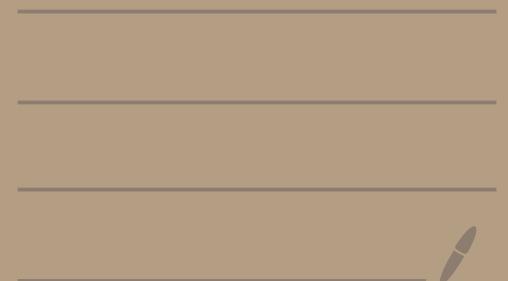


# MARKETS FOR LEMONS

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G. Akerlof : The market for lemons : quality ... . QJE (1970)

## An Example

A good of two qualities: High and Low.

There are  $b$  buyers, each of whom wants to buy one unit.

There are  $S^H$  sellers, each with one unit of  $H$  quality, and  $S^L$  " , " , " , " , " , " , " , " , " , " .

Assumption :  $u^H > c^H > u^L > c^L$

I. Assume that quality is observed.

CE: Two competitive markets:

- The market for good H
- The market for good L.

Supply:

For  $\tau \in \{H, L\}$ :

$$S^{\tau}(P) = \begin{cases} 0 & \text{if } P < c^{\tau} \\ [0, s^{\tau}) & \text{if } P = c^{\tau} \\ s^{\tau} & \text{if } P > c^{\tau}. \end{cases}$$

Demand:

The demands of goods H and L depend on  $(p^L, p^H)$ :

Consumers demand the quality that gives them the largest utility:

$$u^I - p^I = \max \{ u^L - p^L, u^H - p^H \} \geq 0.$$

For  $(p^L, p^H) \in [0, u^L] \times [0, u^H]$ :

$$(D^L(p^L, p^H), D^H(p^L, p^H)) = \begin{cases} (0, b) & \text{if } u^H - p^H = \max \{ u^L - p^L, u^H - p^H \} > 0 \\ \{(x, y) \mid x + y = b\} & \text{if } u^L - p^L = u^H - p^H > 0 \\ (b, 0) & \text{if } u^L - p^L = \max \{ u^L - p^L, u^H - p^H \} > 0 \\ \{(x, y) \mid x + y \leq b\} & \text{if } u^L - p^L = u^H - p^H = 0 \\ (0, 0) & \text{if } \max \{ u^L - p^L, u^H - p^H \} < 0. \end{cases}$$

CE

Assume  $b > \max \{ s^L, s^H \}$ . Then:

- If  $b > s^L + s^H$ , then  $p^L = u^L$ ,  $p^H = u^H$ , and all units trade.
- If  $b < s^L + s^H$ , then  $\begin{cases} u^L - c^L < u^H - c^H \Rightarrow p^L = c^L, p^H = u^H - (u^L - c^L) \\ u^L - c^L > u^H - c^H \Rightarrow p^H = c^H, p^L = u^L - (u^H - c^H) \end{cases}$

Moreover,  $b$  units trade.

CE is Pareto efficient (i.e., maximum surplus is realized).

Example:  $b = 3$ ,  $s^H = s^L = 2$ ,  $u^H = 10$ ,  $u^L = 4$ ,  $c^H = 6$ ,  $c^L = 2$ .

Then:  $p^L = 2$ ,  $p^H = 10 - (4 - 2) = \underline{\underline{8}}$ .

Surplus realized:  $2(10 - 6) + (4 - 2) = 10$

Each buyer: 2.

H sellers: 2

L sellers: 0

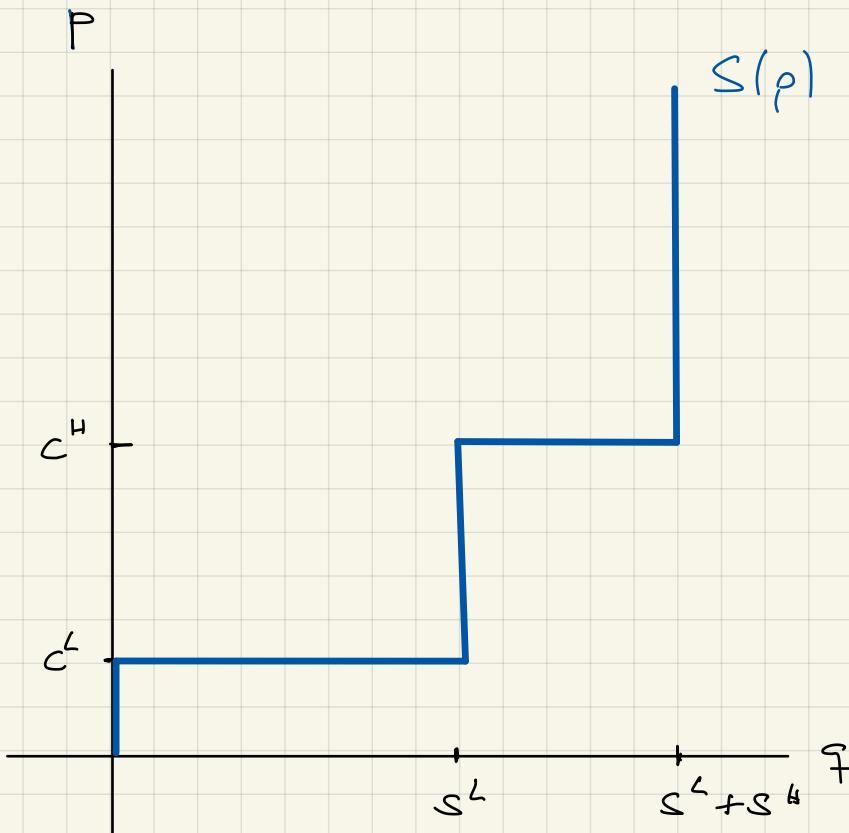
II. Assume quality is known to sellers, but is observed by buyers only upon purchase.

In this case there is a single market where both qualities trade. Hence buyers are uncertain about the quality of the good they are offered.

$$c^E =$$

Supply:

$$S(p) = \begin{cases} 0 & \text{if } p < c^L \\ s^L & \text{if } c^L \leq p < c^H \\ s^L + s^H & \text{if } p > c^H. \end{cases}$$



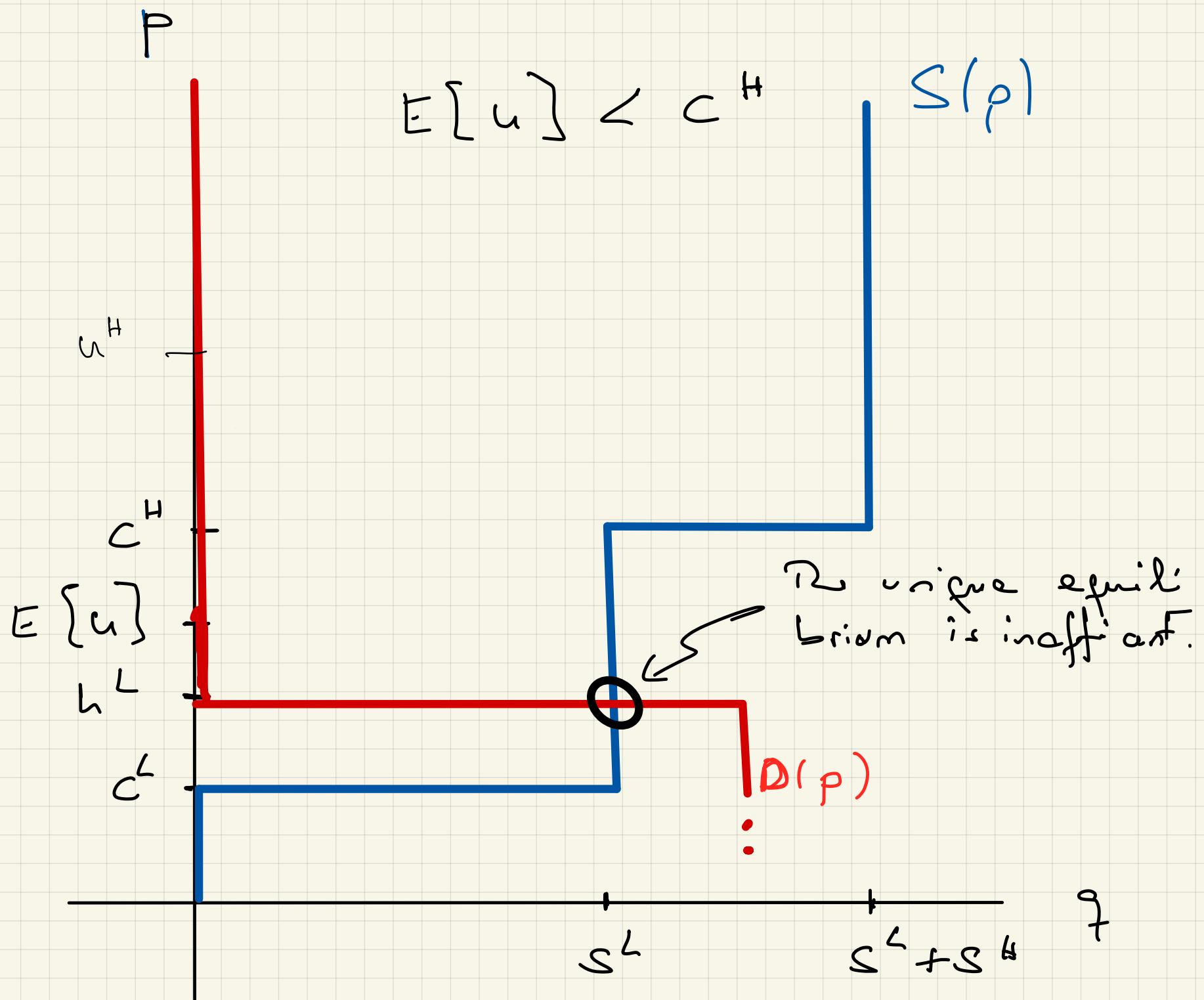
Demand (buys or risk-neutral):

$$E[u] = \frac{s^L}{s^L + s^H} u^L + \frac{s^H}{s^L + s^H} u^H$$

Assume  $E[u] < c^H$ . Hence

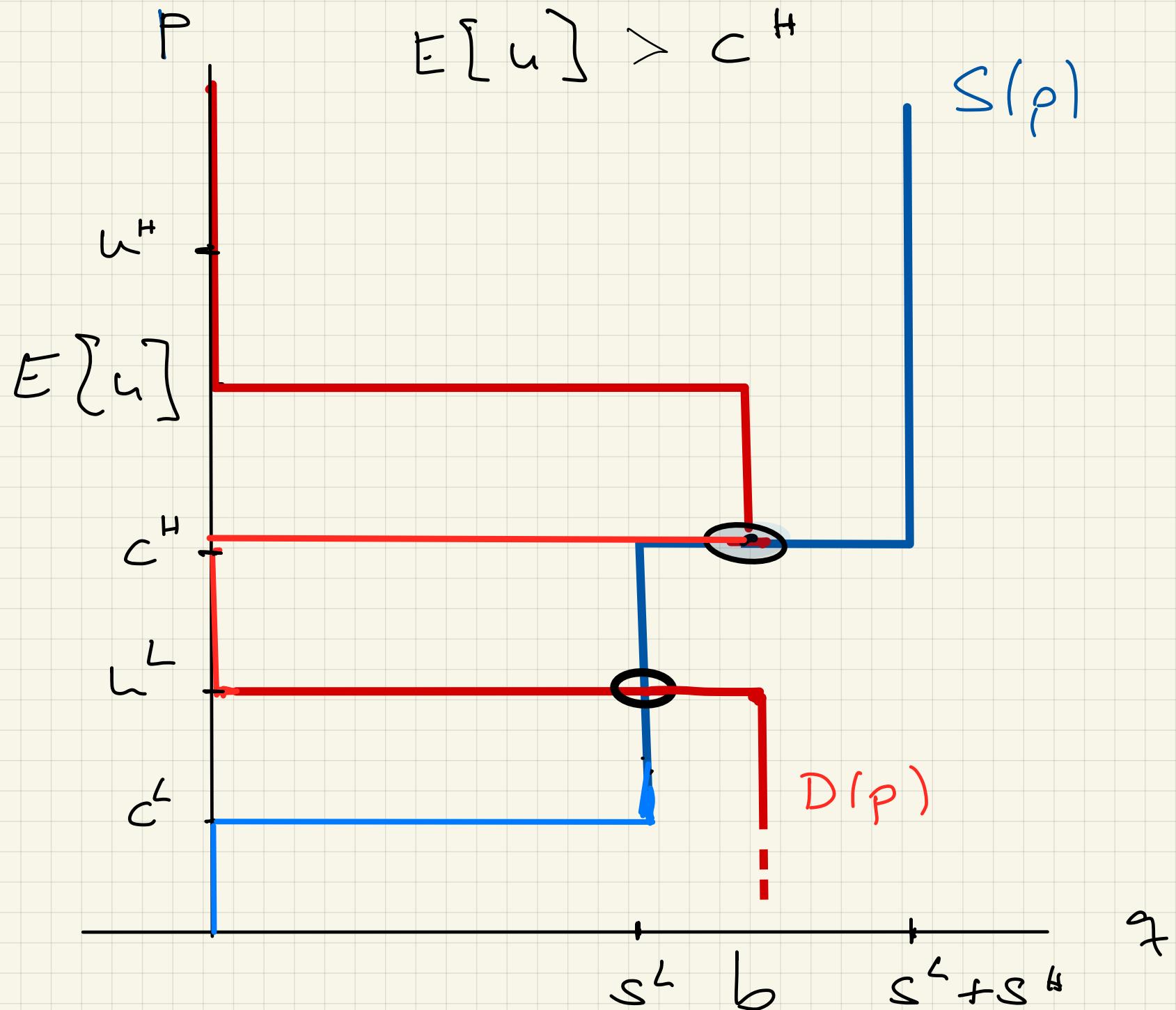
(\*) All sellers supply at risk price

$$D(p) = \begin{cases} 0 & \text{if } p > u^L \\ b & \text{if } c^L \leq p \leq u^L \\ ? & \text{if } p < c^L \end{cases}$$



Assume  $E[u] < c^H$ . Hence

$$D(p) = \begin{cases} 0 & \text{if } p > u^H \\ b & \text{if } c^H \leq p \leq u \\ 0 & \text{if } c^L < p < c^H \\ b & \text{if } c^L \leq p \leq u^L \\ ? & \text{if } p < c^L \end{cases}$$



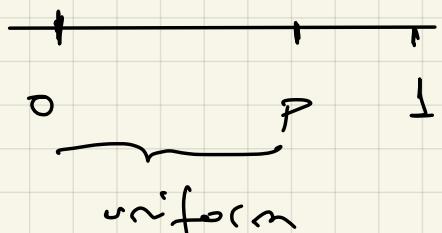
A more extreme example.

A measure  $l$  of sellers. The quality of the good supplied by the seller  $s \in [0, 1]$  is  $q(s) = s$ , and its opportunity cost is  $c(s) = s$ .  
Buyers' values of quality  $s \in [0, 1]$  is  $u(s) = \alpha s$ ,  $\alpha \in (1, 2)$ .

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Supply:  $p \in \mathbb{R}_+$ ,  $S(p) = m - \{p, 1\}$ .

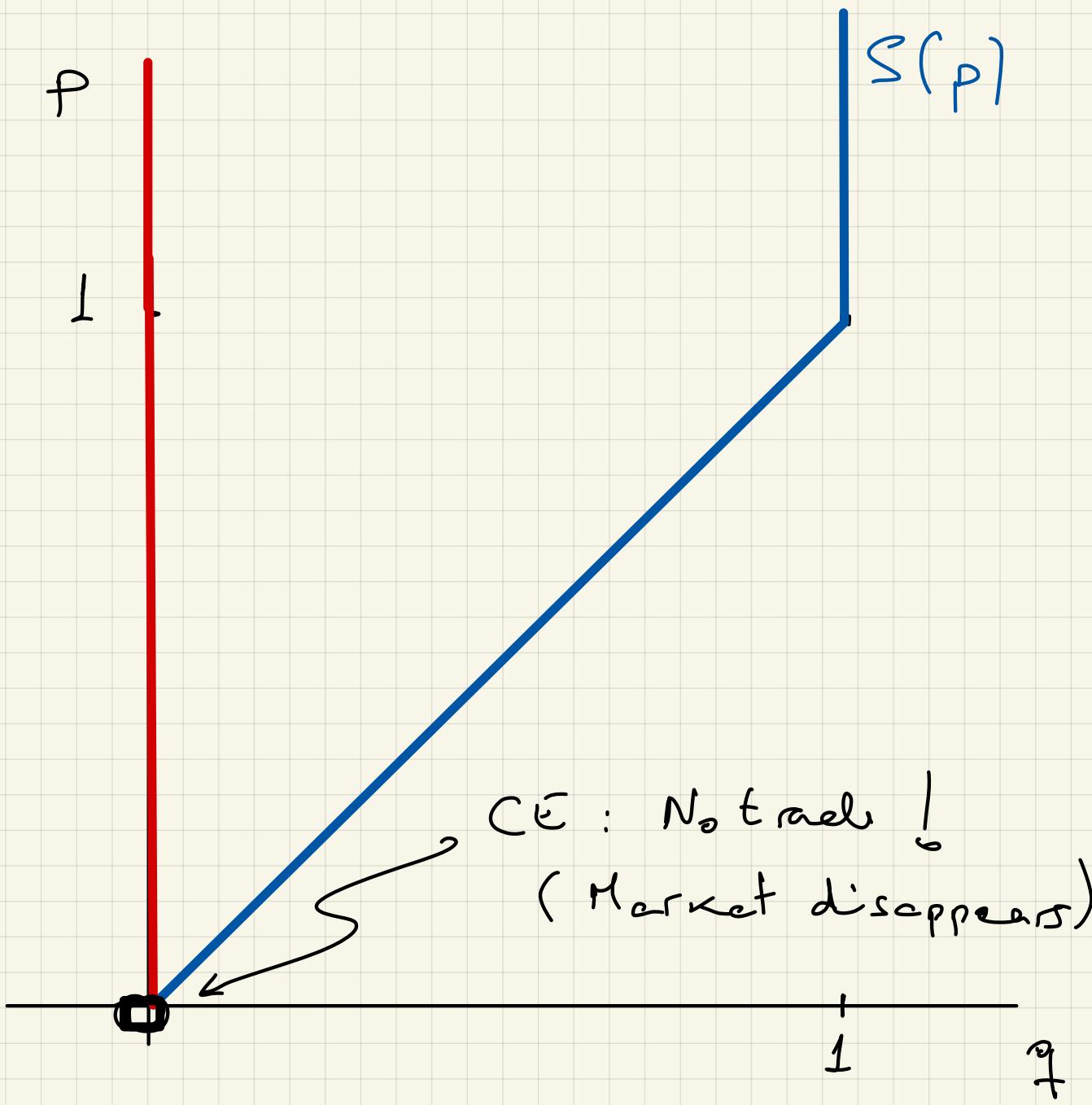
Demand:  $D(p) = \begin{cases} 0 & \text{if } p > E[u(s) | s \leq p] \\ 1 & \text{otherwise.} \end{cases}$



$E[u(s) | s \leq p]$  ?

$$\text{If } p > 1, E[u(s) | s \leq p] = \alpha E[S | s \leq p] = \alpha/2 < 1 < p \Rightarrow D(p) = 0$$

$$\text{If } p \leq 1: E[u(s) | s \leq p] = \alpha E[S | s \leq p] = \alpha \left(\frac{p}{2}\right) < p \Rightarrow D(p) = 0$$



Let  $P \leq 1$ :

$$\begin{aligned} \mathbb{E}[u(s) | S \leq p] &= \mathbb{E}[\alpha s | S \leq p] \\ &= \alpha \mathbb{E}[s | S \leq p] \end{aligned}$$

$$S \sim U[0, 1], \quad f(s) = 1 \quad \left( \int_0^1 f(s) ds = [x]_0^1 = 1 \right)$$

$$\begin{aligned} S | S \leq p : \quad F(s | S \leq p) &= \Pr[S \leq s | S \leq p] \\ &= \frac{\Pr[S \leq \min\{p, s\}]}{\Pr[S \leq p]} \\ &= \begin{cases} 1 & \text{if } s > p \\ \frac{F(s)}{F(p)} = \frac{s}{p} & \text{if } s \leq p. \end{cases} \end{aligned}$$

Hence

$$f(s | S \leq p) = \begin{cases} 0 & \text{if } s > p \\ \frac{1}{p} & \text{if } s \leq p \end{cases}.$$

$$\underline{\mathbb{E}}[S \mid S \leq p] = \int_0^1 s f(s \mid S \leq p) ds$$

$$= \int_0^p \frac{s}{p} ds$$

$$= \frac{1}{p} \left[ \frac{s^2}{2} \right]_0^p = \frac{1}{p} \left( \frac{p^2}{2} \right) = \frac{p}{2}.$$

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