Masters in Economics-uc3m, Microeconomics II

Midterm Exam (March 21, 2023)

Exercise 1. (40 points) Consider a pure exchange economy that operates over two dates, today and tomorrow, and in which there is a single perishable good, consumption, and two consumers. The state of nature tomorrow is uncertain and can be either sunny or cloudy. The initial endowments of consumer 1 are $(4,0,2)$, and her preferences of for consumption today (x), consumption tomorrow if sunny (y_S), and consumption tomorrow if cloudy (y_C) are represented by the utility function $u_1(x, y_S, y_C) = xy_S$, while those of consumer 2 are $(0,4,2)$ and $u_2(x, y_S, y_C) = xy_C$. There are spot markets for consumption at each date. There is also a credit market and a market for a security θ which pays 2 units of good in tomorrow if sunny and nothing otherwise, both of which operate today. Calculate a competitive equilibrium allocation. (Normalize the spot prices to one, i.e., $p_x = p_y = p_z = 1$, and denote by r the interest rate and by q the price of the security θ. Also, use the notation $b_i(r,q)$ and $\theta_i(r,q)$ for consumer i's demands of credit and security. You should verify that the CE interest rate and security price are $(r^*, q^*) = (0, 6/5)$.)

Exercise 2. Ann, Bob, and Conrad share an apartment and must decide the number of hours of cleaning services, x, they will hire. Their preferences are described by a utility function of the form $u(x,y) = y + 2\alpha_i\sqrt{x}$, where y denotes income (in euros) available to spend on other goods. Each is endowed with $\bar{y}_i$ euros, and their preferences parameters are $(\alpha_A, \alpha_B, \alpha_C) = (2,4,6)$. The cost of cleaning services is 6 euros/hour.

(a) (10 points) Calculate the Pareto optimal values of cleaning services.

(b) (13 points) Calculate the hours of cleaning service that will be hired as a result of voluntary contributions.

(c) (13 points) Verify that $s^* = (s_A^*, s_B^*, s_C^*) = (5/3, 2/3, 5/3)$ forms a Nash equilibrium of the game induced by the mechanism (S, ϕ) defined by $S_i = \mathbb{R}$ for $i \in \{A, B, C\}$, and $\phi(s) = (x(s), y_A(s), y_B(s), y_C(s))$, where $x(s) = s_A + s_B + s_C$, $y_A(s) = \bar{y}_A - (2 + s_B - s_C)\,x(s)$, $y_B(s) = \bar{y}_B - (2 + s_C - s_A)\,x(s)$, and $y_C(s) = \bar{y}_C - (2 + s_A - s_B)\,x(s)$. Calculate the resulting allocation and verify that it is the Lindahl equilibrium.

Exercise 3. In a competitive insurance market, there are two types of drivers, the *risky* (H) and the *prudent* (L), which are present in proportions $\lambda \in (0,1)$ and $1-\lambda$, respectively. The probability that a driver has an accident is $p_H = 1/2$ for the risky type and $p_L = 1/4$ for the prudent type. All drivers have the same preferences, represented by the Bernoulli utility function $u(x) = \ln x$, and the sane initial wealth $W = 100€$. An accident generates a loss of $80€$. A driver's type is private information (i.e., not observable).

(a) (12 points) Assume that insurance companies are restricted by law to offer only full insurance policies. Determine the values of λ for which the competitive equilibrium involves all drivers subscribing the pooling full insurance policy and those for which only risky drives subscribe an insurance policy.

(b) (12 points) Calculate the separating policy menu and identify the values of λ for which in the absence of legal constraints it is a competitive equilibrium.

Solutions

Exercise 1. Consumer 1's problem is

$$\max_{(x,y_S,y_C),b,\theta\in\mathbb{R}^3_+\times\mathbb{R}\times\mathbb{R}} xy_S,$$
$$s.t.\ x = 4 + b - q\theta,\ y_S = 2\theta - (1+r)\,b,\ y_C = 2 - (1+r)\,b.$$

Since he does not care about y_C*, we may assume that in equilibrium he will set* $y_C = 0$*, and lend as much as possible conditional on being able to payback if the state is cloudy, i.e., he will set* $b_1(r,q) = 2/(1+r)$*. Then we may write his problem as*

$$\max_{\theta\in\mathbb{R}}(4 + 2/(1+r) - q\theta)(2\theta - 2).$$

Solving the F.O.C. for a solution to this problem we get is true

$$\theta_1(r,q) = \frac{1}{2} + \frac{2}{q} + \frac{1}{q\,(1+r)}.$$

Likewise, consumer 2's problem is

$$\max_{(x,y,y_c),b,\theta\in\mathbb{R}^3_+\times\mathbb{R}\times\mathbb{R}} xy_C,$$
$$s.t.\ x = b - q\theta,\ y_S = 4 + 2\theta - (1+r)\,b,\ y_C = 2 - (1+r)\,b.$$

Since Consumer 2 does not care about y_S*, then for* $q > 0$ *he will set* $y_S = 4 + 2\theta - (1+r)\,b = 0$ *and sell as many units of the security as possible, that is,* $\theta = (1+r)\,b/2 - 2$*, and then choose* b *to solve*

$$\max_{b\in\mathbb{R}}(b - q\,((1+r)\,b/2 - 2))\,(2 - (1+r)\,b)\,.$$

Solving the F.O.C. for a solution to this problem we get

$$b_2(r,q) = \frac{3\,(1+r)\,q - 2}{(1+r)\,((1+r)\,q - 2)},\ \ \theta_2(r,q) = \frac{3\,(1+r)\,q - 2}{2\,(1+r)\,q - 4} - 2.$$

Market clearing requires

$$\begin{aligned} b_1(r,q) + b_2(r,q) &= \frac{2}{(1+r)} + \frac{3\,(1+r)\,q - 2}{(1+r)\,((1+r)\,q - 2)} = 0 \\ \theta_1(r,q) + \theta_2(r,q) &= \left(\frac{1}{2} + \frac{2}{q} + \frac{1}{q\,(1+r)}\right) + \left(\frac{3\,(1+r)\,q - 2}{2\,(1+r)\,q - 4} - 2\right) = 0. \end{aligned}$$

It is readily verified that $(r^*, q^*) = (0, 6/5)$ *solves this system. In the equilibrium allocation consumer 1's borrows* $b_1(0, 6/5) = 2$ *euros and buys* $\theta_1(0, 6/5) = 3$ *units of the security, and her consumption stream is* $(12/5, 4, 0)$*. While consumer 2 borrows* $b_2(0, 6/5) = -2$ *euros (that is, lends 2* euros), *and buys* $\theta_2(0, 6/5) = -2$ *unit (actually, sells 2 units) of the security, and her consumption stream is* $(8/5, 0, 4)$*.*

Exercise 2. (a) Since $MRS_i(x, y) = \alpha_i/\sqrt{x}$*, an interior Pareto optimal allocation satisfies*

$$MRS_A(x, y_A) + MRS_B(x, y_B) + MRS_C(x, y_C) = \frac{2+4+6}{\sqrt{x}} = 6.$$

Solving this equation we get $x^* = 4$*. Thus, in an interior Pareto optimal allocation the number of hours of cleaning service is* $x^* = 4$*.*

(b) Under voluntary contribution, individual i *decides its contribution by solving*

$$\max_{z_i \geq 0} \; \bar{y}_i - z_i + 2\alpha_i \sqrt{\frac{z_i + z_{-i}}{6}},$$

were z_{-i} *is the sum of the contributions of individuals other than* i*. Hence in an interior solution*

$$-1 + \frac{\alpha_i}{6\sqrt{\frac{z_i + z_{-i}}{6}}} = 0.$$

Hence individual i*'s reaction function is*

$$z_i = \max\{\frac{a_i^2}{6} - z_{-i}, 0\}.$$

Let us show that in a Nash equilibrium (NE)

$$z_A + z_B + z_C \geq 6.$$

To show this simply note that if $z_C < 6 - z_A - z_B$*, then Conrad will increase its contribution according to her reaction function. Moreover,* $z_A = 0$*, for if* $z_A > 0$*, then* $z_A = 4/6 - z_B - z_C \leq 4/6 - (6 - z_A) = z_A - 16/3$*, i.e.,* $z_A = -16/6 < 0$*, a contradiction. Likewise,* $z_B = 0$*, since* $z_B = 0$ *implies* $z_B = 16/6 - z_A - z_C \leq 16/6 - (6 - z_B) = z_B - 10/3$*, i.e.,* $z_B = -10/6 < 0$*, a contradiction.*

Thus, the unique NE is

$$(z_A^{NE}, z_B^{NE}, z_C^{NE}) = (0, 0, 6),$$

and the therefore $x^{NE} = 6/6 = 1$*.*

(c) We verify that $s_A^* = 5/3$ *maximizes Ann's payoff given* $(s_B^*, s_C^*) = (2/3, 5/3)$ *by solving the problem*

$$\max_{s \in \mathbb{R}} \; \bar{y}_A - (2 + 2/3 - 5/3)\,(s + 2/3 + 5/3) + 2\alpha_A \sqrt{s + 2/3 + 5/3}.$$

The F.O.C. for a solution to this problem is

$$-1 + \frac{2}{\sqrt{s + 7/3}} = 0.$$

whose solution is $s^* = 5/3$*. Verifying that the strategies of Bob and Conrad also maximize their respective payoff given the others' strategies is analogous.*

The equilibrium allocation is therefore $x = 4$*,*

$$\begin{aligned} y_1 &= \bar{y}_1 - (2 + 2/3 - 5/3)\,4 = \bar{y}_1 - 4 \\ y_2 &= \bar{y}_2 - (2 + 5/3 - 5/3)\,4 = \bar{y}_2 - 8 \\ y_3 &= \bar{y}_3 - (2 + 5/3 - 2/3)\,4 = \bar{y}_3 - 12, \end{aligned}$$

which is the Lindahl allocation.

Exercise 3. (a) If all drivers subscribe the same policy, the probability that a driver randomly selected from has an accident is

$$\bar{p}(\lambda) = \lambda p_H + (1-\lambda)p_L = \lambda\left(\frac{1}{2}\right) + (1-\lambda)\left(\frac{1}{4}\right) = \frac{1+\lambda}{4}.$$

and therefore the fair premium of the policy is

$$\bar{I}(\lambda) = 80\bar{p}(\lambda) = 20(1+\lambda).$$

In order for drivers of type L to be willing to subscribe this policy we must have

$$\frac{1}{4}\ln(100-80) + \frac{3}{4}\ln 100 \leq \ln\left(100 - \bar{I}(\lambda)\right),$$

that is,

$$\ln 20^{1/4}100^{3/4} \leq \ln\left(100 - \bar{I}(\lambda)\right) \Leftrightarrow 20^{1/4}100^{3/4} \leq 100 - \bar{I}(\lambda)$$

or

$$\lambda \leq \frac{100 - 20^{1/4}100^{3/4}}{20} - 1 \simeq 0.6563.$$

(b) The separating policies are $(\tilde{I}_H, \tilde{D}_H) = (40, 0)$, *and* $(\tilde{I}_L, \tilde{D}_L) = (\left(80 - \tilde{D}_L\right)/4, \tilde{D}_L)$, *where* $\tilde{D}_L$ *is the solution to the equation*

$$\frac{1}{2}\ln\left(100 - x - \frac{80-x}{4}\right) + \frac{1}{2}\ln\left(100 - \frac{80-x}{4}\right) = \ln 60,$$

or equivalently

$$\left(100 - x - \frac{80-x}{4}\right)\left(80 + \frac{x}{4}\right) = 60^2.$$

Solving this equation we get

$$\tilde{D}_L = \frac{80}{3}\sqrt{37} - \frac{320}{3} \simeq 55.54,\ \tilde{I}_L = \frac{1}{4}\left(80 - \left(\frac{80}{3}\sqrt{37} - \frac{320}{3}\right)\right) \simeq 6.115.$$

For this menu to be a separating equilibrium the pooling policy cannot be preferred by the low risk drivers to the separating policy,

$$\begin{aligned} \ln(100 - 20(1+\lambda)) &\leq \frac{1}{4}\ln\left(100 - \tilde{I}_L - \tilde{D}_L\right) + \frac{3}{4}\ln\left(100 - \tilde{I}_L\right) \\ &\Updownarrow \\ 100 - 20(1+\lambda) &\leq \left(100 - \tilde{D}_L - \tilde{I}_L\right)^{\frac{1}{4}}\left(100 - \tilde{I}_L\right)^{\frac{3}{4}} \end{aligned}$$

that is,

$$\lambda \geq \frac{100 - \left(100 - \tilde{D}_L - \tilde{I}_L\right)^{\frac{1}{4}}\left(100 - \tilde{I}_L\right)^{\frac{3}{4}}}{20} - 1 \simeq 0.24730.$$