Microeconomics-Final Exam: Test (uc3m, June 2, 2022)

Name: Group:

You have 45 minutes. Mark your answer with an "x." You get 2 points for each correct answer, -0.66 for each incorrect answer, and zero points for each question you do not answer.

1. The Pareto preferences are defined as $(x, y) \succsim_P (x', y') \Leftrightarrow \{x \geq x', y \geq y'\}$. Therefore:

☐ do not satisfy axiom $A.2$ (transitivity) ☒ do not satisfy axiom $A.1$ (completeness)
☐ satisfy axioms $A.1$, $A.2$ and $A.3$ ☐ do not satisfy axiom $A.3$ (monotonicity).

2. If the marginal rate of substitution of a consumer is $MRS(x, y) = y/4$, his monetary income is $I = 3$ euros and prices are $p_x = p_y = 1$, then his optimal consumption bundle is:

☐ $(3, 0)$ ☐ $(2, 1)$ ☐ $(1, 2)$ ☒ $(0, 3)$.

3. A consumer considers car and gas as perfect complements and wishes to consume as much as possible of both. Hence, the income and substitution effects, IE and SE, of an increase in the price of gas over its demand are:

☐ $IE = 0$, $SE < 0$ ☒ $IE < 0$, $SE = 0$ ☐ $IE < 0$, $SE < 0$ ☐ $IE > 0$, $SE = 0$.

4. A consumer's preferences over bundles in $\mathbb{R}^2_+$ satisfy $(x, y) \succsim (x', y') \Leftrightarrow x > x'$ or $\{x = x',\ y \geq y'\}$, her income is $I > 0$ and the prices are $p_x = p_y = 1$. Hence, the compensated variation of 1 euro tax per unit of x this consumer is:

☐ $2I$ euros ☐ 2 euros ☒ I euros ☐ zero.

5 and 6. In 2021 prices were $(p_x, p_y) = (1, 2)$, while they are $(p'_x, p'_y) = (2, 3)$ in 2022. All households have the same income I and preferences represented by a utility function $u(x, y) = x + \alpha y$, where $\alpha \in (0, \infty)$.

5. Identify the true CPI of a household with preference parameter $\alpha < 3/2$.

☐ $\frac{3\alpha}{2}$ ☐ $\frac{2I}{3}$ ☐ $\frac{3}{2}$ ☒ 2.

6. The statistical service calculates the CPI (of the Laspeyres type) using data of expenditures of households A, B and C, whose preference parameters are $\alpha_A = 1$, $\alpha_B = 4$ and $\alpha_C = 8$. Hence, the CPI corresponding to 2021 is

☐ $\frac{3}{2}$ ☐ $\frac{3I}{2}$ ☒ $\frac{5}{3}$ ☐ $\frac{5I}{3}$.

7. Lolita is a competitive cow that produces milk using oatmeal (O) and hay (H) according to the production function $F(O, H) = \min\{2O, \sqrt{H}\}$. Thus, as a milk producer Lolita has:

☐ decreasing returns to scale ☐ constant returns to scale
☒ diseconomies of scale ☐ decreasing marginal cost.

8, 9 and 10. An individual whose preferences are represented by the Bernoulli utility function $u(x) = \ln x$, receives a job offer that pays $w_A = 64$ if the economy accelerates its growth (A), $w_B = 16$ if it maintains its actual growth rate (B) and $w_C = 4$ if it enters into a recession (C). The probabilities of scenarios A, B and C are $p_A = 1/4, p_B = 1/2$ and $p_C = 1/4$, respectively. In her current job she receives a fixed salary $\bar{w} = 14$.

8. What is the minimal counteroffer of (fixed) wage increase by the firm in which she currently works that will discourage her from accepting this job offer?

☒ 2 ☐ 4 ☐ 8 ☐ 0

9. If the firm in which she currently works does not counteroffer, then what is the value of perfect information (VPI) for the worker?

☐ $VPI = 0$ ☐ $VPI = 4$ ☒ $4 < VPI < 5$ ☐ $VPI = 5$

10. What would be the certainty equivalent (EC) of the job offer and the value of perfect information (VPI) if the worker was risk neutral?

☐ $EC = 14, VIP = 0$ ☐ $EC = 25, VIP = 5$ ☐ $EC = 20, VIP = 5$ ☒ $EC = 25, VIP = 2,5$

11. (CANCELED) A firm produces a good with labor and capital according to the production function $F(L, K) = \sqrt{L(K-2)}$. If the input prices are $w = 1$ and $r = 4$, then the firm's conditional input demand for $q > 0$ satisfies:

☐ $L(q) = q^2$ ☐ $K(q) = q^2 + 2$ ☐ $L(q) = \frac{q}{2}$ ☐ $K(q) = 2q + 2$.

12. The production function of a firm is $F(L, K) = \sqrt{LK}$. The input prices are $w = 1$ and $r = 4$. In the short run capital is fixed to $\bar{K} = 4$. Then, the firm's short run cost functions satisfy:

☐ $C'(q) = 4q$ ☐ $C(q) = 4(q+1)^2$ ☐ $\frac{CV(q)}{q} = 4\sqrt{q}$ ☒ $\frac{C(q)}{q} = \frac{16}{q} + \frac{q}{4}$.

13. In a long run competitive equilibrium the price is:

☐ greater than the marginal cost of the most efficient technology
☐ greater than the average cost of the most efficient technology
☒ independent of the market demand
☐ smaller the larger the number of firms in the market.

14. A firm that produces a good with total costs $C(q) = 10q$ monopolizes a market in which the elasticity of demand is constant and equal to -2. Therefore, in equilibrium the price is

☐ 10 ☒ 20 ☐ 2 ☐ 5.

15. With respect to the monopoly equilibrium, the effects over the surpluses of producers (PS) and consumers (CS) of the introduction of a maximum price below the monopoly equilibrium price are:

☐ Both PS and CS may increase ☒ PS decreases and CS increases
☐ Both PS and CS may decrease ☐ PS decreases, but the effect on CS is ambiguous.

Microeconomics-Final Exam: Exercises (uc3m, June 2, 2022)

Name: Group:

You have two hours to solve all the exercises.

1. A worker has a monetary (non labor) income of 40 euros and 12 hours for labor and leisure. Her preference over leisure and food are represented by the utility function $u(h, y) = y + 48 \ln h$, where h denotes the hours of leisure she enjoys and y the number of units of food she eats. The wage is w euro/hour and the price of food is p euro/unit.

(a) (15 points) Calculate the worker's demand of leisure, $h(w, p)$, and food, $y(w, p)$, and her labor supply, $l(w, p)$.

(b) (10 points) In 2021 the price of food was $p = 1$ and the worker's wage was $w = 8$. The price of food is expected to increase by 20% , while her wage is expected to remain at $w = 8$. Determine the compensated variation of the increase of the prices of food.

Solution. An interior solution to the workers problem must solve the system:

$$\begin{aligned} MRS(h, y) &= \frac{48}{h} = \frac{w}{p} \\ wh + py &= 40 + 12w \end{aligned}$$

whose solution is

$$h(w, p) = \frac{48p}{w}, \; y(w, p) = \frac{40 + 12w}{p} - 48.$$

Since

$$\frac{48p}{w} \leq 12 \Leftrightarrow \frac{w}{p} \geq 4$$

for $w/p < 4$ the workers problem has a corner solution. Hence, the demands of leisure and food are

$$h(w, p) = \begin{cases} 12 & \text{if } \frac{w}{p} < 4 \\ \frac{48p}{w} & \text{if } \frac{w}{p} \geq 4 \end{cases}, \; y(w, p) = \begin{cases} \frac{40}{p} & \text{if } \frac{w}{p} < 4 \\ \frac{40 + 12w}{p} - 48 & \text{if } \frac{w}{p} \geq 4 \end{cases}$$

and her supply of labor is

$$l(w, p) = \begin{cases} 0 & \text{if } \frac{w}{p} < 4 \\ 12 - \frac{48p}{w} & \text{if } \frac{w}{p} \geq 4. \end{cases}$$

(b) At prices $(w, p) = (8, 1)$ *worker enjoys* $h(8, 1) = 6$ *hours of leisure, consumes* $y(8, 1) = 88$ *units of food, and works* $l(8, 1) = 6$ *hours. Her utility is* $u(6, 88) = 88 + 48 \ln 6$. *To calculate the compensated variation of the increase of the price of food from* $p = 1$ *to* $p' = 1\,(1, 2) = 12/10$, *we solve the system*

$$\begin{aligned} \frac{48}{h} &= \frac{8}{\frac{12}{10}} \\ y + 48 \ln h &= 88 + 48 \ln 6 \end{aligned}$$

identifying the cheapest bundle that at the wage $w = 8$ *of food price* $p' = 1.2$ *provides the identical utility to that of the bundle* $(6, 88)$. *The solution to this system is*

$$(\hat{h}, \hat{y}) = (\frac{36}{5}, 88 + 48(\ln 6 - \ln \frac{36}{5}).$$

To buy $\hat{y}$ *unit of food the worker needs an income equal to*

$$\left(88 + 48(\ln 6 - \ln \frac{36}{5})\right) \frac{12}{10} = 95, 1$$

Since the worker's income is

$$40 + w\left(12 - \hat{h}\right) = 40 + 8\left(12 - \frac{36}{5}\right) = 78, 4$$

the compensated variation is

$$VC = 95, 1 - 78, 4 = 16, 7.$$

2. The available technology allows to produce a good according to the production function $F(L,K) = \sqrt[3]{L(K-\alpha)}$ for $K \geq \alpha$, and $F(L,K) = 0$ if $K < \alpha$. The parameter $\alpha \in \mathbb{R}_+$ is the minimal amount of capital that is required to produce the good and is different for different firms. The input prices are $w = r = 1$.

(a) (15 points) Calculate the total, marginal and average cost functions as well as the supply function of a competitive firm with this technology.

(b) (10 points) Calculate the competitive equilibrium if the good's market demand is $D(p) = 240/p$, and there are 10 firms producing the good with the given technology, and for 5 of this firms $\alpha = 1$, while the remaining 5 firms $\alpha = 8$. Calculate the long run competitive equilibrium with free entry and no patents, identifying the total output and the number of firms with technology $\alpha = 1$ and $\alpha = 8$.

Solution: (a) We have

$$MRTS(L,K) = \frac{K-\alpha}{L}.$$

For $q > 0$ the conditional input demands solve the system

$$\begin{aligned} \frac{K-\alpha}{L} &= 1 \\ \sqrt[3]{L}\sqrt[3]{K-\alpha} &= q, \end{aligned}$$

whose solution is

$$L(q) = q^{\frac{3}{2}},\ K(q) = q^{\frac{3}{2}} + \alpha.$$

For $q = 0$, the solution to the cost minimization problem is $K = L = 0$.

Therefore, the total cost function is

$$C(q) = wL(q) + rK(q) = \begin{cases} 2q^{\frac{3}{2}} + \alpha & \text{if } q > 0 \\ 0 & \text{if } q = 0, \end{cases}$$

and for $q > 0$ the average and marginal cost functions are

$$\frac{C(q)}{q} = 2\sqrt{q} + \frac{\alpha}{q},\ C'(q) = 3\sqrt{q}.$$

To calculate the supply of the firm we solve the equation $p = C'(q)$ and check the second order condition for profit maximization,

$$C''(q) = \frac{3}{2\sqrt{q}} > 0,$$

which holds. The closing condition requires to check $p = CMa(q) \geq CMe(q)$. Solving the equation

$$C'(q) = 3\sqrt{q} = 2\sqrt{q} + \frac{\alpha}{q} = \frac{C(q)}{q}$$

we identify the level of production for which a firm begins to have diseconomies of scale, and the minimal average cost,

$$q^*_\alpha = \alpha^{\frac{2}{3}},\ AC^*_\alpha = \frac{C(q^*_\alpha)}{q^*_\alpha} = 3\sqrt[3]{\alpha}.$$

Hence, the supply of the competitive firm is

$$s(p,\alpha) = \begin{cases} 0 & \text{if } p < 3\sqrt[3]{\alpha} \\ \{0, \alpha^{\frac{2}{3}}\} & \text{if } p = 3\sqrt[3]{\alpha} \\ \dfrac{p^2}{9} & \text{if } p > 3\sqrt[3]{\alpha}. \end{cases}$$

(b) To calculate the competitive equilibrium we note that the supply of the firms with $\alpha = 1$ is positive for $p \geq 3\sqrt[3]{1} = 3$, while the supply of the firm with $\alpha = 8$, is positive for $p \geq 3\sqrt[3]{8} = 6$. Assuming the equilibrium price satisfies $p \geq 6$, the supply is

$$S(p) = 5s(p,1) + 5s(p,8) = \frac{10p^2}{9} = \frac{240}{p} = D(p)$$

and equilibrium condition is

$$D(p) = \frac{240}{p} = \frac{10p^2}{9} = S(p),$$

whose solution is $p^* = 6$. At this price the total output $Q^* = 40$, and each firm produces $q^* = 4$. Clearly prices $p \leq 6$ create an excess demand and therefore the price $p^* = 6$ is the unique equilibrium.

To calculate the long run competitive equilibrium, we note that the firms with $\alpha = 1$ reach their minimum average cost, $AC_1^* = 3\sqrt[3]{1} = 3$ for $q_1^* = 1^{\frac{2}{3}} = 1$, while the firms with $\alpha = 8$ reach their minimum average cost, $AC_8^* = 3\sqrt[3]{8} = 6$ for $q_8^* = 8^{\frac{2}{3}} = 4$. Therefore, equilibrium price in the long run is $p_L^* = 3$, on only the firms with the "technology" $\alpha = 1$ remain in the market in the long run. The demand at the price $p_L^* = 3$ is

$$Q_L^* = D(3) = \frac{240}{3} = 80$$

and the number of firms with $\alpha = 1$ in the market in the long run is

$$n_L^* = \frac{80}{q_1^*} = 80.$$

3. A firm monopolizes an electricity market in which the demand of business consumers is $D_B(p) = \max\{300 - p, 0\}$ while the demand of households is $D_H(p) = \max\{200 - p, 0\}$. The firm produces electricity with total cost $C(q) = 10^4 + 50q$ euros.

(a) (10 points) Calculate the monopoly equilibrium without price discrimination. Also, calculate the efficiency loss and the Lerner index.

(b) (10 points) Determine the outputs and prices in the monopoly equilibrium with third degree price discrimination and discuss its effects over the monopoly profits and the surplus of business consumers and households.

Solution:(a) The total demand

$$D(p) = \begin{cases} 500 - 2p & \text{if } p < 200 \\ 300 - p & 200 \leq p < 300 \\ 0 & \text{if } p \geq 300. \end{cases}$$

Hence, its inverse is

$$P(q) = \begin{cases} 300 - q & \text{if } 0 \leq q < 100 \\ 250 - \frac{q}{2} & \text{if } 100 \leq q < 500 \\ 0 & \text{if } q \geq 500. \end{cases}$$

The monopoly's revenue is $I(q) = P(q)q$, and its marginal revenue is

$$I'(q) \begin{cases} 300 - 2q & \text{if } 0 \leq q > 100 \\ 250 - q & \text{if } 100 \leq q < 500 \\ 0 & \text{if } q \geq 500. \end{cases}$$

The monopoly equilibrium is obtained by solving the equation $I'(q) = C'(q)$. Assuming $q < 100$, we have the equation

$$300 - 2q = 50,$$

whose solution is $q = 125 > 100$. Thus, in equilibrium $q > 100$, and therefore the equation $I'(q) = C'(q)$ is

$$250 - q = 50;$$

whose solution is $q = 200$. Hence the monopoly equilibrium is $q_M = 200$ and $p_M = 250 - \frac{200}{2} = 150$. The monopoly sells 150 units to business consumers and 50 units to households.

The monopoly's Lerner is

$$L = \frac{p_M - C'(q_M)}{p_M} = \frac{150 - 50}{150} = \frac{2}{3}.$$

The maximum surplus is obtained by setting $P(q) = C'(q) = 50$. Hence, the efficiency loss is

$$EL = \frac{1}{2}\left(D(50) - D(p_M)\right)(p_M - 50) = \frac{1}{2}(400 - 200)(150 - 50) = 10^4.$$

(b) With third degree price discrimination the monopoly's problem is

$$\max_{q_E, q_H \geq 0} I_E(q_E) + I_H(q_H) - C(q_E + q_H) = P_E(q_E)q_E + P_H(q_H)q_H - 50(q_E + q_H),$$

with $P_E(q) = \max\{300 - q, 0\}$ *and* $P_H(q) = \max\{200 - q, 0\}$. *Solving the system*

$$\begin{aligned} 300 - 2q_E &= 50 \\ 200 - 2q_H &= 50. \end{aligned}$$

we get $q_E = 125$, $q_H = 75$, $P_E(q_E) = 175$, $P_H(q_H) = 125$.

The surplus of the households increases since they pay a lower price and consume more electricity.

The surplus of the business consumers decreases since these consumers pay more and consume less than without price discrimination.

The monopoly's profit increases since the revenue increases and the cost remains the same.