}$$

Solving this system we get $\tilde{h} = \sqrt[3]{10\,(120)^2\,/8^2} \simeq 13.1$. *Hence the substitution effect over leisure is*

$$13.1 - 10 = 3.1.$$

That is, leisure increases by 3.1 *hours, and hence labor supply but that amount. Therefore, the substitution effect on labor supply is* $SE = -3.1$. *The total effect on labor supply is*

$$TE = l(4,72) - l(6,72) = 6 - 8 = -2.$$

Hence the income effect is

$$IE = TE - SE = -2 - (-3.1) = 1.1.$$

CT4.A worker has daily a non-labor income of M euros and has 12 hours for leisure and labor activities. Her preferences for leisure-consumption are represented by the utility function $u(h,c) = hc^2$, where h denotes the number of hours of leisure she enjoys and c her consumption measured in euros. The hourly wage is w euros.

A. (15 points) Describe the worker's problem, including its budget constraints, and calculate her demand of leisure and consumption, $h(M,w)$ and $c(M,w)$.

B. (5 points) Calculate and graph her labor supply for $M = 48$.

C. (10 points) For $w = 4$ and $M = 48$, determine the substitution and income effects over her labor supply of a 25% tax on labor income.

Solution

A. *The worker's problem is:*

$$\begin{array}{rcl} \max\limits_{h,c}\ u(h,c) & = & hc^2 \\ c + wh & \leq & M + 12w \\ 0 & \leq & h \leq 12,\ c \geq 0 \end{array}$$

The worker's marginal rate of substitution is

$$RMS(h,c) = \frac{c}{2h},$$

A solution to the worker's problem solves the system

$$\begin{array}{rcl} \dfrac{c}{2h} & = & w \\ c + wh & = & M + 12w \end{array}$$

Solving the system we get

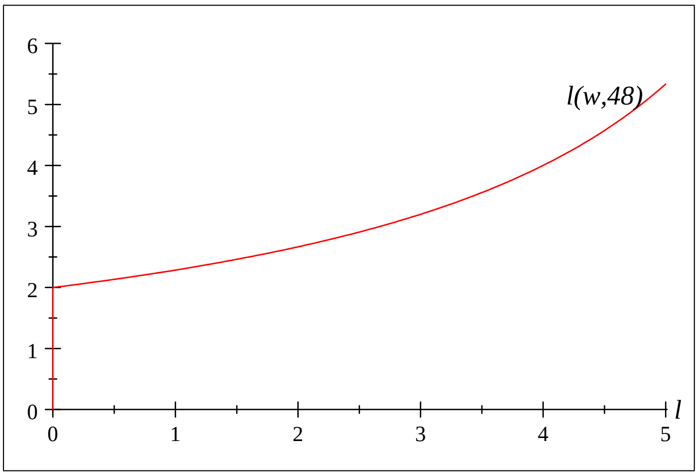
$$h^* = 4 + \frac{M}{3w};\ c^* = \frac{2M}{3} + 8w.$$

For $0 \leq h^* \leq 12$ *to hold we must have* $M/3w \leq 8$; that is, $w \geq M/24$. *If* $w < M/24$, *the solution to the problem is* $h^* = 12$, $c^* = M$. *Therefore, the worker's demand functions are*

$$h^* = h\left(w, M\right) = \begin{cases} 4 + \frac{M}{3w} & \textit{if } w \geq \frac{M}{24} \\ 12 & \textit{if } w < \frac{M}{24} \end{cases};\ c^* = c\left(w, M\right) = \begin{cases} \frac{2M}{3} + 8w & \textit{if } w \geq \frac{M}{24} \\ M & \textit{if } w < \frac{M}{24} \end{cases}.$$

B. *The worker's labor supply is*

$$l(w, 48) = 12 - h(w, 48) = \begin{cases} 8 - \dfrac{16}{w} & \textit{if } w \geq 2 \\ 0 & \textit{if } w < 2. \end{cases}$$



C. *The tax on labor income amounts to wage reduction from* $w = 4$ *to* $w = (1 - 0.25)\,4 = 3$ *euros/hour. For* $w = 4$*, worker's optimal bundle is* $(h\,(4,48)\,, c\,(4,48)) = (8,64)$ *and her utility is* $u(8,64) = 8^5$*. To calculate the substitution effect over the labor supply we need to identify leisure and consumption that allow the worker to preserve its welfare at the smallest income. That is, the solution to the system:*

$$\begin{aligned} 8^4 &= hc^2 \\ \frac{c}{2h} &= 3 \end{aligned}$$

Solving this system we get $\tilde{h} = \sqrt[3]{8^5/36} \simeq 9.7$*. Hence the substitution effect over leisure is*

$$9.7 - 8 = 1.7.$$

That is, leisure increases by 1.7 *hours, and hence labor supply but that amount. Therefore, the substitution effect on labor supply is* $SE = -1.7$*. The total effect on labor supply is*

$$TE = l(3,48) - l(4,48) = 2.66 - 4 = -1.33.$$

Hence the income effect is

$$IE = TE - SE = -1.33 - (-1.7) = 0.37.$$

TF. A firm produces a good according to the production function $F(L,K)=\sqrt{LK}$. Currently, it has 1 unit of capital, a quantity it cannot change in the short run. The input prices are $w=1$ and r.

A. (10 points) Calculate and graph the short run firm's total, average, average variable, and marginal cost functions. Also, calculate the supply of a competitive firm with this technology.

B. (10 points) Assume now that the firm monopolizes two markets in which the demands are $D_A(p)=\max\{A-p,0\}$ and $D_B(p)=\max\{A-p/2,0\}$, respectively. Calculate the price, quantity traded and consumer surplus in each market, as well as the monopoly profit, in the monopoly equilibrium with third degree price discrimination.

C. (8 points) Calculate the price, the quantity traded and the consumer surplus in each market, as well as the monopoly profit, in the monopoly equilibrium without price discrimination. ¿Who are the winners and losers relative to the situation in part B.

Solution. *(Replace A and r with the numbers in your exercise.)*

A. *Labor demand:*

$$F(L,1)=q\Leftrightarrow L(q)=q^2.$$

Cost functions:

$$C(q)=r+q^2,\ CMa(q)=2q,\ CM_e(q)=\frac{r}{q}+q,\ CM_eV(q)=q.$$

Competitive supply:

$$S(p)=p/2.$$

B. *Inverse demand functions:*

$$P_A(q)=\max\{A-q,0\},\ P_B(q)=\max\{2(A-q),0\}.$$

The monopoly equilibrium outputs in markets A and B form a solution to the system

$$\begin{aligned} A-2q_A &= 2\left(q_A+q_B\right)\\ 2A-4q_B &= 2\left(q_A+q_B\right). \end{aligned}$$

Solving we get:

$$(q_A^D,p_A^D)=(\frac{A}{10},\frac{9A}{10}),\ (q_B^D,p_B^D)=(\frac{3A}{10},\frac{14A}{10})$$

Hence the consumer surpluses and profits are

$$CS_A^D=\frac{A^2}{200},CS_B^D=\frac{9A^2}{100},\ \pi^D=\frac{A}{10}\left(\frac{9A}{10}\right)+\frac{3A}{10}\left(\frac{14A}{10}\right)-r-\left(\frac{A}{10}+\frac{3A}{10}\right)^2=\frac{7A^2}{20}-r.$$

C. *The total demand and its inverse are:*

$$D(p)=\begin{cases} 2A-\frac{3}{2}p & \text{if } p\le A\\ A-\frac{p}{2} & \text{if } A<p\le 2A\\ 0 & \text{if } p>2A \end{cases},\quad P(q)=\begin{cases} 2(A-q) & \text{if } q\in[0,\frac{A}{2}]\\ \frac{4}{3}A-\frac{2}{3}q & \text{if } q\in(\frac{A}{2},A]\\ 0 & \text{if } q>A \end{cases}.$$

Without price discrimination the monopoly output solves the equation

$$MR(q) = P'(q)q + P(q) = MC(q).$$

For $q \in (\frac{A}{2}, A]$ *we have*

$$MR(q) - MC(q) = \frac{4}{3}(A - q) - 2q < \frac{4A}{3} - \frac{10}{3}\left(\frac{A}{2}\right) = -\frac{A}{3} < 0.$$

Therefore the solution to the monopoly problem is $q < A/2$ *satisfying*

$$2A - 4q = 2q \Leftrightarrow q^{ND} = \frac{A}{3}.$$

The monopoly price is

$$p^{ND} = 2\left(A - \frac{A}{3}\right) = \frac{4A}{3}.$$

The consumer surpluses are

$$CS_A^{ND} = 0 < CS_B^D,\ CS_B = \frac{A^2}{9} > CS_B^D,$$

and the monopoly profit is

$$\pi^{ND} = \frac{4A}{3}\left(\frac{A}{3}\right) - r - \left(\frac{A}{3}\right)^2 = \frac{A^2}{3} - r < \pi^D.$$