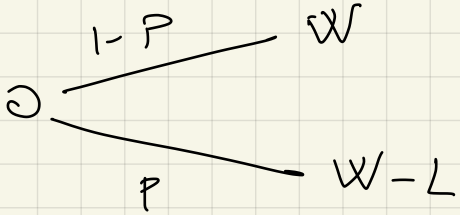
Competitive Insurance Markets

THE MARKET FOR INSURANCE.

M. Rothschild and J. Stiglitz, QJE (1976).

- A population of individuals face the risk of a wealth loss L with probability $p \in (0,1)$; that is, face the lottery



- Their preferences are represented by a Bernoulli utility function $u : \mathbb{R} \to \mathbb{R}$, such that $u' > 0$, $u'' < 0$.

- There is a competitive insurance market where firms offer policies (I, D), where

  I : Premium

  D : Deductible

If an individual subscribes the policy (I, D), then he faces the lottery

$$(I, D) \quad \circ \begin{cases} \xrightarrow{\;1-p\;} \; W - I := x_n \\[2ex] \xrightarrow{\;\;p\;\;} \; W - I - D := x_a \end{cases}$$

Hence

$$Eu(I, D) = (1-p)\, u(x_n) + p\, u(x_a) := U(x_n, x_a)$$

Examples:

① $(I, D) = (0, L)$ (No insurance)

② $(I, D) = (I, 0)$ (Full insurance)

③ $(I, D) = (pL, 0)$ (Full insurance at fair premium)

④ $(I, D) = (I, \frac{L}{2})$ (Partial insurance)

Typically, $D < 0$ is not allowed.

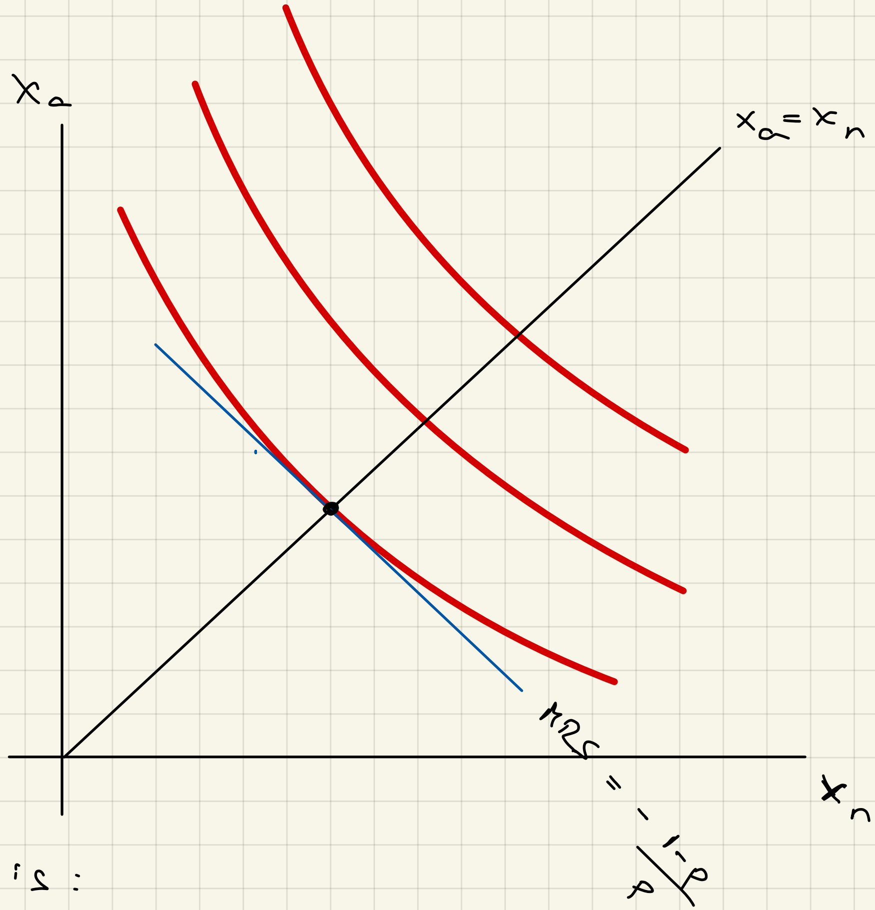
Preferences for insurance policies:

Let us calculate the $MRS(x_n, x_a)$:

$$U(x_n, x_a) = (1-p)\, u(x_n) + p\, u(x_a)$$

Hence

$$-\frac{dx_a}{dx_n} = MRS(x_n, x_a) = \frac{\frac{\partial U}{\partial x_n}}{\frac{\partial U}{\partial x_a}} = \frac{1-p}{p} \frac{u'(x_n)}{u'(x_a)}$$



Exercise: Check that $u' > 0$, $u'' < 0$ imply that indifference curves are convex, that is:

$$\frac{d^2 x_a}{d x_n^2} > 0.$$

Fair odds Policies

$$I = p\,(L - D)$$

Fair Odds Line:

$$X_n = W - I \iff I = W - X_n$$

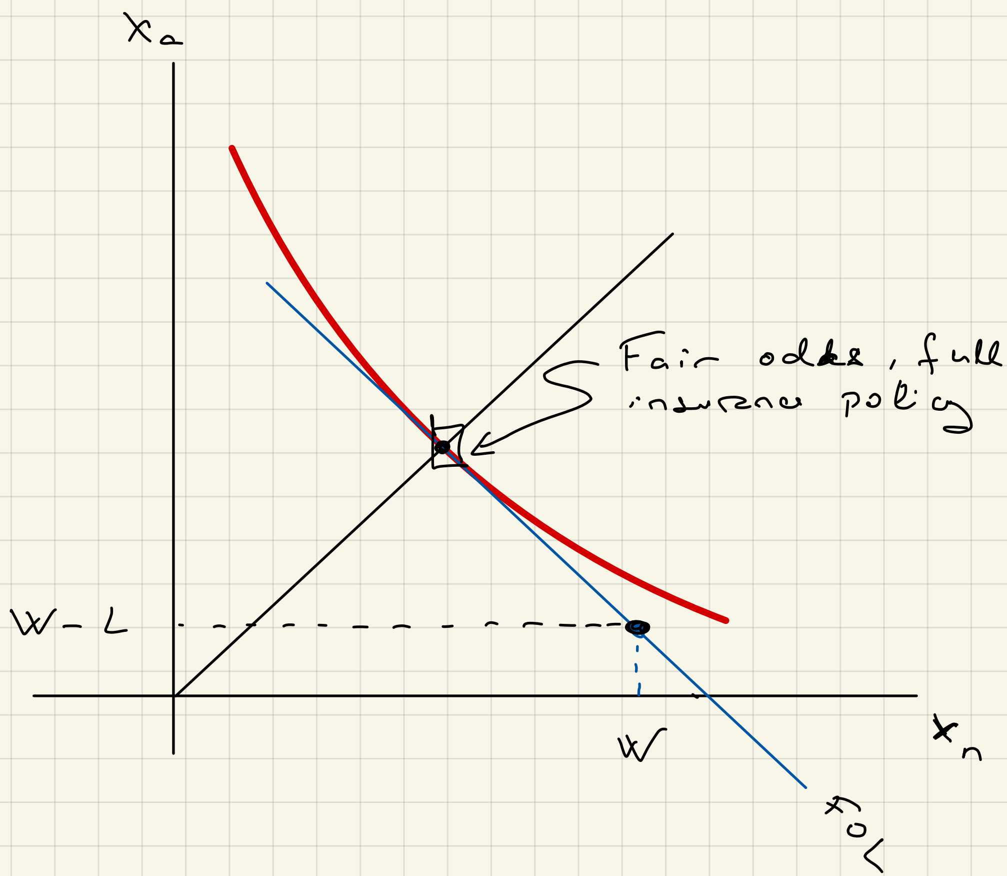
$$X_a = W - I - D = X_n - D \iff D = X_n - X_a$$

Hence

$$W - X_n = p\left[L - (X_n - X_a)\right]$$

i.e.,

$$\boxed{X_a = \left(\frac{W}{p} - L\right) - \frac{1-p}{p}\,X_n}$$



Note: Policies below (above) the fair-odds line make profits (losses).

CE :

In a CE a policy (I, D) is subscribed only if

(1) $I = p(L-D)$

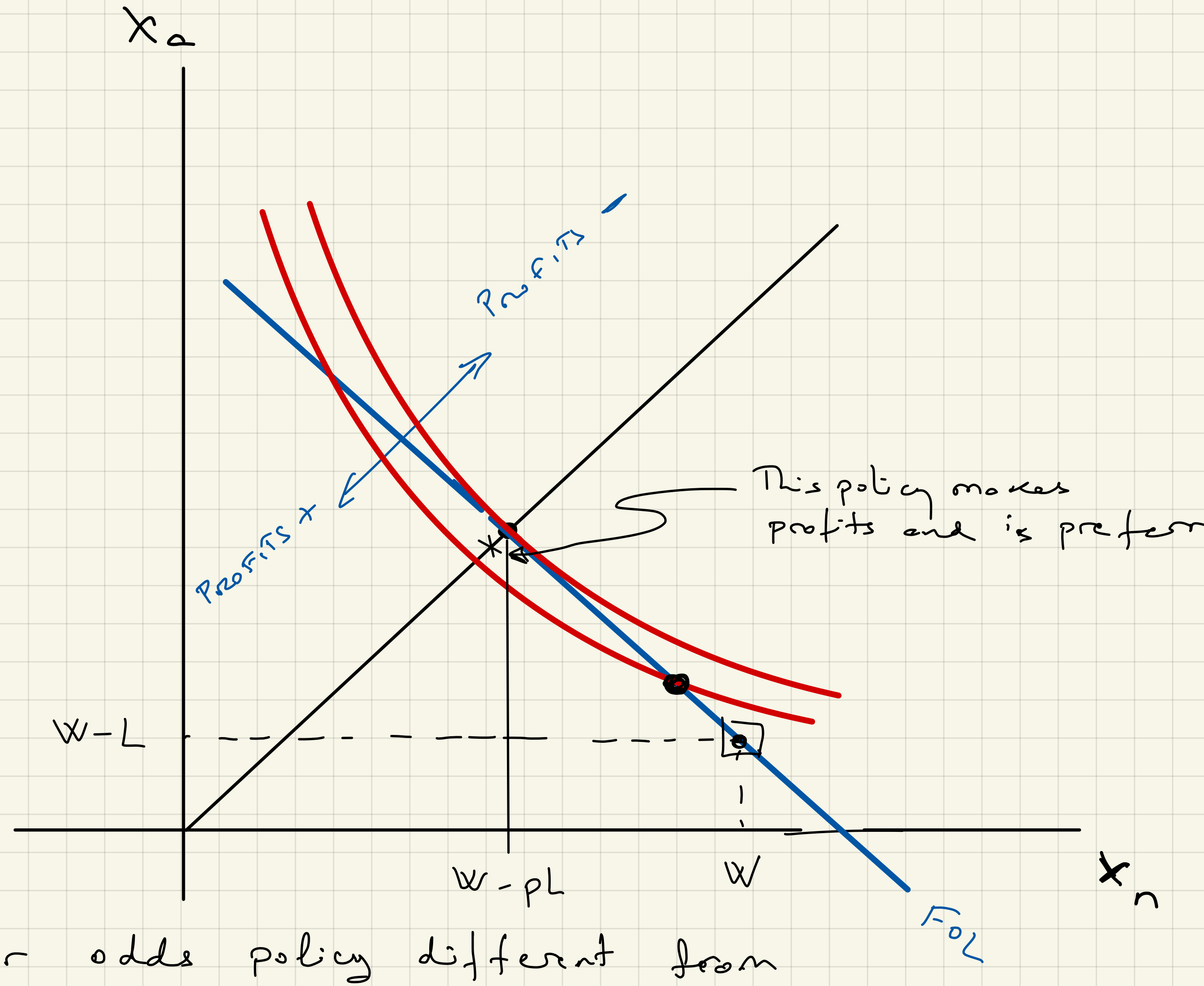
(2) $\nexists\ (\tilde{I}, \tilde{D})$ such that

- $\tilde{I} > p(L-\tilde{D})$
- $Eu(\tilde{I}, \tilde{D}) > Eu(I, D)$.

Proposition. In a CE all individuals subscribe the policy

$$(I^*, D^*) = (pL, 0).$$

Proof

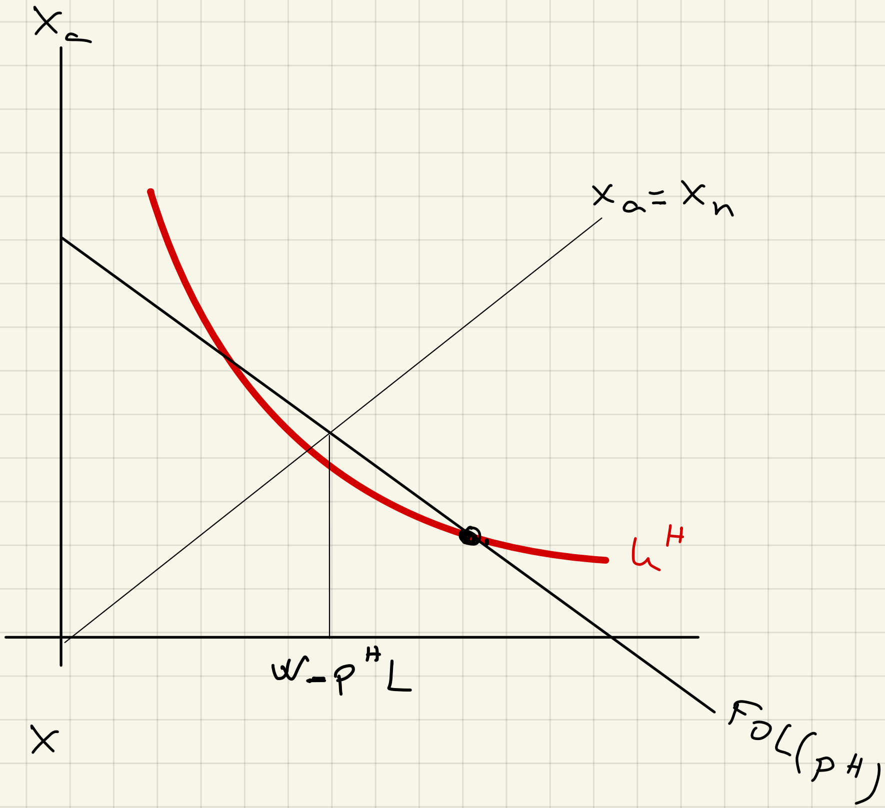


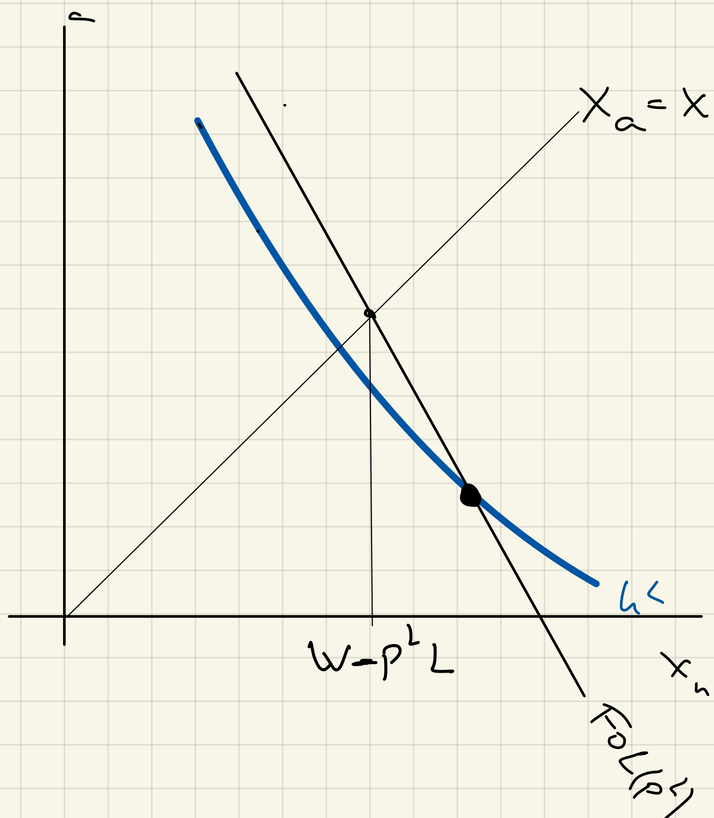
Any fair odds policy different from $(pL, 0)$ is upset by a policy $(pL+\varepsilon, 0)$, $\varepsilon > 0$, which makes profits.

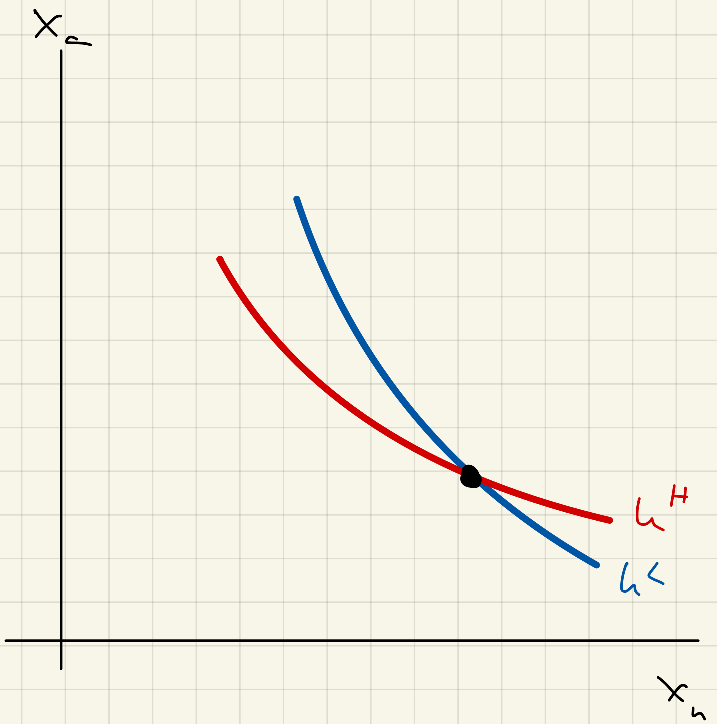
Adverse Selection

Assume that for a fraction $\lambda \in (0,1)$ of individuals the probability of the wealth loss is p^H, while the remaining fraction $1-\lambda$ of individuals this probability is p^L, where

$$\boxed{0 < p^L < p^H}$$







$$MRS_L(x_n, x_a) = \frac{1-p^L}{p^L} \frac{u'(x_n)}{u'(x_a)}$$
$$> \frac{1-p^H}{p^H} \frac{u'(x_n)}{u'(x_a)}$$
$$= MRS_H(x_n, x_a)$$

Note:

$$\frac{d}{dp}\left(\frac{1-p}{p}\right) = \frac{d}{dp}\left(\frac{1}{p} - 1\right) = -\frac{1}{p^2} < 0.$$

If insurance companies could recognize the individual of either type, then in a CE they will offer to each type her full insurance fair-odds policy.

What if insurance companies cannot distinguish between high and low risk individuals?

Which policies will they offer in a CE?

Will they offer a single (pooling) policy to both types?

Proposition A CE in which the same policy is subscribed by both types (i.e., a pooling equilibrium) does not exist.

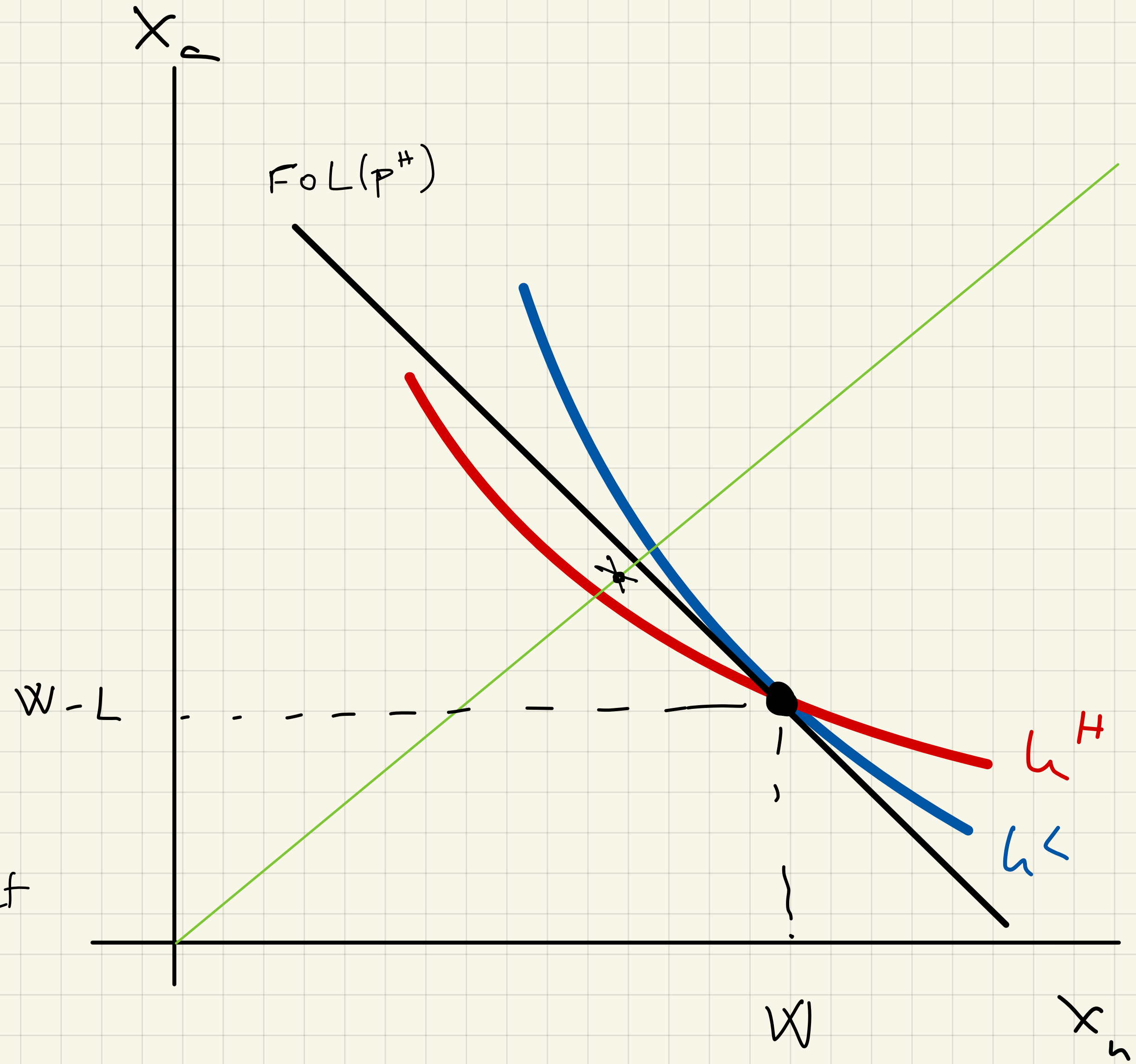
Proof

(a) Offering the single policy $(0, L)$

is not a CE.

A company offering the policy $(p^H L + \varepsilon, 0)$

will attract all the high risk individuals and make positive profits.

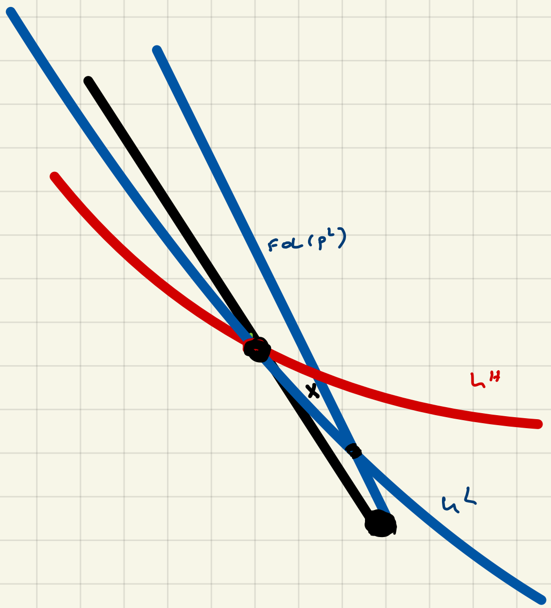


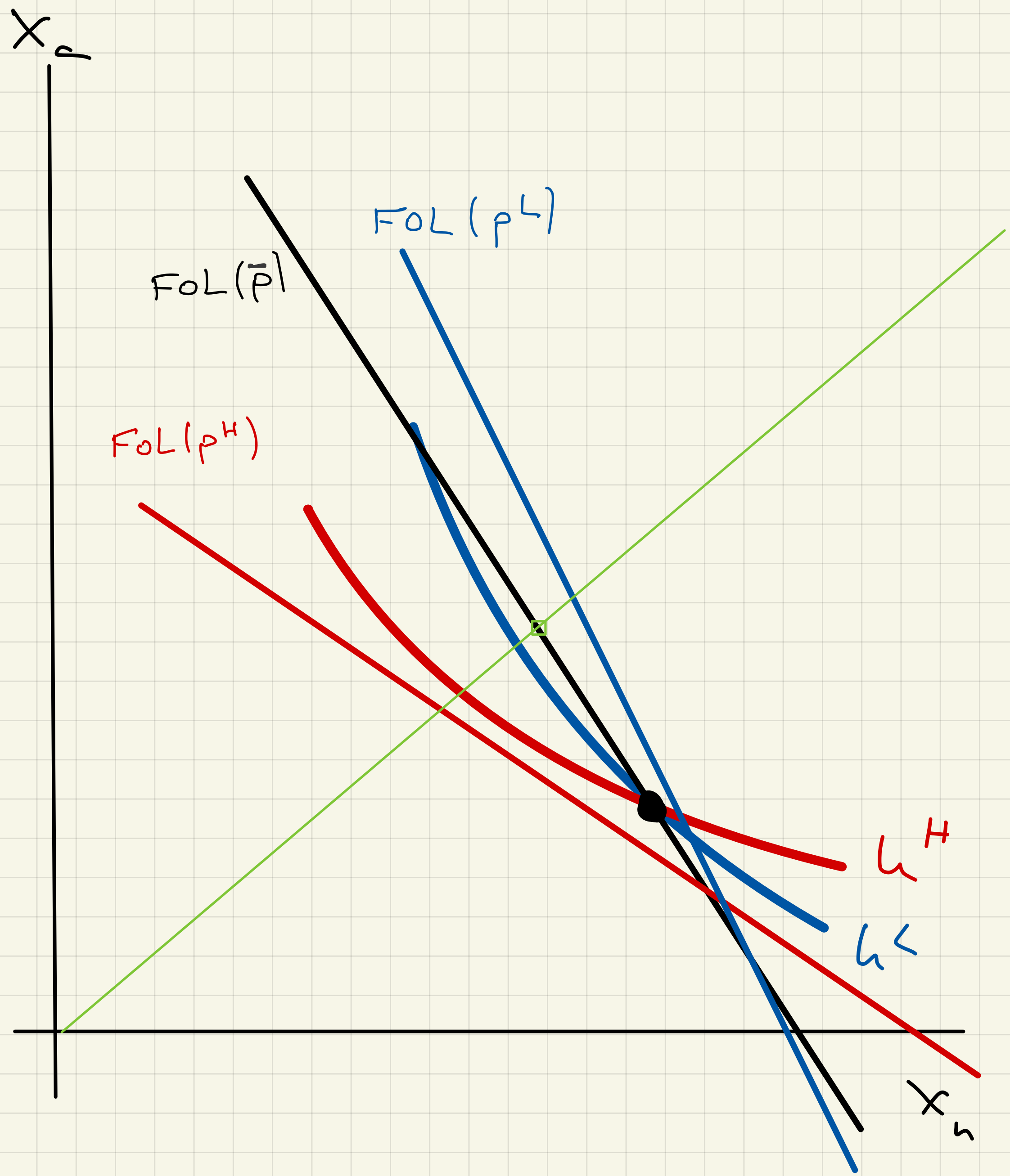
Note. It may also attract the low risk individuals.

(b) Any other fair odds policy $(\bar{p}(L-D), D)$, with

$$\bar{p} = \lambda p^H + (1-\lambda) p^L,$$

will be upset by a policy aimed to attracting the low risk individuals



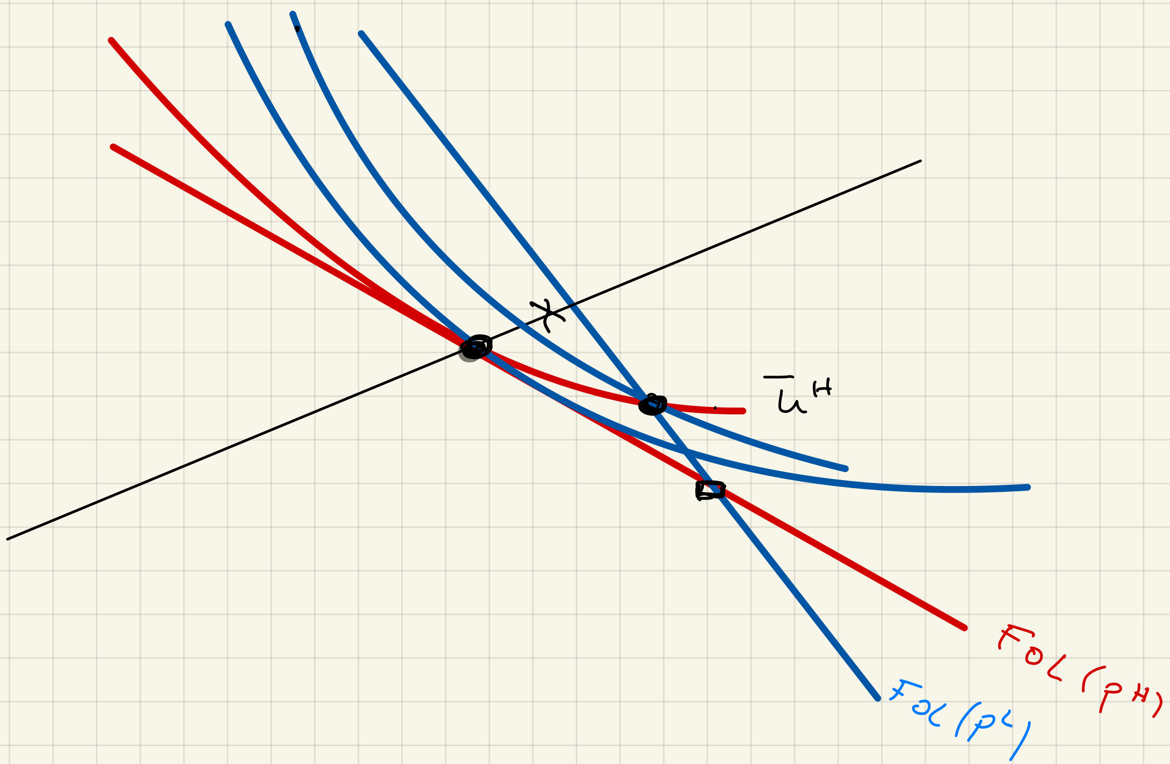


A SEPARATING EQUILIBRIUM

$\{(I^H, D^H), (I^L, D^L)\}$, ① $I^H = p^H L$, $D^H = 0$

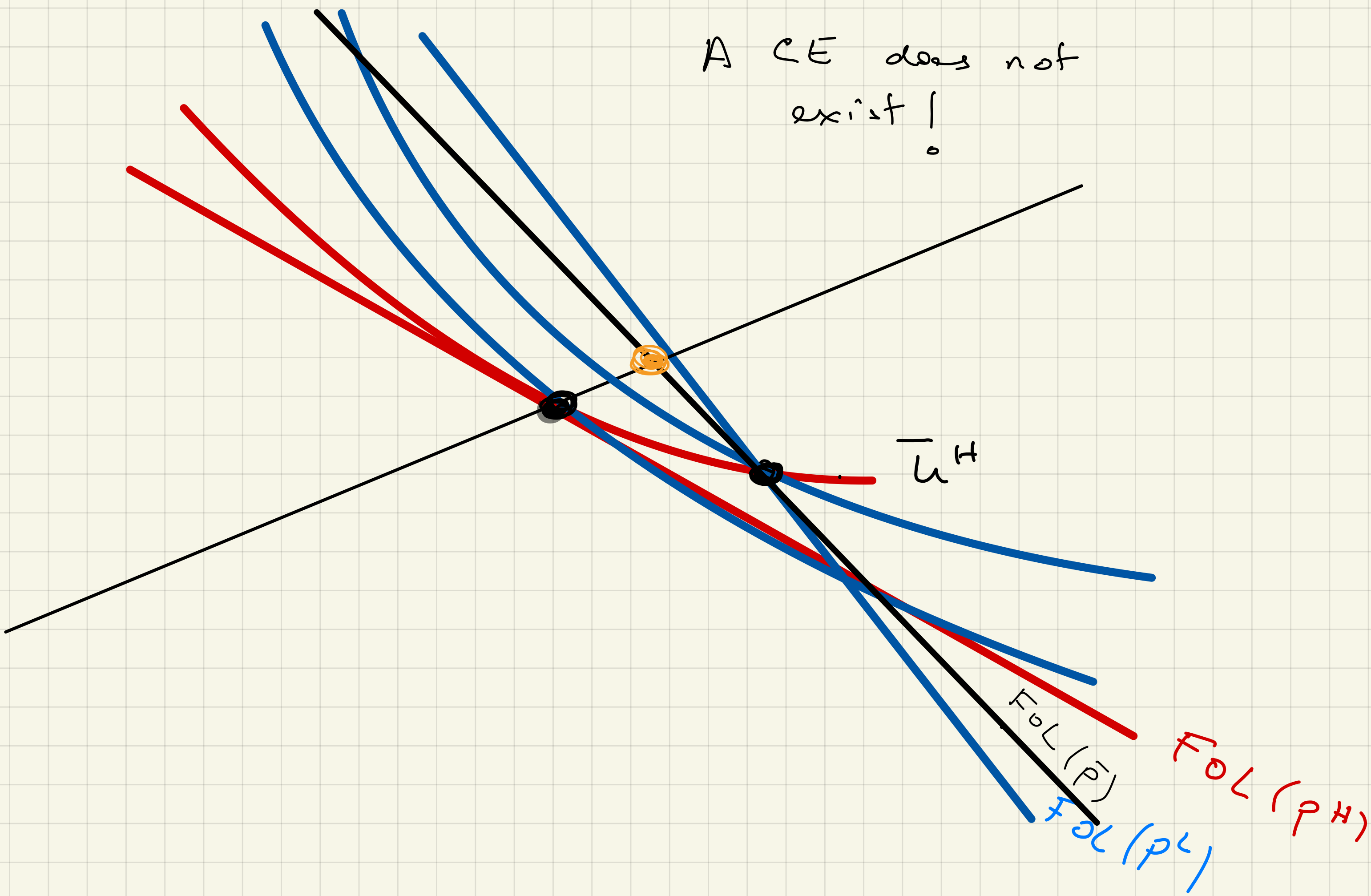
② $I^L = p^L (L - D^L)$

$E_H u(I^L, D^L) := p^H u(W - I^L - D^L) + (1-p^H) u(W - I^L) = E_H u(I^H, D^H) = u(W - I^H)$.



For this menu to be a CE, λ must be sufficiently large that.

$$u(W-\bar{p}L) \leq p^L u(W-I^L-D^L) + (1-p^L)\, u(W-I^L)$$



Conclusion

The existence of markets with <u>adverse selection</u>, which is a particular case of asymmetric information, may result in <u>perfectly competitive markets</u> producing

- inefficient outcomes,

or even

- cease to exist altogether!

$W = L = 1, \quad u(x) = \sqrt{x}, \quad p^L = \frac{1}{4}, \quad p^H = \frac{1}{2}, \quad \lambda \in (0,1).$

WHEN TYPES ARE OBSERVABLE, THE CE POLICIES ARE:

$$(I^H, D^H) = \left(\frac{1}{2}, 0\right), \quad (I^L, D^L) = \left(\frac{1}{4}, 0\right)$$

THE <u>POOLING POLICY</u> IS $(\bar{I}, \bar{D}) = (\bar{p}(\lambda) L, 0)$,

WHERE:

$$\bar{p}(\lambda) = \frac{\lambda}{2} + \frac{(1-\lambda)}{4} = \frac{1+\lambda}{4}.$$

FOR $\lambda = 1/4$, FOR EXAMPLE, $\bar{p}(1/4) = \frac{5}{16}$, AND

$$(\bar{I}, \bar{D}) = \left(\frac{5}{16}, 0\right)$$

WITH THIS POLICY, THE EXPECTED UTILITIES OF AGENTS ARE

$$\bar{U}_H = \bar{U}_L = \sqrt{1 - \frac{5}{16}} = \frac{\sqrt{11}}{4} \simeq 0.83.$$

BUT THIS POLICY IS "DESTABILIZED" BY, E.G., THE POLICY

$$(\tilde{I}^L, \tilde{D}^L) = \left(\frac{1}{8}, \frac{1}{2}\right).$$

THE EXPECTED UTILITIES OF AGENTS THAT SUBSCRIBE THIS POLICY ARE

$$\tilde{U}_L = \frac{3}{4}\sqrt{\frac{7}{8}} + \frac{1}{4}\sqrt{\frac{3}{8}} \simeq 0.854$$

$$\tilde{U}_H = \frac{1}{2}\sqrt{\frac{7}{8}} + \frac{1}{2}\sqrt{\frac{3}{8}} \simeq 0.773.$$

HENCE LOW RISK INDIVIDUALS PREFER THIS POLICY TO THE POOLING POLICY, WHILE HIGH RISK INDIVIDUALS PREFER THE POOLING POLICY.

$$(\tilde{\tilde{I}}, \tilde{\tilde{D}}) = \left(\frac{1}{8} + \frac{1}{32}, \frac{1}{2}\right)$$

$$\tilde{\tilde{U}}_L = \frac{3}{4}\sqrt{\frac{27}{32}} + \frac{1}{4}\sqrt{\frac{11}{32}} = 0.835$$

$$\tilde{\tilde{U}}_H = \frac{1}{2}\sqrt{\frac{27}{32}} + \frac{1}{2}\sqrt{\frac{11}{32}} = 0.75$$

THE SEPARATING POLICIES ARE:

$(I^H, D^H) = (\frac{1}{2}, 0)$, $(I^L, D^L) = \left(p^L(L-D^*), D^*\right)$, WHERE D SOLVES:

$$\frac{1}{2}\sqrt{1-\frac{1-D}{4}} + \frac{1}{2}\sqrt{1-\frac{1-D}{4}-D} = \frac{1}{\sqrt{2}},$$

WHERE $\frac{1}{\sqrt{2}} = U_H^S$ IS THE EXPECTED UTILITY OF A HIGH RISK INDIVIDUAL WHO SUBSCRIBES THE POLICY (I^H, D^H). SOLVING THIS EQUATION WE GET

$$\boxed{D^* = \sqrt{3} - 1 \simeq .732.}$$

THE SEPARATING "MENU" FORMS A CE PROVIDED THE (EXP.) UTILITY OF LOW RISK INDIVIDUALS OF THE POOLING POLICY, $(\bar{p}(\lambda)L, 0)$, WHERE

$$\bar{p}(\lambda) = \frac{\lambda}{2} + \frac{1-\lambda}{4} = \frac{1+\lambda}{4},$$

WHICH IS

$$u\left(1 - \frac{1+\lambda}{4}\right) = u\left(\frac{3-\lambda}{4}\right) = \frac{\sqrt{3-\lambda}}{2},$$

IS LESS THAN THE EXPECTED UTILITY OF THE POLICY (I^L, D^L),

$$\begin{aligned} Eu(I^L, D^L) &= p^L u(1 - I^L - D^L) + (1-p^L) u(1 - I^L) \\ &= \frac{1}{4}\sqrt{1 - \frac{2-\sqrt{3}}{4} - (\sqrt{3}-1)} + \frac{3}{4}\sqrt{1 - \frac{2-\sqrt{3}}{4}} \\ &= \frac{\sqrt{6}}{8}(\sqrt{3} \cancel{+} 1). \end{aligned}$$

THAT IS

$$\frac{\sqrt{6}}{8}(\sqrt{3}+1) \geq \frac{\sqrt{3-\lambda}}{2} \iff \lambda \geq \frac{3}{4}(2-\sqrt{3}) \simeq 0.2$$

NOTE: IF THE POOLING POLICY (WHICH IS PREFERED TO THE SEPARATING POLICIES BY THE HIGH RISK INDIVIDUALS) IS PREFERED TO THE SEPARATING POLICY BY THE LOW RISK INDIVIDUAL (AS WELL) THEN:

- THE POOLING POLICY IS PARETO SUPERIOR TO THE SEPARATING POLICIES, AND
- THE POLICY $(\bar{p}(\lambda)+\varepsilon, 0)$, FOR $\varepsilon > 0$ SMALL, DE-STABILIZES THE SEPARATING POLICIES — THAT IS, THE SEPARATING POLICIES DO NOT FORM A CE.

THUS, WHEN THE SEPARATING MENU IS <u>NOT</u> A CE ....

MAKING THE POOLING POLICY <u>MANDATORY</u> MAKES EVERYONE BETTER OFF.

(EX. 3 IN LIST 2 IS ANALOGOUS.)

Exercise 2b. NYC used to be a pickpocket's playground. In a typical day, a fraction $p_L = 1/4$ of *alert* tourists reported that his wallet was stolen, while this fraction was $p_H = 1/2$ for *inattentive* tourists. Each tourist typically carried $W = 150$ euros in his wallet for the daily expenses, and the typical loss was $L = 100$. Tourists' preferences are described by the Bernoulli utility function $u(x) = \ln x$.

(a) (10 points) Assume that there is a competitive insurance market where tourist may subscribe a policy covering this risk. Determine the policies that will be offered assuming that insurance companies can tell whether a tourist is of the alert or the inattentive type.

Solution. Since the market is competitive, under complete information companies will offer the fair premium full insurance policy to each type; that is, they will offer the policy

$$(I_H, 0) = (100p_H, 0) = (50, 0)$$

to the inattentive tourists, and the policy

$$(I_L, 0) = (100p_L, 0) = (25, 0)$$

to the alert tourists.

(b) (20 points) Assume now that insurance companies cannot tell whether a tourist subscribing a policy is of the alert or the inattentive type, and that there are twice as many inattentive tourists than alert tourists. Which insurance policies will be offered? (To solve an equation you will encounter, these formulae will be useful: $a \ln x + b \ln y = \ln(x^a y^b)$; also, the solution to the equation $ax^2 + bx + c = 0$ is $x = (-b \pm \sqrt{b^2 - 4ac})/(2a)$.)

Solution. As established in class, in a competitive equilibrium, when it exists, insurance companies offer separating fair policies $(I_H, 0) = (50, 0)$ *and* $(\hat{I}_L, \hat{D}_L)$, *where*

$$\hat{I}_L = (100 - \hat{D}_L)p_L,$$

and $\hat{D}_L$ *is such that the inattentive tourists are indifferent between the two policies, that is*

$$\frac{1}{2} \ln \left(150 - (100 - \hat{D}_L)p_L - \hat{D}_L\right) + \frac{1}{2} \ln \left(150 - (100 - \hat{D}_L)p_L\right) = \ln (150 - 50)\,.$$

This equation may be written ~~*for* $x = D_L$~~ *as*

$$\left(150 - (100 - \hat{D}_L)/4 - \hat{D}_L\right)\left(150 - (100 - \hat{D}_L)/4\right) = (100)^2\,.$$

Solving this equation we get

$$\hat{D}_L = 100(2\sqrt{13} - 5)/3 \simeq 73.703.$$

Hence

$$\hat{I}_L = \frac{1}{4}\left(100 - \hat{D}_L\right) = \frac{1}{4}\left(100 - 100(2\sqrt{13} - 5)/3\right) \simeq 6.574\,1.$$

For these policies to form a competitive equilibrium the alert tourist must prefer the policy $(\hat{I}_L, \hat{D}_L)$ *to the pooling policy* $(100\bar{p}, 0)$, *where*

$$\bar{p} = \frac{2}{3}p_H + \frac{1}{3}p_L = \frac{5}{12}.$$

The expected utility of an alert tourist with the policy $(\hat{I}_L, \hat{D}_L)$ *is*

$$\frac{1}{4}\ln\left(150 - \left(200(4 - \sqrt{13})/9\right) - \left(100(2\sqrt{13} - 5)/3\right)\right) + \frac{3}{4}\ln\left(150 - \left(200(4 - \sqrt{13})/9\right)\right) \simeq 4.766$$

and his expected utility with the pooling policy is

$$\ln\left(150 - (100)\,\frac{5}{12}\right) \simeq 4.685.$$

Hence the policies $\{(I_H, 0), (\hat{I}_L, \hat{D}_L)\}$ *form a competitive equilibrium in this market.*

(c) (10 points) Assume that the market is monopolized by a single company, which by law must offer a single insurance policy to all tourists. (That is, the firm cannot "screen" tourists with a menu of policies.) Which policy will this company offer? (Hint. Should the firm offer full insurance? Should it offer a policy intended for both types of tourists or a policy that attracts only inattentive tourist?) Determine which tourists win and lose in this situation relative to that of part (b).

Solution. The company must decide whether to offer a policy that only inattentive tourist subscribe or one which both types of types of tourists subscribe. Obviously, in either case the company will offer full insurance since it can extract more surplus from the risk averse tourists.

If the firm offers a policy that both types subscribe, it has to offer it at the maximum premium the alert tourists are willing to pay, that is,

$$\ln(150 - x) = \frac{1}{4}\ln(150 - 100) + \frac{3}{4}\ln(150),$$

Solving this equation we get $\bar{I} = 150 - \sqrt[4]{50\,(150)^3} \simeq 36$. *The monopoly's expected profit per tourist is*

$$\bar{I} - \bar{p}L = 36 - \frac{5}{12}(100) = -\frac{17}{3}.$$

(The probability that the average tourist suffers the loss ~~given~~ is $\bar{p}$.) Hence the monopoly will not offer this policy.

If the firm offers a policy that only inattentive tourists subscribe, then it charges the premium that solves the equation

$$\begin{aligned} \ln(150 - I_H) &= \frac{1}{2}\ln(150 - 100) + \frac{1}{2}\ln(150) \\ &\Leftrightarrow \\ (150 - I_H)^2 &= 3(50)^2 \\ &\Leftrightarrow \\ I_H &= 150 - 50\sqrt{3} \simeq 63.397. \end{aligned}$$

Since only $2/3$ *of the tourist are inattentive, the monopoly's expected profit*

$$\frac{2}{3}\left(\bar{I}_H - p_H L\right) = \frac{2}{3}\left(\left(150 - 50\sqrt{3}\right) - \frac{1}{2}(100)\right) \simeq 8.93.$$

Hence the monopoly will offer the policy $\left(\bar{I}_H, 0\right)$.

(d) (Question added after the exam.) Assume that the fraction of inattentive tourists is $\lambda \in (0,1)$. For which values of λ would the monopoly of part (c) offer a pooling policy?

Solution. The aggregate probability is now

$$\bar{p}(\lambda) = \frac{\lambda}{2} + \frac{1-\lambda}{4} = \frac{1+\lambda}{4}.$$

If the firm offers a policy that both types subscribe, $\left(\bar{I}, 0\right)$, *where* $\bar{I} = 150 - \sqrt[4]{50\,(150)^3} \simeq 36.025$, *the monopoly's expected profit per tourist is*

$$\bar{I} - \bar{p}(\lambda) L = 150 - \sqrt[4]{50\,(150)^3} - \frac{1+\lambda}{4}(100).$$

If the firm offers a policy that only inattentive tourists subscribe, $\left(\bar{I}_H, 0\right)$, *where* $I_H = 150 - 50\sqrt{3}$, *then the monopoly's expected profit*

$$\frac{\lambda}{2}\left(\bar{I}_H - p_H L\right) = \frac{\lambda}{2}\left(\left(150 - 50\sqrt{3}\right) - \frac{1}{2}(100)\right).$$

Solving

$$\frac{\lambda}{2}\left(\left(150 - 50\sqrt{3}\right) - \frac{1}{2}(100)\right) = 150 - \sqrt[4]{50\,(150)^3} - \frac{1+\lambda}{4}(100)$$

we get.

$$\bar{\lambda} = \frac{\sqrt[4]{168750000} - 125}{25\sqrt{3} - 75} \simeq 0.34779$$

Thus, for $\lambda < \bar{\lambda}$ *the monopoly offers the policy* $\left(\bar{I}_H, 0\right)$, *which all tourists subscribe. Note that alert tourist are worse off, while inattentive tourist are better off, than in the competitive equilibrium of the market identified in part (b).*