Universidad Carlos III — Microeconomics
Ph.D. in Economics — 2000-01

Final Exam

Short Questions: Answer any two of the following questions:

S1 (10). Consider an economy where there are two goods, x and y, two consumers, and no production is possible. Individual 1 prefers lexicographically good x over y, whereas Individual 2's preferences are described by a continuous, strictly increasing and strictly concave utility function. Determine whether a competitive equilibrium exists.

S2 (10). Show that in a pure exchange economy where individuals preferences satisfy local non-satiation every competitive equilibrium allocation is Pareto optimal.

S3 (10).Consider an economy where there are two goods, x and y, and two periods, today and tomorrow. There is uncertainty about the state of nature tomorrow, which can be either sunny (S) or cloudy (C). There are not markets for contingent trade, but there are spot markets at each date, and two securities, a and b. Security a pays 2 units of x if the state is S, and it pays nothing in C. Security b pays one unit of x in each state. Show that a Radner equilibrium allocation is Pareto optimal, and find formulas relating the spot market and securities prices to the Arrow-Debreu prices.

S4 (10). Consider an economy where there are 3 individuals whose preferences over income, y, and a public good, x, are represented by twice continuously differentiable, increasing and concave utility functions, $u^i(x, y^i)$ for $i = 1, 2, 3$. There is a constant returns to scale technology for producing public good that yields one unit of public goof for each unit of income used as input. Determine whether the allocations induced by the mechanism (S, ϕ), defined by $S_i = \mathbb{R}$ and $\phi(s) = (\rho(s), \tau(s))$, where $\rho(s) = s_1 + s_2 + s_3$, $\tau^1(s) = (\frac{1}{3} + s_2 - s_3)\rho(s)$, $\tau^2(s) = (\frac{1}{3} + s_3 - s_1)\rho(s)$, and $\tau^3(s) = (\frac{1}{3} + s_1 - s_2)\rho(s)$, are Pareto optimal.

Longer Questions: answer all of the questions.

L1 (40). Anna and Bob consume only peanuts (in pounds), x, and beer (in pints), y. Their preferences are represented by the utility functions $u^A(x,y) = y + \ln(x+1)$, and $u^B(x,y) = x + 2y$, and their endowments are $\omega^A = (2,0)$ and $\omega^B = (2,6)$.

(a) Calculate and graph (in an Edgeworth box) the set of Pareto optimal allocations.

(b) Calculate the set of competitive equilibrium prices and allocations.

Now suppose that Anna discovers a technology which allows her to transform peanuts into beer at a constant rate of one pint per pound of peanuts used as input.

(c) Are the allocations identified in (a) still Pareto optimal? If they are not, calculate and graph the new set of Pareto optimal allocations.

(d) Assume now that Anna sets up a firm to exploit the new technology, and that the firm behaves as a price taker profit maximizer. Calculate the set of competitive equilibrium prices and allocations.

L2 (20). There are four persons, two goods (x and y), and no production is possible. Everyone has the same preference, described by the utility function $u(x,y) = xy$. The initial endowments are $\omega^1 = \omega^2 = (2,0)$, and $\omega^3 = \omega^4 = (0,2)$. Determine the core.

L3 (20 points) Two fishermen have free access to a lake. The amount of fish (in pounds) caught weakly by each fisherman $i \in \{1,2\}$ depends on number of days he boats out, z_i, and on the total number of days both of then boat out, $z = z_1 + z_2$, according to the function $q_i = (10 - z)z_i$. The fish caught is sold in the local market at a price of two euros per pound. Both fishermen care about their income, y, and their leisure time (measured in number of days a week), x, and their preferences are described by the utility function $u(x,y) = 2x + y$.

(a) Calculate the number of days each fisherman will boat out, and the resulting allocation of income and leisure. If this allocation is not Pareto optimal, find a Pareto optimal Pareto superior allocation, and a scheme of taxes that would induce this new allocation.

(b) Suppose that the right to fish in the lake is allocated to one of the two fishermen using a second price auction. The fisherman who wins the auction decides how many days he boats out; the money collected goes to the fisherman who losses the auction. Determine whether the resulting allocation is Pareto optimal.