MICROECONOMICS II
2002-2003

Final Exam

Tuesday 25 March

Allocate your time efficiently in order to maximize your score.
Give complete and clear answers. Good luck.

1. (**Walrasian equilibrium, Pareto optima and the core**) 20 points
   Consider an exchange economy with two consumers and two commodities. The consumption sets are $X_i = \Re^2_+$ and the preferences of the consumers are represented by the utility functions:

$$\begin{aligned} u_1(x_{11}, x_{12}) &= \min\{x_{11}, 2x_{12}\}; \\ u_2(x_{21}, x_{22}) &= \min\{3x_{21}, x_{22}\}. \end{aligned}$$

   The aggregate endowment in this economy is $\omega = (12, 12)$.

   (a) Calculate the set of Pareto optimal allocations.

   (b) Assume that consumer 1 has all of good 1 and consumer 2 has all of good 2. Find the WE. Calculate the core of the economy.

   (c) Describe the set of individual endowments, consistent with the aggregate endowment, that result in a WE with positive prices for both goods.

2. (**Externalities in Consumption**) 30 points
   Consider a production economy with two commodities. All firms have access to the following constant returns to scale technology

$$Y = \{y \in \Re^2 : y = \lambda(-1, 1), \lambda \geq 0\}$$

   There are I identical consumers with preferences represented by the utility function:

$$u_i(x) = x_{i1} x_2^A,$$

   where $x_2^A = \frac{\sum_{i=1}^{I} x_{i2}}{I}$. Individual endowments are $\omega_i = (k, 0)$. Normalize prices so that $p_2 = 1$.

(a) Find the WE.

(b) Find the set of Pareto optimal allocations.

(c) Is there any I such that WE allocations are Pareto optimal?

3. (**Competition with Fixed Costs**) 30 points

Fixed costs leading to decreasing average costs are usually regarded as inimical to competition, but consider the following. There are two commodities, pizza and money. Firm j supplies *delivered pizza to consumer* i according to a cost function (in terms of money) given by

$$c_j(q_{ji}) = \begin{cases} 0 & \text{if } q_{ij} = 0 \\ 4 + q_{ji} & \text{if } q_{ij} > 0 \end{cases}$$

where $q_{ij} > 0$ is the quantity that j delivers to i, there is a unit constant marginal cost of producing each pizza and 4 is a fixed delivery cost that is independent of the number of pizzas delivered to i. A firm can deliver pizza to any number of consumers.

Consumer i has quasilinear utility function $u_i(x_i, m_i) = \phi_i(x_i) + m_i$ with

$$\phi_i(x_i) = \begin{cases} 10x_i & \text{for } 0 \leq x_i \leq 1 \\ 10 & \text{otherwise} \end{cases}$$

where x_i is the amount of pizzas consumed.

Thus, all consumers have the same preferences and all firms have the same costs. For the following, assume two consumers and two firms.

(a) Verify that the cost function for delivered pizza is not convex. Given that the price of money is unity and the unit price of pizza is p, define a Walrasian equilibrium (WE) for the economy. Demonstrate that there is no WE.

(b) Calculate the social contribution (or marginal product) of each participant and show that they add up to the maximum total gains from trade. Would the social contribution of a consumer/firm change if there were n consumers and n firms?

(c) Speficy a two-part price schedule $p(z)$ for delivered pizza such that if firms and consumers take that price schedule as given, there is a WE where each agent receives his/her social contribution. (Hint: you know from part (a) that linear price schedules, i.e. $p(z) = p \cdot z$, will not work).

(d) Modify the cost function above so that there is a fixed cost for *producing* (rather than for delivering) pizza. Suppose

$$c_j(q_j) = \begin{cases} 0 & \text{if } q_j = 0 \\ 4 + q_j & \text{if } q_j > 0 \end{cases}$$

where q_j is the total number of pizzas produced by firm j.

Compute the social contributions of all participants and compare their sum to the maximum total gains from trade. Would these social contributions change if there were n consumers and n firms?

(e) Explain the differences, if any, between your answers to (b) and (d). How would large numbers affect the competitivity of the model in part (d)? What is the "approximate" PCE for (d) as $n \to \infty$?

4. (20 points) Prove that a WE allocation for a exchange economy $\mathcal{E} = \{(X_i, \succsim_i, \omega_i)\}$ belongs to the core $C(\mathcal{E})$.