Microeconomics II
2003-2004

Final Exam

Monday 22 March

Allocate your time efficiently in order to maximize your score.
Give complete and clear answers. Good luck.

1. (**Walrasian equilibrium, Pareto optima and the core**) 20 points
Consider an exchange economy with two consumers and two commodities. The first good is *indivisible* so that its consumption can take only positive integer values, while the second good is perfectly divisible. The consumption sets are then $X_i = N \times \Re_+$ where $N = \{0, 1, 2, ...\}$. The preferences of the consumers are represented by the utility functions:

$$u_1(x_{11}, x_{12}) = x_{11} + x_{12};$$
$$u_2(x_{21}, x_{22}) = \min\{x_{21}, x_{22}\}.$$

The endowments are $\omega_1 = (2, 1/2)$ and $\omega_1 = (0, 3/2)$.

   (a) Define the set of feasible allocations and represents these allocations in an Edgeworth box diagram.

   (b) If this economy has Pareto optimal allocations, represent the set of Pareto optimal allocations in the Edgewoth box.

   (c) If these economy has Core allocations, represent the Core in the Edgewoth box.

   (d) Does this economy have a Walrasian equilibrium? If so, find the Walrasian equilibrium and represent it in the Edgewoth box (indicating the WE allocation, price ratio and budget lines).

2. (**Externalities in Consumption**) 30 points
Consider a production economy with two commodities. All firms have access to the following constant returns to scale technology

$$Y = \{y \in \Re^2 : y = \lambda(-1, 1), \lambda \geq 0\}$$

There are I identical consumers with preferences represented by the utility function:

$$u_i(x) = x_{i1} x_2^A,$$

where $x_2^A = \frac{\sum_{i=1}^{I} x_{i2}}{I}$. Individual endowments are $\omega_i = (k, 0)$. Normalize prices so that $p_2 = 1$.

(a) Find the WE.

(b) Find the set of Pareto optimal allocations.

(c) Is there any I such that WE allocations are Pareto optimal?

3. (**Competition with Fixed Costs**) 30 points

Fixed costs leading to decreasing average costs are usually regarded as inimical to competition, but consider the following. There are two commodities, pizza and money. Firm j supplies *delivered pizza to consumer* i according to a cost function (in terms of money) given by

$$c_j(q_{ji}) = \begin{cases} 0 & if q_{ij} = 0 \\ 4 + q_{ji} & if q_{ij} > 0 \end{cases}$$

where $q_{ij} > 0$ is the quantity that j delivers to i, there is a unit constant marginal cost of producing each pizza and 4 is a fixed delivery cost that is independent of the number of pizzas delivered to i. A firm can deliver pizza to any number of consumers.

Consumer i has quasilinear utility function $u_i(x_i, m_i) = \phi_i(x_i) + m_i$ with

$$\phi_i(x_i) = \begin{cases} 10x_i & for 0 \le x_i \le 1 \\ 10 & otherwise \end{cases}$$

where x_i is the amount of pizzas consumed.

Thus, all consumers have the same preferences and all firms have the same costs. For the following, assume two consumers and two firms.

(a) Verify that the cost function for delivered pizza is not convex. Given that the price of money is unity and the unit price of pizza is p, define a Walrasian equilibrium (WE) for the economy. Demonstrate that there is no WE.

(b) Calculate the social contribution (or marginal product) of each participant and show that they add up to the maximum total gains from trade. Would the social contribution of a consumer/firm change if there were n consumers and n firms?

(c) Speficy a two-part price schedule $p(z)$ for delivered pizza such that if firms and consumers take that price schedule as given, there is a WE where each agent receives his/her social contribution. (Hint: you know from part (a) that linear price schedules, i.e. $p(z) = p \cdot z$, will not work).

(d) Modify the cost function above so that there is a fixed cost for *producing* (rather than for delivering) pizza. Suppose

$$c_j(q_j) = \begin{cases} 0 & if q_j = 0 \\ 4 + q_j & if q_j > 0 \end{cases}$$

where q_j is the total number of pizzas produced by firm j.

Compute the social contributions of all participants and compare their sum to the maximum total gains from trade. Would these social contributions change if there were n consumers and n firms?

(e) Explain the differences, if any, between your answers to (b) and (d). How would large numbers affect the competitivity of the model in part (d)? What is the "approximate" PCE for (d) as $n \to \infty$?

4. (20 points) Prove that a WE allocation for a exchange economy $\mathcal{E} = \{(X_i, \succsim_i, \omega_i)\}$ belongs to the core $C(\mathcal{E})$.