PUBLIC GOODS AND EXTERNALITIES: AN EXAMPLE

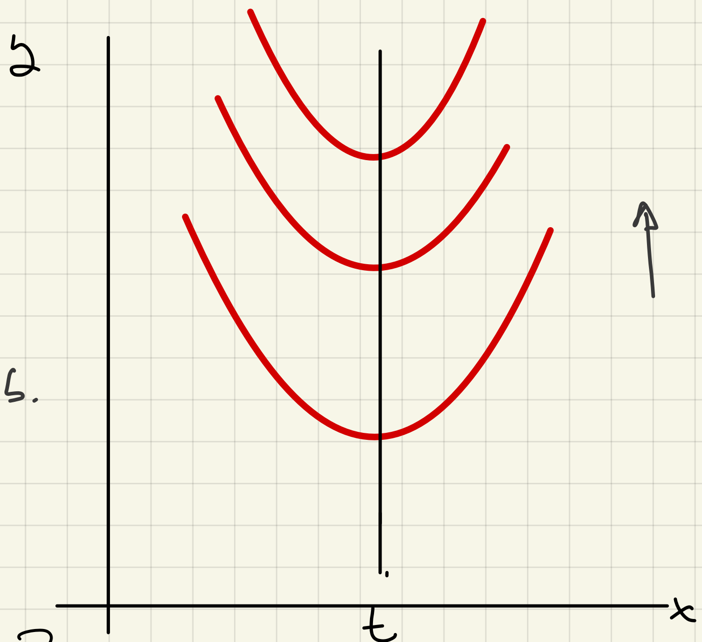
Ann and Bob share an apartment. Central heating is provided free of charge.

Their preferences for room temperature (x) and income (y) are represented by utility functions

$$u_i(x,y) = y - \alpha_i (t_i - x)^2,$$

where $t_A = 25$, $\alpha_A = 3/2$

$t_B = 20$, $\alpha_B = 1$.



Thus, the apartment's temperature is a public good to Ann and Bob.

At which temperature should be set the apartment's thermostat?

Let us try and identify "good outcomes". To begin,

Let us set the temperature to some intermediate value $t \in [t_B, t_A]$,

$$\text{e.g.,} \quad x = 21^\circ.$$

Is this temperature Pareto optimal?

Since

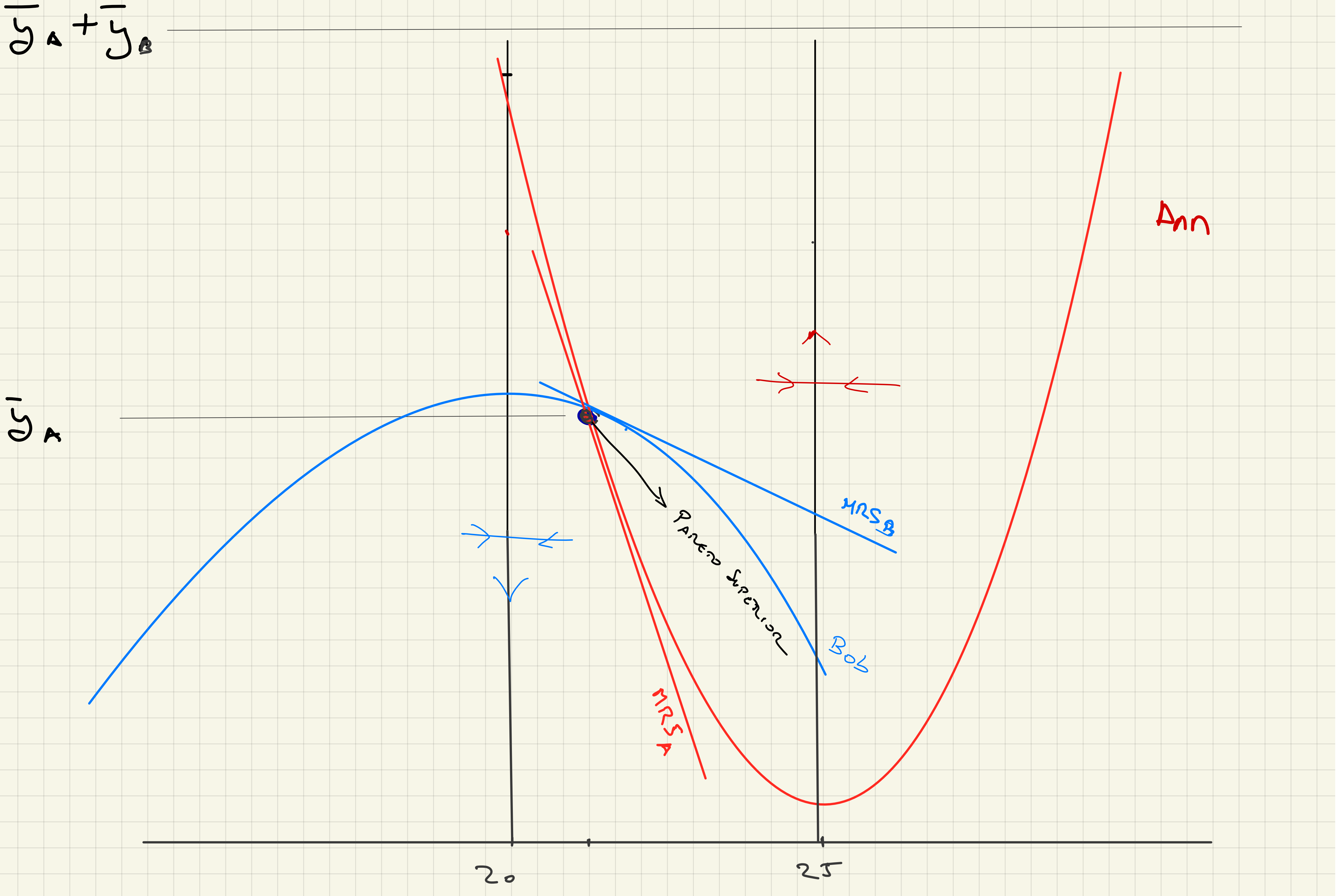
$$MRS_i(x,y) = 2\alpha_i(t_i - x),$$

we have

$$MRS_A(21,y) = 75 - 3(21) = 12$$

$$MRS_B(21,y) = 40 - 2(21) = -2.$$

How to interprete these numbers?



$\bar{y}_A + \bar{y}_B$
Ann
$\bar{y}_A$
MRS B
Pareto superior
Bob
MRS A
20
25

Ann proposes to increase the temperature by 1°, and offers Bob 5 euros as compensation

Would Bob accept ?

$$u_B(\bar{y}_B, 21) = \bar{y}_B - (20-21)^2 = \bar{y}_B - 1.$$

$$u_B(\bar{y}_B + 5, \underline{22}) = \bar{y}_B + 5 - (20-22)^2 = \bar{y}_B + 1.$$

Woud Ann make such offer?

$$u_A(\bar{y}_A, 21) = \bar{y}_A - \frac{3}{2}(25-21)^2 = \bar{y}_A - 24.$$

$$u_A(\bar{y}_A - 5, 22) = \bar{y}_A - 5 - \frac{3}{2}(25-22)^2 = \bar{y}_A - 18.5$$

Both are better off !

Note $\qquad MRS_A(21) + MRS_B(21) = 12 - 2 = 10 > 0$

They should raise the temperature so long as

$$MRS_A(x) + MRS_B(x) > 0$$

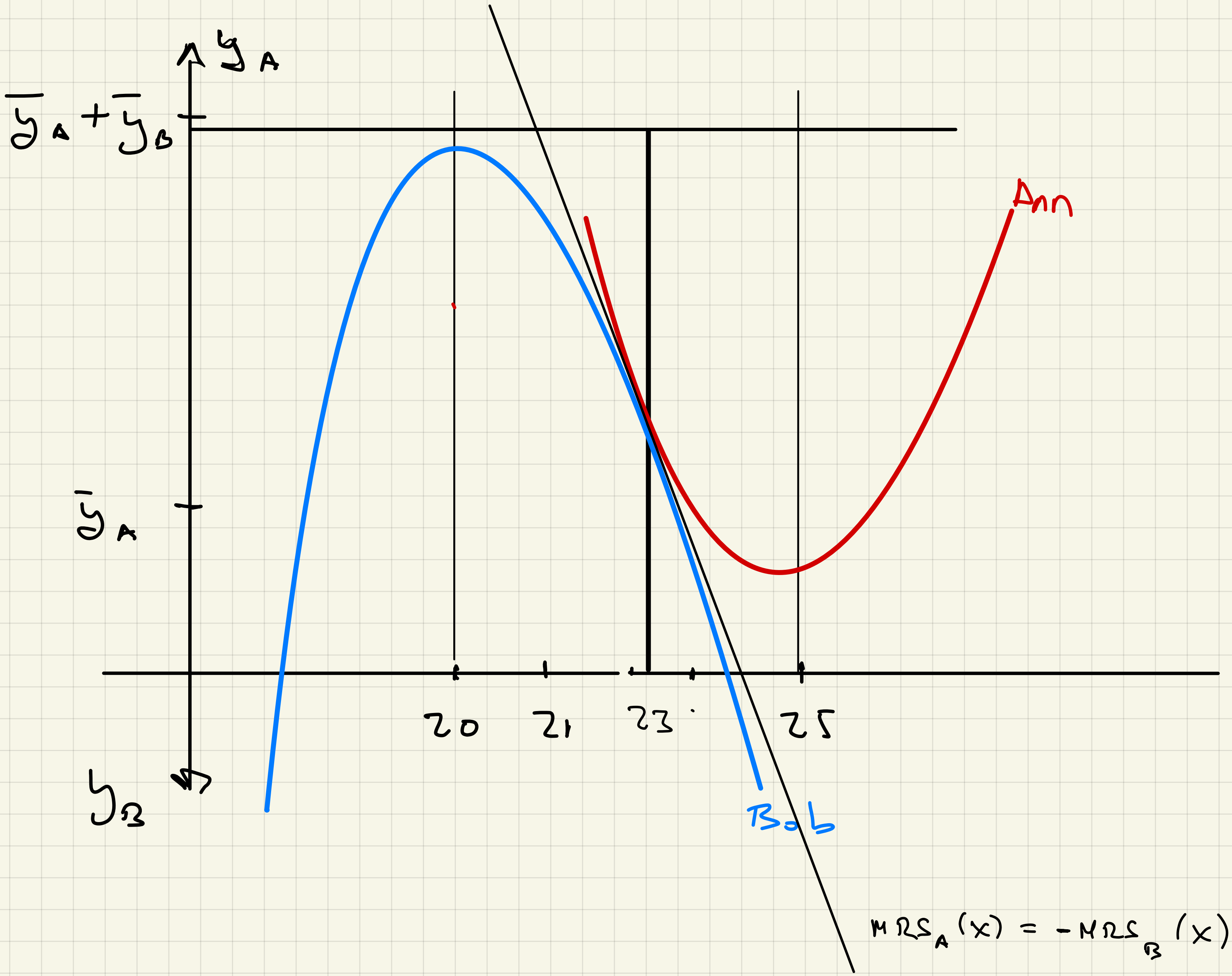
Likewise, if $MRS_A(x) + MRS(B) < 0$, they can both improve by reducing the temperature (and accord some compensations).

<u>PO</u> requires:

$$MRS_A(x) + MRS_B(x) = 0$$

$$75 - 3x + 40 - 2x = 0$$

i.e., $\qquad x^* = \frac{115}{5} = \underline{23^\circ}$



y_A
$\bar{y}_A + \bar{y}_B$
$\bar{y}_A$
y_B
Ann
Bob
20
21
23
25
$MRS_A(x) = -MRS_B(x)$

... but how is the apartment's temperature decided?

And how to solve this problem?

BARGAINING

Set $x^* = 23°$. Bargain over the gains to be have.

Assume Ann owns the apartment and has the right to set the temperature.

Absent interactions:

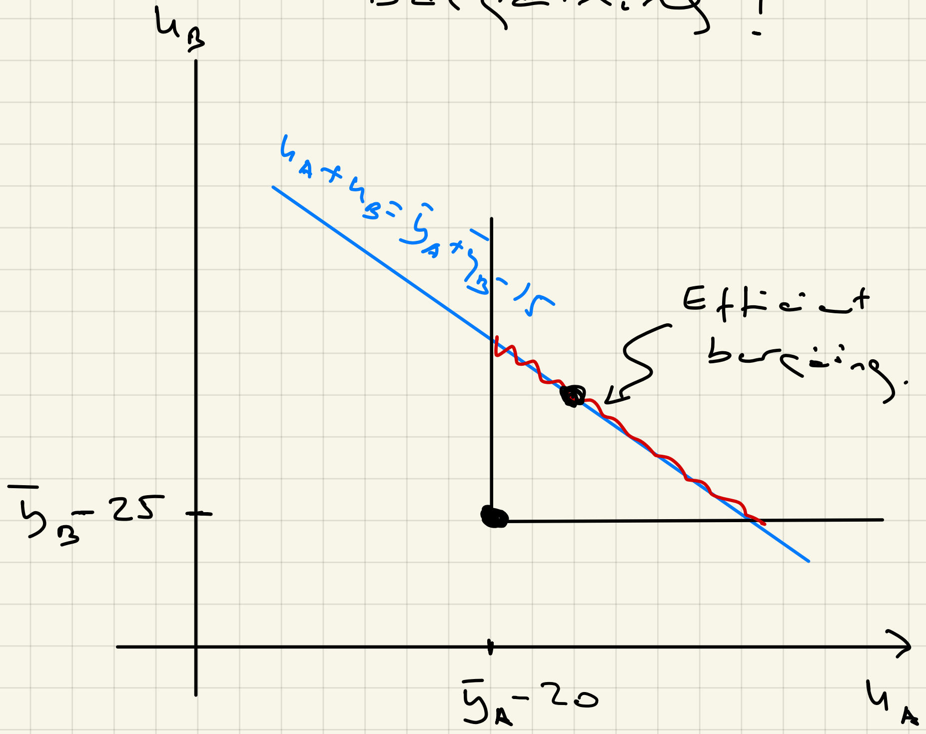
$\bar{x} = 25°$; $\bar{u}_A = \bar{y}_A$, $\bar{u}_B = \bar{y}_B - 25$.

Sadly, for $x = 23$,

$$u_A + u_B = \bar{y}_A - \frac{3}{2}(25-23)^2 + \bar{y}_B - (20-23)^2$$

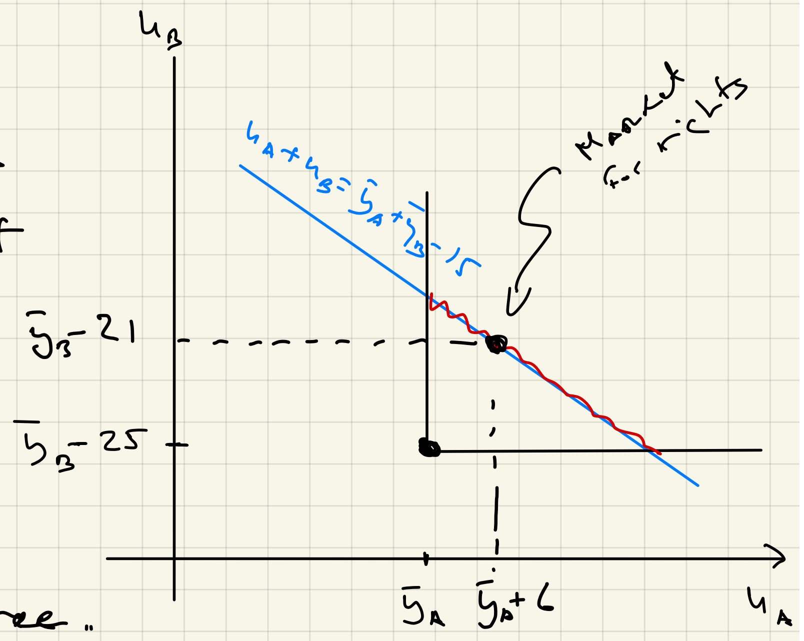
$$= \bar{y}_A + \bar{y}_B - 15 > \bar{y}_A + \bar{y}_B - 25 = \bar{u}_A + \bar{u}_B.$$

Perhaps a compromise may be reached by bargaining!



A MARKET SOLUTION

Assume instead that starting from Ann's ideal temperature (25°), a market is created where Ann and Bob may supply (demand) "rights" to lower the temperature.



p : price of lowering the temperature 1 degree.

Ann
$$\max_{r} \ \bar{y}_A + pr - \frac{3}{2}\left[25 - (25 - r)\right]^2 \Rightarrow r_A(p) = \frac{p}{3}$$

Bob
$$\max_{r} \ \bar{y}_B - pr - \left[20 - (25 - r)\right]^2 \Rightarrow r_B(p) = 5 - \frac{p}{2}$$

Market Clearing. $\frac{p}{3} = 5 - \frac{p}{2} \iff p^* = 6 \ ; \ r^* = 2.$

Market Outcome: $x^* = 23°$; $y_A^* = \bar{y}_A + 12$, $u_A^* = \bar{y}_A + 6$.

$y_B^* = \bar{y}_B - 12$, $u_B^* = \bar{y}_B - 21$.

Exercises:

(1) If there is another apartment's resident, Conrad, whose preferences parameters are $\alpha_c = 1$, $t_c = 22$, what is the apartment's optimal temperature?

(2) If the cost of maintaining the temperature at t degrees is

$$c(t) = \frac{t^2}{4}$$

what is the apartment's optimal temperature?

(3) What would be the temperature if Ann, Bob and Conrad vote, and the thermostat is set at the median temperature?

Exercise 5. A certain restaurant is known for refusing to give separate checks to customers: After a group has ordered and eaten together at this restaurant, the group is presented with a single check for the entire amount the group has eaten. It has been suggested that the restaurant does this because with a single check those who dine in groups will be likely to simply divide the charge equally, each person paying the same amount irrespective of who ordered what; and that diners, knowing that they will ultimately divide the charge equally, will order more than they would have ordered had each paid for his own order. Analyze this suggestion in the following setting: There are n diners in a group. Each has a utility function of the form $u(x_i, y_i) = y_i + \ln x_i$, where x_i is the amount of food that i ordered and y_i is the amount of money that i has after leaving the restaurant. The restaurant charges p dollars per unit of food, and the restaurant's profit is an increasing function of the amount of food it sells at the price p. Each diner knows when he orders his food that the group will divide the check equally. Compare the outcome under this arrangement with that resulting when each diner pays for his own order. What are the restaurant's profit and the diners' welfare in each case?

Restaurant Example (Exercise 2.5)

① Separate Checks

$$\max_{x \geq 0} \quad \bar{y}_i - px + \ln x$$

F.O.C. $\quad -p + \frac{1}{x} = 0 \Rightarrow \boxed{x_i(p) = \frac{1}{p}}, \quad \boxed{n\, x_i(p) = \frac{n}{p}}$

② Common Equally Divided Check.

$$\max_{x_i} \quad \bar{y}_i - \frac{p}{n}\left(\sum_{j \neq i} x_j + x_i\right) + \ln x_i$$

$$-\frac{p}{n} + \frac{1}{x_i} = 0 \Rightarrow \boxed{\tilde{x}_i(p) = \frac{n}{p}.} \quad \boxed{n\,\tilde{x}_i(p) = \frac{n^2}{p}}$$

Yes, more food is consumed under this scheme!

Exercise 6. Alpha and Beta have just terminated their marriage. They hold no animosity toward one another, and each is concerned about the welfare of their only child, little Joey. Their preferences are described by the utility functions $u_A(x, y_A) = x^{\alpha} y_A$ and $u_B(x, y_B) = x^{\beta} y_B$, where y_A and y_B denote the number of dollars "consumed" directly by the respective parents in a year, and x denotes the number of dollars per year consumed by Joey. Joey's consumption is simply the sum of the support contributions by Alpha and Beta, $s_A + s_B$. These contributions will be voluntary: Neither parent has sought a legal judgment against the other. Assume throughout that $\alpha = 1/4$ and $\beta = 1/3$, and that Alpha's and Beta's annual incomes are \$48,000 is \$40,000, respectively.

(a) Suppose that Alpha is unwilling to contribute anything toward Joey's support, so that Beta must provide. What levels of x and y_B will Beta choose?

(b) Actually, Alpha, as well as Beta, is willing to contribute to Joey's support. What will be their equilibrium contributions to Joey's support?

(c) Find an allocation of the parents' incomes that will make them both happier than the one in (b).

(d) Determine the conditions that characterize the Pareto optimal allocations.

(e) Indicate some of the difficulties that a neutral third party (e.g., a judge) might encounter in attempting to implement some method for arriving at a Pareto optimal allocation of the parents' incomes.

(a)

$$\left.\begin{array}{l}\max\limits_{x\in[0,40]} \; x^{1/3}(40-x)\end{array}\right\} \quad \text{FOC: } \frac{x^{-2/3}}{3}(40-x) = x^{1/3}$$

$$\iff 40 = 4x \iff \boxed{x^* = 10}$$

(b)

$$A: \left.\max\limits_{z\in[0,48]} \; (z+z_B)^{1/4}(48-z)\right| \quad \text{FOC: } \frac{(z+z_B)^{-3/4}}{4}(48-z) = (z+z_B)^{1/4}$$

$$\iff z_A = \frac{48-4z_B}{5}.$$

$$B: \left.\max\limits_{z\in[0,40]} \; (z+z_A)^{1/3}(40-z)\right\} \quad \text{FOC: } \frac{(z+z_A)^{-2/3}}{3}(40-z) = (z+z_A)^{1/3}$$

$$\iff z_B = \frac{40-3z_A}{4}$$

<u>EQUILIBRIUM</u>: $z_A^* = 4,\ z_B^* = 7 \Rightarrow \boxed{x_v^* = 11}$. $u_A^* = 11^{1/3}(48-4) \simeq 80.13$

$u_B^* = 11^{1/3}(40-7) \simeq 73.39$

(c) $\hat{z}_A = 4.5,\ \hat{z}_B = 7.5 \Rightarrow \hat{x} = 12$

$\hat{u}_A = 80.96, \quad \hat{u}_B = 74.4$

(d)

$$MRS_A(x, y_A) = \frac{y_A}{4x}$$

$$+$$

$$MRS_B(x, y_B) = \frac{y_B}{3x}$$

$$c'(x) = 1$$

Optimality:

$$\boxed{\frac{y_A}{4} + \frac{y_B}{3} = x}$$

Feasibility:

$$\boxed{x + y_A + y_B = 88}$$

(e) How to elicit the information about the preferences of A and B.

<u>Lindahl Equilibrium:</u> Prices: $p_A = p \in [0,1]$, $p_B = 1-p$

(Hence $p_A + p_B = 1$ – Feasibility)

$$MRS_A(x, y_A) = p$$

$$MRS_A(x, y_B) = 1-p$$

$$y_A = 48 - px$$

$$y_B = 40 - (1-p)x$$

$$p^* = 2/3, \quad x^* = \frac{72}{5} = 14.4$$

$$y_A = \frac{192}{5} = 38.4$$

$$y_B = \frac{176}{5} = 35.2$$

Exercise 7. 100 men have access to a common grazing area. Each man can choose to own either no cows, one cow, or two cows to provide milk for his family. The more cows the grazing land is required to support, the lower is each cow's yield of milk; specifically, a man who owns x_i cows obtains

$$Q_i = (250 - x)x_i$$

quarts of milk per year, where $x = \sum_{j=1}^{100} x_j$ is the total number of cows in the grazing land supports. Each man wants to obtain as much milk as he can, but no man has the resources to own more than two cows.

(a) How many cows do you predict each man will own? (Justify your answer.)

(b) Assume that the men can make transfers of milk among themselves. Is your prediction in (a) Pareto efficient for the 100 men? If so, verify it. If not, find a pattern of cow ownership and transfer payments that yields a Pareto optimal allocation of milk to the men that makes everyone strictly better off than in (a).

(c) Now suppose that there are only two men whose cows share a common grazing area, and that each cowls daily yield of milk, in quarts, depends on how many cows in total are grazing according to the following table

Total cows grazing:	1	2	3	4
Each cowls daily yield:	8	5	3	2

Which allocations of milk are Pareto efficient and individually rational (i.e., such that each man is at least as well off as he would be by "unilateral" action)? What are all the patterns of cow ownership and transfer payments that will support these allocations? Determine all the core allocations of milk to the two men.

(d) For the situation described in part (c), answer all the questions posed in part (a).

(a)

$$Q_i(x_i, X_-) = (250 - x_i - X_-)\, x_i$$

$$\frac{\partial Q_i}{\partial x_i} = 250 - X_- - 2x_i \geq 48 > 0 \qquad (\text{For } x_i \leq 2 \text{ and } X_- \leq 198)$$

Hence $x_i^o = 2$, $X^* = 200$; $Q_i = 100$, $Q = 100^2$

A Pareto superior allocation: $\hat{x}_i = 1 \Rightarrow \hat{Q}_i = 150$, $\hat{Q} = 15.000$

(b)

$$\max_{X \in [0,200]} (250 - X)\, X \quad \left\{ \begin{array}{l} 250 - 2X = 0 \\ X^P = 125 \ , \quad Q^P = 125^2 \\ Q|_{\text{PER CAPITA}} = 156{,}25 \end{array} \right.$$

(c), (d):

	1	2
1	5, 5 **	3, 6
2	6, 3	4, 4 *

** Pareto Optimal

(*) Nash Equilibrium