UC3M Mathematics for Economics II (mock final exam)

Name: $_$

Question:	1	2	3	4	5	Total
Points:	20	20	20	20	20	100
Score:						

1

(a) (10 points) Consider the matrix

	(2	0	0		
A =		-4	-4	2		
	ĺ	2	3	1)	

Prove that A is diagonalizable, write its diagonal form and the matrix P.

(b) (10 points) Classify the quadratic form $Q(x, y, z) = 5x^2 + 2y^2 + 2z^2 - 4xy + 4xz + 8yz$ restricted to the subset $C = \{(x, y, z) : x + z = 0\}$.

2

- (a) (10 points) Let the set $B = \{(x, y) : 0 \le x \le 8, 0 \le y \le x, xy \le 16\}$. Make a drawing of this set.
- (b) (10 points) Calculate the double integral

$$\iint_B x dx dy.$$

3

- (a) (10 points) Calculate the area of the set A of \mathbb{R}^2 defined by $A = \{(x, y) : x \le 0, 0 \le y \le \frac{1}{(4-x)^2}\}.$
- (b) (10 points) Consider the improper integral

$$\int_0^4 \frac{xdx}{4-x^2}$$

Study whether it converges and, in this case, compute its value.

|4|

(a) (10 points) The sequence $\{x_n\}_{n=1}^{\infty}$ satisfies $x_{n+1} = x_n^2 + 1$ for all $n = 1, 2, \ldots$ and $x_1 = \frac{1}{2}$. Justify why the sequence cannot be convergent.

(b) (10 points) Calculate
$$\lim_{n\to\infty} \left(\sqrt{n+4n^2}-2n\right)$$
.

|5|

(a) (10 points) Study the convergence of the series $\sum_{n=1}^{\infty} \frac{n^n}{4^n n!}$.

(b) (10 points) Calculate the sum of the series $\sum_{n=1}^{\infty} \left(\frac{1}{n!} - \frac{1}{(n+1)!} \right)$.