Universidad Carlos III
Ph.D. in Economics

Microeconomics II
2001-2002

Final Exam

(A) Short Questions: For each statement, either verify that it is true or provide a counterexample.

A1. (10) If some consumer has a lexicographic preference, then a competitive equilibrium will not exist.

A2. (10) In a two-good and two-person pure exchange economy where each individual's preference is represented by a utility function of the form $u^i(x,y) = \alpha_i x + y$, where $0 < \alpha_1 < \alpha_2$, both the competitive price ratio and the competitive allocation are unique.

A3. (10) Consider a two-person, two-good pure exchange economy where individuals' preferences are represented by the utility function $u^i(x,y) = xy$ for $i \in \{A, B\}$, and their endowments are $(\bar{x}^A, \bar{y}^A) = (3,1)$ and $(\bar{x}^B, \bar{y}^B) = (1,3)$. Is the allocation $[(1.8, 1.8), (2.2, 2.2)]$ in the core of the economy? Is it in the core of the replica economy $E(2)$?

(B) Longer Questions

B1. (30) There are only two goods in the world, bread and wheat, quantities of which will be denoted by x and y, respectively. The economy consist of 2 consumers and one firm. The firm can produce bread form wheat according to the production function $f(z) = \sqrt{z}$. Consumer 1 is endowed with 2 units of wheat and no bread, and has preferences represented by the utility function $u^1(x,y) = 2x + y$. Consumer 2 owns neither wheat nor bread, but he is the sole owner of the firm; his preferences are described by the utility function $u^2(x,y) = xy$.

(a) Characterize and represent in a diagram the set of interior Pareto optimal allocations.

(b) Calculate the competitive equilibrium prices and allocations.

(c) Would Consumer 2 be better off if the firm behaved as a monopolist? What if the firm set the prices so as to maximize Consumer 2's welfare?

B2. (20) Consider an economy where there are n individuals who are endowed with one unit of time and who care only about their consumption of food. Food can be produced using time as input according to a constant return to scale technology that yields K units of food per unit of time. The coefficient K is the amount of "knowledge" in the economy. For simplicity, assume that $K = \sum_{i=1}^{n} k_i$, where each k_i is the amount of time each person devotes to gaining knowledge.

(a) Assuming that there is no market for knowledge, determine the amount of time each individual will devote to gaining knowledge and producing food. What would be the impact of an increment in the number of individual over the per-capita consumption of food?

(b) Determine whether the total allocation of time devoted to gaining knowledge and to food production in (a) is Pareto optimal. If it is not, how much (total) time devoted should be devoted to these two activities? What is the efficiency loss (in units of food) in (a)? How does the the per-capita loss of food consumption depend upon the number of individuals?

(c) Assume now that the right to use the above technology is assigned to a firm. This firm demands time as input (to be used in both gaining knowledge and food production) and supplies food. The ownership of this firm is equally distribuited among consumers. Determine whether the competitive equilibrium allocation is Pareto optimal?

B3. (20) Consider an economy that extends over two periods, today and tomorrow, and where there are two consumers, A and B, and a single perishable good. The state of nature tomorrow can either be H or L. Consumers' preferences over consumption today and tomorrow are represented by the utility functions $u^A(c_0, c_H, c_L) = c_0 + .3\,(c_H)^2 + 5c_H - .2(c_L)^2 + 5c_L$, and $u^B(c_0, c_H, c_L) = c_0 + .1\,(c_H)^2 + 5c_H - .2(c_L)^2 + 5c_L$. Both A and B are endowed with twelve units of the good in each of the two periods.

(a) Suppose that the only market is a credit market. What will be the competitive equilibrium interest rate and how much will each person borrow or save? Is the resulting allocation Pareto optimal?

(b) Suppose that, in addition to the credit market in (a), there is another market in which one can buy or sell insurance today against the occurrence of event H. (Each unit of such insurance is a contract in which the seller agrees to pay the buyer one unit of consumption tomorrow if event H occurs.) Let p denote the market price of the insurance (in units of consumption today). Determine the competitive equilibrium prices (i.e., the interest rate and the price p of insurance) and allocation, and determine whether this allocation Pareto is optimal.