Microeconomics-Final Exam: Test (uc3m, June 2021)

Name: Group:

You have 30 minutes. Mark your answer with an "x." You get 2 points for each correct answer, -0.66 for each incorrect answer, and zero points for each question you do not answer.

1. It is known that a consumer's preferences $\succeq$ satisfy axioms $A.1$, $A.2$ and $A.3$, and that $A = (0,2) \succ B = (1,1)$. Therefore, we can infer the following relation between these bundles and $C = (1,2)$:

☒ $C \succ B$ ☐ $C \backsim A$ ☐ $C \backsim B$ ☐ $C \succ A$.

2. If a consumer's preferences satisfy monotonicity (axiom $A.3$), then her indifference curves

☐ are convex ☒ are decreasing ☐ do not cross ☐ are continuous.

3. Which axiom is not satisfied by the preferences represented by the utility function $u(x,y) = 2\sqrt{x} - y$?

☐ A.1 (completeness) ☐ A.2 (transitivity) ☒ A.3 (monotonicity) ☐ A.4 (convexity).

4. The demand at prices $(p_x, p_y) \gg 0$ of a consumer who prefers lexicographically good x to good y is:

☐ $x(p_x,p_y,I) = y(p_x,p_y,I) = \dfrac{I}{2(p_x+p_y)}$ ☒ $x(p_x,p_y,I) = \dfrac{I}{p_x}$, $y(p_x,p_y,I) = 0$

☐ $x(p_x,p_y,I) = \dfrac{I}{2p_x}, y(p_x,p_y,I) = \dfrac{I}{2p_y}$ ☐ indeterminate.

In 2019 prices were $(p_x, p_y) = (1,2)$, and households A and B consumed the bundles $(x_A, y_A) = (2,1)$ and $(x_B, y_B) = (4,4)$, respectively. In 2021 prices are $(p'_x, p'_y) = (3,2)$. Assume that the bundles of households A and B are used to identify the consumption bundle of the representative agent. Then:

5. the (plutocratic) consumer price index (CPI), that is, the CPI resulting from the version of the Laspeyres formula used by the Instituto Nacional de Estadística (Spain), is:

☐ 3/2 ☐ 11/6 ☒ 7/4 ☐ 2.

6. while a democratic CPI is:

☐ 3/2 ☒ 11/6 ☐ 7/4 ☐ 2.

7. The risk premium of the lottery that pays $x = (0,8)$ with probabilities $p = (1/4, 3/4)$ for an individual whose preferences are represented by the Bernoulli utility function u is 2. If the preferences of another individual are represented by the Bernoulli utility function $v = 2u$, then her certainty equivalent of the lottery is:

☐ 2 ☒ 4 ☐ 6 ☐ 0

8. The expected utility and risk premium of the lottery l that pays $x = (0, 2, 4)$ with probabilities $p = (3/8, 1/2, 1/8)$ for a consumer whose preferences over lotteries are represented by the Bernoulli utility function $u(x) = x^2$ are:

☐ $Eu(l) = 2,\ RP(l) = 1$	☐ $Eu(l) = 2,\ RP(l) = 1/2$
☒ $Eu(l) = 4,\ RP(l) = -1/2$	☐ $Eu(l) = 4,\ RP(l) = -1$.

9. An individual whose preferences are represented by the Bernoulli utility function $u(x) = x$ receives a job offer the pays wages depending on whether the economy accelerates its growth (A), maintains its actual growth rate (B) or enters into a recession (C). In each of these scenarios the wages are 24, 8, and 0, respectively. The probabilities of scenarios A, B and C are $p_A = 1/4, p_B = 1/2$ and $p_C = 1/4$, respectively. Currently, the individual has a job that pays a fixed salary equal to 10. Therefore, the value of perfect information for this individual is:

☒ $\frac{7}{2}$ ☐ $\frac{7}{4}$ ☐ $\frac{5}{2}$ ☐ $\frac{5}{4}$.

10. Lolita is a competitive cow that produces milk using oats (A) and hay (H) according to the production function $F(O, H) = O + \sqrt{H}$. Therefore, as a milk producer Lolita has

☐ decreasing marginal cost	☐ constant returns to scale
☒ diseconomies of scale	☐ a concave total cost function.

11. A firm produces a good with labor and capital according to the production function $F(L, K) = \min\{\sqrt{L}, 2K\}$. If the prices of labor and capital are $w = 1$ and $r = 4$, respectively, then for $q > 0$ its cost functions satisfy:

☒ $MC(q) = 2\,(q + 1)$ ☐ $C(q) = 4 + q^2$ ☐ $AC(q) = \frac{4}{q} + 3$ ☐ $MC(q) = 4 + 2q$.

12. If a firm's cost functions satisfy $AC(q) > MC(q)$ for all q, then the firm has:

☒ decreasing average cost	☐ constant returns to scale
☐ diseconomies of scale	☐ increasing marginal cost.

13. The two existing technologies, A and B, allow to produce a good with costs $C_A(q) = 3q^2 + 12q + 3$ and $C_B(q) = 5q^2 + 20$, respectively. If the market demand is $D(p) = \max\{36 - p, 0\}$, then in the long run competitive equilibrium (with free entry and free use of technologies A and B) the price, p_L^*, and the number of firms of each type, (n_A^*, n_B^*), satisfy:

☐ $p_L^* = 18 = n_A^* + n_B^*$	☒ $p_L^* = 18 = n_A^*,\ n_B^* = 0$
☐ $p_L^* = 20, n_A^* = 0,\ n_B^* = 16$	☐ $p_L^* = 20, n_A^* + n_B^* = 16$.

14. If a monopoly produces the good with zero cost and the market demand is $D(p) = \max\{1 - \frac{p}{4}, 0\}$, then the monopoly's Lerner index is

☐ $L = 0$ ☐ $L = \frac{1}{4}$ ☐ $L = \frac{1}{2}$ ☒ $L = 1$.

15. A monopoly produces the good with cost $C(q) = q^2$, and the market demand is $D(p) = \max\{12 - p, 0\}$. The effect of a price cap of $\bar{p} = 8$ euros/unit is:

☐ an increase in the deadweight loss of the monopoly	☐ a decrease of the total surplus
☒ an increase of the consumers surplus	☐ a decrease of the monopoly's output.

Microeconomics-Final Exam: Exercises (uc3m, June 2021)

Name: Group:

You have two hours to solve all the exercises.

1. The preferences of a consumer over energy (x, in kilowatts hour) and other goods (y, in euros) are represented by the utility function $u(x,y) = xy^2$. Since y is measured in euros, $p_y = 1$. Let us denote the energy price by p_x (euros/kWh) and the consumer's income by I (euros).

(a) (10 points) Calculate the consumer's ordinary demand functions , $x(p_x, I)$ and $y(p_x, I)$. (Verify the existence of corner solutions or show that there are not any.)

For $I > 0$, the consumer, by buying the bundle $(\bar{x}, \bar{y}) = (I/(2p_x), I/2) \gg 0$, assures a positive utility, $u(\bar{x}, \bar{y}) = I^2/(4p_x) > 0$. Therefore the utility of the optimal bundle (x^*, y^*) satisfies $u(x^*, y^*) \geq u(\bar{x}, \bar{y}) > 0 = u(0, y) = u(x, 0)$; that is, the solution to the consumer's problem is interior.

We have

$$MRS(x,y) = \frac{y}{2x}.$$

For $I > 0$ the solution to the consumer's problem (which we know is interior) solves the system

$$\begin{aligned} \frac{y}{2x} &= \frac{p_x}{1} \\ xp_x + yp_y &= I. \end{aligned}$$

Solving this system we get the ordinary demand functions:

$$\begin{aligned} x(p_x, I) &= \frac{I}{3p_x} \\ y(p_x, I) &= \frac{2I}{3}. \end{aligned}$$

(b) (15 points) If the consumer's income is $I = 12$ and $p_x = 1$, calculate income and substitution effects over the demand of energy (x) and the revenue of a 1 euro tax per kilowatt-hour of energy consumption.

We calculate the optimal bundle at price $p_x = 1$ and income $I = 12$:

$$x_A = x(1, 12) = \frac{12}{3\,(1)} = 4;\; y_A = y(1, 12) = \frac{2\,(12)}{3} = 8.$$

The consumer's utility for this bundle is $u_A(4, 8) = 4\,(8)^2 = 256$. With the tax the effective price of energy is $p'_x = 2$ and the optimal bundle is

$$x_C = x(2, 12) = \frac{12}{3\,(2)} = 2;\; y_C(2, 12) = \frac{2\,(12)}{3} = 8,$$

and the tax revenue is

$$R = 1 \times x_C = 2$$

To calculate the substitution effect SE we identify the bundle (x_B, y_B) that solves the system:

$$\begin{aligned} \frac{y_B}{2x_B} &= \frac{p'_x}{1} = 2 \\ x_B\,(y_B)^2 &= 256. \end{aligned}$$

The solution is $(x_B, y_B) = (2\sqrt[3]{2}, 8\sqrt[3]{2})$. Hence,

$$SE = x_B - x_A = 2\sqrt[3]{2} - 4 = -2\left(2 - \sqrt[3]{2}\right)$$

and the income effect is

$$IE = TE - SE = (2 - 4) + 2\left(2 - \sqrt[3]{2}\right) = -2\left(\sqrt[3]{2} - 1\right)$$

(c) (5 points) Assuming that the consumer's income is $I = 12$ and taking $(p_x, p_y) = (1, 1)$ as the prices in the base period, and $(p'_x, p'_y) = (2, 1)$ as the prices in the current period, calculate calculate the consumer's true price index.

In part (b)we calculated the optimal bundle at prices $(p_x, p_y) = (1, 1)$, $(x_A, y_A) = (1, 4)$. Also, the calculations in part (b) identify $(x_B, y_B) = (2\sqrt[3]{2}, 8\sqrt[3]{2})$, the cheapest bundle that allows the consumer to maintain the utility $u_A(4, 8) = 256$ at prices $(p'_x, p'_y) = (2, 1)$. Hence the consumer's true price index, CPI^*, is

$$CPI^* = \frac{p'_x\left(2\sqrt[3]{2}\right) + p'_y\left(8\sqrt[3]{2}\right)}{12} = \frac{2\left(2\sqrt[3]{2}\right) + \left(8\sqrt[3]{2}\right)}{12} = \sqrt[3]{2} \simeq 1.26.$$

2. A firm produces a good with labor L and capital K according to the production function $F(L,K) = \sqrt{L} + K$. The prices of labor and capital are $w = 1$ and $r = 3$, respectively.

(a) (10 points) Calculate the firm's long run supply .

The conditional input demands solve the system

$$\begin{aligned} MRTS(L,K) &= \frac{w}{r} \Leftrightarrow \frac{1}{2\sqrt{L}} = \frac{1}{3} \\ F(L,K) &= q \Leftrightarrow \sqrt{L} + K = q, \end{aligned}$$

Hence,

$$L(q) = \begin{cases} q^2 & \text{if } q \leq 3/2 \\ \frac{9}{4} & \text{if } q > 3/2. \end{cases}, \quad K(q) = \begin{cases} 0 & \text{if } q \leq 3/2 \\ q - \frac{3}{2} & \text{if } q > 3/2. \end{cases}$$

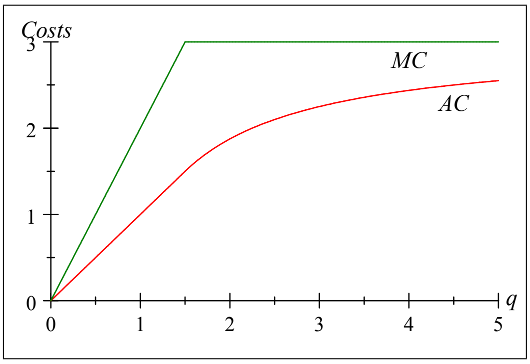
The cost function is

$$C(q) = L(q) + rK(q) = \begin{cases} q^2 & \text{if } q \leq 3/2 \\ \frac{9}{4} + 3\left(q - \frac{3}{2}\right) & \text{if } q > 3/2. \end{cases}$$

The marginal and average cost functions are

$$MC(q) = \begin{cases} 2q & \text{if } q \leq 3/2 \\ 3 & \text{if } q > 3/2 \end{cases}, \quad AC(q) = \begin{cases} q & \text{if } q \leq 3/2 \\ 3 - \frac{9}{4q} & \text{if } q > 3/2 \end{cases}$$

As we see, the firm has diseconomias of scale since $C''(q) = CMa'(q) \geq 0$, and $MC(q) > AC(q)$ for all q. The graph shows the functions MC and AC.



Hence the firm's supply is its marginal cost curve. (Since $C''(q) \geq 0$ and $CMa(q) > CMe(q)$, the second order and shutting down conditions hold.) In particular, for $p > 3$ the firm's profit maximization problem has no solution – the firm would supply infinite).

$$s(p) = \begin{cases} \frac{p}{2} & \textit{if } p < 3 \\ [3/2, \infty) & \textit{if } p = 3 \\ \textit{indeterminate} & \textit{if } p > 3. \end{cases}$$

(b) (10 points) Currently (and in the short run) the firm's capital is $\bar{K} = 1$. Calculate firm' short run supply.

To calculate the firm's supply we note that $F(L, 1) = 1$ and therefore the short run conditional demand of labor is

$$\bar{L}(q) = \begin{cases} 0 & \text{if } q < 1 \\ (q-1)^2 & \text{if } q \geq 1. \end{cases}$$

Therefore, the short run cost function is

$$\bar{C}(q) = r\bar{K} + \bar{L}(q) = \begin{cases} 3 & \text{if } q < 1 \\ 3 + (q-1)^2 & \text{if } q \geq 1 \end{cases}$$

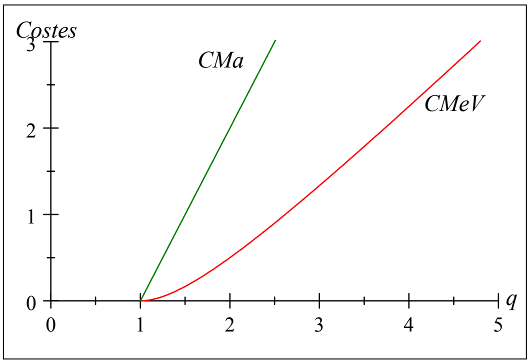
and the marginal and average variable cost functions are

$$\overline{MC}(q) = \begin{cases} 0 & \text{if } q < 1 \\ 2\,(q-1) & \text{if } q \geq 1 \end{cases}, \quad \overline{AVC}(q) = \begin{cases} 0 & \text{if } q < 1 \\ \dfrac{(q-1)^2}{q} & \text{if } q \geq 1 \end{cases}.$$

As we see,,the firm has diseconomias of scala since $C''(q) = CMa'(q) \geq 0$, and $MC(q) > AC(q)$ for all q, since

$$\overline{MC}(q) - \overline{AVC}(q) = 2\,(q-1) - \frac{(q-1)^2}{q} = (q-1)\left(\frac{q+1}{q}\right) > 0.$$

The graph shows the functions $\overline{MC}$ and $\overline{AVC}$.



Again, since the firm has diseconomies of scale, the firm's short supply is its marginal cost curve – obviously, the second order and shutting down conditions hold for profit maximization hold; that is,

$$p = 2\,(q-1) \Leftrightarrow \bar{s}(p) = \frac{p}{2} + 1.$$

3. (20 points) A firm owns a patent of a drug whose demand is $D^w(p) = \max\{10-p, 0\}$ in *westeros* and $D^e(p) = \max\{2-p, 0\}$ in *easteros*. The firm produces the drug at zero cost. Calculate the monopoly equilibrium with and without (third degree) price discrimination. Who benefits (consumers of either westeros or easteros, the monopolist) from price discrimination?

The inverse demands are

$$p^w(q) = \max\{10-q, 0\},\ p^e(q) = \max\{2-q, 0\},$$

respectively. Therefore, with third degree price discrimination the monopoly supplies the quantities satisfying the system

$$\begin{aligned} 10 - 2q^w &= 0 \\ 2 - 2q^e &= 0, \end{aligned}$$

whose solution is $q^w = 5$ *and* $q^e = 1$*. The equilibrium prices are* $p^w = 5$ *and* $p^e = 1$*. The consumer surpluses and the monopoly profits are*

$$EC^w = 12.5,\ EC^e = 0,5,\ \pi = 26.$$

To calculate the monopoly equilibrium without price discrimination we calculate the aggregate demand. For prices above 10 the demand is zero; for prices between 2 and 10 there is a positive demand only in westeros; and for prices below 2 there is a positive demand in both markets. Hence, the aggregate demand is

$$p(q) = \begin{cases} 0 & \text{if } q > 12 \\ 6 - \dfrac{q}{2} & \text{if } 8 < q \leq 12 \\ 10 - q & \text{if } q \leq 8. \end{cases}$$

The monopolist revenue is $I(q) = p(q)q$ *and its marginal revenue is*

$$IMa(q) = \begin{cases} 0 & \text{if } q > 12 \\ 6 - q & \text{if } 8 < q \leq 12 \\ 10 - 2q & \text{if } q \leq 8. \end{cases}$$

The output in the monopoly equilibrium solves the equation $IMa(q) = CMa(q)$*. Since* $IMa(q) = 6 - q < 0$ *if* $8 < q \leq 12$*, the solution to the monopoly problem is* $q \leq 8$ *and hence solves the equation*

$$10 - 2q = 0$$

which solution is $q_M = 5$*. The equilibrium price is* $p_M = 5$*. The consumer surpluses and the monopoly profits without price discrimination are:*

$$\overline{EC}^w = 12.5,\ \overline{EC}^e = 0,\ \bar{\pi} = 25.$$

We see that the consumers of westeros are equally well off with and without price discrimination, while both the monopolist and the consumers of easteros worse off than with price discrimination.