Microeconomic Theory I, Problem Set 2

Due date: Wednesday September 30
From MWG: Ch. 1, C.1, C.2, D.1, D.3, D.4; Ch. 2, F.12

Exercise 1 *This problem concerns WARP and* **Walrasian budget sets***. A Walrasian budget set is the set of goods affordable when prices are p and there is wealth w to spend. In the case of two goods, the typical Walrasian budget set is*

$$B_{p,w} = \{x \in \mathbb{R}^2_+ : p \cdot x \leq w\}.$$

For this problem, assume that we have a single-valued choice structure,

$$C(B_{p,w}) = \{x(p,w)\}.$$

Throughout, assume that $p \cdot x(p,w) = w$.

1. *On the same graph, carefully draw the three budget sets $B_{p,w}$ when $(p^A, w^A) = ((4,2), 72)$, $(p^B, w^B) = ((1,2), 36)$, $(p^C, w^C) = ((1,1), 20)$. (You may want this graph to fill as much as half a page.)*

2. *Pick three points $x(p^A, w^A)$, $x(p^B, w^B)$, and $x(p^C, w^C)$ that do* **not** *violate WARP. Explain your choices (recalling that $p \cdot x(p,w) = w$).*

3. *Pick three points $x(p^A, w^A)$, $x(p^B, w^B)$, and $x(p^C, w^C)$ with the property that any pair of them* **do** *violate WARP. Explain your choices (recalling that $p \cdot x(p,w) = w$).*

4. *If (p^C, w^C) is changed from $((1,1), 20)$ to $((1,1), 30)$, is it possible to find three points with the property that any pair of them violate WARP? Explain (recalling that $p \cdot x(p,w) = w$).*

Exercise 2 *Consider the following Walrasian demand function: $L = 2$, and for all $(p,w) \in \mathbb{R}^{L+1}_+$,*

$$\begin{pmatrix} x_1(p,w) \\ x_2(p,w) \end{pmatrix} = \begin{pmatrix} f(p_1,p_2) \\ g(w,p_1,p_2) \end{pmatrix}. \tag{1}$$

Throughout this exercise we allow $x_2(p,w)$ to be negative.

1. *Show that $x(p,w) = (x_1(p,w), x_2(p,w)))$ satisfy homogeneity of degree 0 and Walras law if and only if $x(p,w)$ takes the following form:*

$$\begin{pmatrix} x_1(p,w) \\ x_2(p,w) \end{pmatrix} = \begin{pmatrix} h\left(\frac{p_1}{p_2}\right) \\ \frac{w - p_1 h\left(\frac{p_1}{p_2}\right)}{p_2} \end{pmatrix}. \tag{2}$$

   for some non negative function h. For the rest of the exercise, suppose that $x(.,.)$ can be written as above, and let us furthermore assume that h is positive, differentiable, and strictly decreasing.

2. *Compute the matrix of price effects. Identify if there is a Giffen good?*

3. *What does the income expansion paths look like? What does that mean in terms of income effects? Compute the vector of income effect. Is there an inferior good?*

4. *Compute the Slutsky matrix. Check that $S(p,w)p = 0$ and $p^T \times S(p,w) = 0$. Show that $S(p,w)$ is semidefinite negative.*

5. *Does the demand function $x(\cdot)$ satisfy the WARP?*