MICROECONOMICS II
2006-2007

Final Exam

Monday 19 March

Allocate your time efficiently in order to maximize your score.
Give complete and clear answers. Good luck.

1. (**Walrasian equilibrium, Pareto optima and the core**) 20 points
Consider an exchange economy with two consumers and two commodities. The first good is *indivisible* so that its consumption can take only positive integer values, while the second good is perfectly divisible. The consumption sets are then $X_i = N \times \Re_+$ where $N = \{0, 1, 2, ...\}$. The preferences of the consumers are represented by the utility functions:

$$\begin{aligned} u_1(x_{11}, x_{12}) &= x_{11} + x_{12}; \\ u_2(x_{21}, x_{22}) &= \min\{x_{21}, x_{22}\}. \end{aligned}$$

The endowments are $\omega_1 = (2, 1/2)$ and $\omega_1 = (0, 3/2)$.

   (a) Define the set of feasible allocations and represents these allocations in an Edgeworth box diagram.

   (b) If this economy has Pareto optimal allocations, represent the set of Pareto optimal allocations in the Edgewoth box.

   (c) If these economy has Core allocations, represent the Core in the Edgewoth box.

   (d) Does this economy have a Walrasian equilibrium? If so, find the Walrasian equilibrium and represent it in the Edgewoth box (indicating the WE allocation, price ratio and budget lines).

2. (**Externalities in Consumption**) 30 points
Consider a production economy with two commodities. All firms have access to the following constant returns to scale technology

$$Y = \{y \in \Re^2 : y = \lambda(-1, 1), \lambda \geq 0\}$$

There are I identical consumers with preferences represented by the utility function:

$$u_i(x) = x_{i1} x_2^A,$$

where $x_2^A = \frac{\sum_{i=1}^{I} x_{i2}}{I}$. Individual endowments are $\omega_i = (k, 0)$. Normalize prices so that $p_2 = 1$.

(a) Find the WE.

(b) Find the set of Pareto optimal allocations.

(c) Is there any I such that WE allocations are Pareto optimal?

3. (**Perfectly Competitive Equilibrium**) 30 points
Consider a quasilinear model with a single non-money commodity. There are 20 identical consumers with utility function $u_i(x_i, m_i) = \phi(x_i) + m_i$ where

$$\phi(x_i) = \begin{matrix} 2x_i - \frac{x_i^2}{2} & \text{for } 0 \leq x_i \leq 4 \\ 0 & \text{otherwise} \end{matrix}$$

Assume for simplicity that all endowments are zero. There are $J > 20$ firms with constant marginal cost of production up to capacity 1,

$$c_j(q_j) = \begin{matrix} q_j & \text{for } 0 \leq q_j \leq 1 \\ & \text{otherwise} \end{matrix} .$$

(a) Find Walrasian Equilibria for this economy, and illustrate equilibria in a demand and supply diagram.

(b) Show that the WE is also a PCE. What is the firm's profit? Show that each consumer faces a perfectly elastic supply in the relevant region of the consumer's demand.

(c) Suppose now that firm 1 has the opportunity of undertaking a cost-saving technical innovation at fixed costs F that would (i) lower the marginal cost of production from 1 to3/4, and ii) allow to produce up to a capacity of 16 units. Explain why the innovator is a perfect competitor.

(d) Explain why seller 1 will undertake the innovation if and only if the private profitability of the innovation agrees with its social value.

4. (20 points) Prove that a WE allocation for a exchange economy $\mathcal{E} = \{(X_i, \succsim_i, \omega_i)\}$ belongs to the core $C(\mathcal{E})$.