May 6, 2019

MATHEMATICS FOR ECONOMICS II (2018-19)

ECONOMICS, LAW–ECONOMICS, INTERNATIONAL STUDIES–ECONOMICS

SHEET 6. DOUBLE INTEGRALS

(1) Compute the following double integrals:

$$a)\int_0^1\int_0^1 xydxdy \qquad b)\int_0^1 dx\int_0^1 (2x+y)dy \qquad c)\int_0^1\int_0^1 xy(x+y)dxdy$$

$$d)\iint_{[0,1]\times[1,4]}\sqrt{xy}dxdy \quad e)\int_0^1\int_0^1 (e^x+e^y)dxdy \quad f)\int_0^1 dy\int_0^1 e^{x+y}dxdy$$

(2) Compute the following double integrals:

$$(a)\iint_A xdydx \qquad (b)\iint_B (x+y)dxdy \qquad (c)\iint_C xe^{-x^2/y}dxdy$$

$$(d)\iint_D \frac{1}{(x+y)^2}dydx \quad (e)\iint_E \frac{x^2}{y^2}dydx \qquad (f)\iint_F \frac{x^2}{y^2}dxdy$$

where

$A=\{(x,y)\,:\,x\geq 1, 2x^2-2\leq y\leq x^2+x\}$,
B the quadrilateral of vertexes $(1,1)$, $(2,2)$, $(3,1)$, $(6,2)$,
$C=\{(x,y)\,:\,0\leq x\leq\sqrt{y}, 1\leq y\leq 2\}$,
$D=[3,4]\times[1,2]$,
$E=\{(x,y)\,:\,1\leq y\leq x, 1\leq x\leq 2\}$,
$F=\{(x,y)\,:\,1\leq x\leq y, 1\leq y\leq 2\}$.

(3) (a) Using that $dxdy=\rho\, d\rho d\theta$ in polar coordinates, compute $\iint_A e^{-x^2-y^2}dxdy$, where $A=\{(x,y):0\leq x^2+y^2\leq R^2\}$.

(b) Compute from (a): $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-x^2-y^2}dxdy$.

(c) Deduce from (b) the value of $\int_{-\infty}^{\infty} e^{-x^2}dx$.

(d) From (a), using that $\{(x,y):0\leq x^2+y^2\leq a^2\}\subset[-a,a]^2\subset\{(x,y):0\leq x^2+y^2\leq 2a^2\}$, compute approximately $\int_{-a}^{a} e^{-x^2}dx$.

(e) From (d), and supposing that $0<a<b$, compute approximately $\int_a^b e^{-x^2}dx$.

(f) From (d), and supposing that $a<0<b$, compute approximately $\int_a^b e^{-x^2}dx$.

(4) Find the area limited at the right by the circle of radius 2 and at the left by the line $x=1$.