Microeconomic Theory I, Problem Set 1

Exercise 1 Show that if \succeq is transitive, then the strict $PR \succ$ derived from \succeq is also transitive. Show that if \succeq is transitive, then for all $x, y, z \in X$, $\{x \succeq y \text{ and } y \succ z\}$ implies $x \succ z$.

Exercise 2 Let \succeq be a PR on $X = \{a, b, c\}$ such that

$$a \succ b, b \succ c, and c \succ a.$$
 (1)

Does (1) completely characterize \geq ? Can \succeq be rational? Can it be acyclic? Can it be complete?

Exercise 3 Let C be a choice function on $X = \{a, b, c\}$ such that $\mathcal{B} = 2^X$ and

$$a \in C(\{a, b\}), b \in C(\{b, c\}), and c \in C(\{c, a\}).$$
 (2)

NB: note that $a \in C(\{a,b\})$ only means that a is one element of $C(\{a,b\})$, it does not mean that it is the only one, because b could also belong to $C(\{a,b\})$. So in the questions below, you can decide whether $C(\{a,b\})$ is equal to $\{a\}$ or $\{a,b\}$ depending on the property you want to satisfy.

- 1. Can C be nonempty? Can C be consistent? Can C satisfy WARP?
- 2. Can C be rationalized by a PR?Can C be rationalized by a complete PR? Can it be rationalized by a transitive PR?
- 3. Can C be single valued (i.e., for all $B \in 2^X$, C(B) is single-valued)? Can C be single valued and satisfy WARP?
- 4. Can C be single-valued and consistent?
- 5. Can C be single-valued and rationalized by a PR?