UC3M
Mathematics for Economics II (final exam)
June 29, 2022

Niu: _____ Group: _____

Name: _____

Question:	1	2	3	4	5	Total
Points:	20	20	20	20	20	100
Score:						

1

(a) (10 points) It is known that

$$\begin{vmatrix} 1 & 0 & 1 & a \\ 2 & -1 & 1 & b \\ 3 & 4 & 0 & c \\ 1 & -1 & 0 & d \end{vmatrix} = 10.$$

Find the value of the determinant

$$\begin{vmatrix} 1 & 0 & 2a+6 & 1 \\ 2 & -1 & 2b & 1 \\ 3 & 4 & 2c & 0 \\ 1 & -1 & 2d & 0 \end{vmatrix}.$$

(b) (10 points) Find an echelon matrix equivalent to the 4×5 matrix A below.

$$A = \begin{pmatrix} 1 & -1 & 2 & 2 & 1 \\ 3 & 3 & 0 & 2 & -1 \\ 2 & -4 & 6 & 1 & -1 \\ -1 & -3 & 2 & -3 & -2 \end{pmatrix}$$

Use the echelon matrix to calculate the rank of A.

2

Consider the following matrix.

$$A = \begin{pmatrix} 2/3 & -1/3 \\ -1/3 & 2/3 \end{pmatrix}.$$

(a) (10 points) Calculate de eigenvalues and eigenvectors of A and prove that A is diagonalizable. Find a diagonal form D of A and an associated matrix P. Justify your answers.

(b) (10 points) Using the results obtained in part (a) above, calculate the nth–power of A, that is, calculate

$$\begin{pmatrix} 2/3 & -1/3 \\ -1/3 & 2/3 \end{pmatrix}^n,$$

where n is an arbitrary positive integer. Justify your answer.

3

(a) (10 points) Classify the quadratic form $Q(x,y,z) = \frac{a}{3}x^2 + ay^2 + \frac{27}{a}z^2 + axy + 3yz$, where $a \neq 0$ is a parameter.

(b) (10 points) Draw the plane set $D = \{(x,y) : 0 \leq x \leq 2,\ y \leq x,\ y \leq 2 - x\}$ and find the value of the double integral

$$\iint_D \sqrt{x+y}\, dx\, dy.$$

4

(a) (10 points) Study the convergence of the improper integral

$$\int_3^4 \ln(x-3)\,dx$$

and find its value if it results to be convergent.

Hint: Calculate $\lim_{x\to 3^+}(x-3)\ln(x-3)$ using L'Hopital's Rule.

(b) (10 points) Study the convergence of the improper integral

$$\int_5^\infty \frac{1}{25+x^2}\,dx$$

and find its value if it results to be convergent.

5

(a) (10 points) Calculate the following limit

$$\lim_{n\to\infty} \frac{\sqrt{n^2+7}-n}{\sqrt{n^2+13}-n}.$$

Justify your answer.

(b) (10 points) State the Theorem of Leibniz about alternating series.
Check that the series

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{\sqrt[3]{n}}.$$

fulfils all the hypotheses of the Theorem of Leibniz. If S is the sum of the above series, and S_{26} denotes the partial sum of the first 26 terms of the series,

$$\frac{1}{\sqrt[3]{1}} - \frac{1}{\sqrt[3]{2}} + \cdots + \frac{1}{\sqrt[3]{25}} - \frac{1}{\sqrt[3]{26}},$$

which is the bound to $|S - S_{26}|$ provided by the Theorem of Leibniz?