

Applied Economics

Quasi-experiments: Instrumental Variables and Regression Discontinuity

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Policy evaluation with quasi-experiments

- In a quasi-experiment or natural experiment there is a source of randomization that is “as if” randomly assigned, but this variation was not the result of an explicit randomized treatment and control design. We distinguish two types of quasi experiments:
 - A case in which treatment (D) is “as if” randomly assigned (perhaps conditional on some control variables X).
 - A case in which a variable (Z) that influences treatment (D) is “as if” randomly assigned (perhaps conditional on X), then Z can be used as an instrumental variable for D in an IV regression that includes the control variables X .
 - The article by Angrist is an example of this case.

Lifetime Earnings and the Vietnam Era Draft Lottery: Evidence from Social Security Administrative Records, Angrist, AER(1990)

- Did military service in Vietnam have a negative effect on earnings?
- A negative relationship between earnings and veteran status does not imply that veteran status causes lower earnings.
- Simple comparisons of earnings by veteran status give a biased measure of the effect of treatment on the treated (unless veteran status is independent of potential earnings).
- Comparisons of earnings controlling for observed characteristics make sense if veteran status is independent of potential earnings after these observed variables are taken into account.

Effect of veteran status on earnings

Let y represent earnings, D_i denote Vietnam-era veteran status, and X_i a set of controls:

Consider estimating the following conditional expectation by OLS

$$E[Y_i|D_i, X_i] = \beta_0 + \alpha D_i + \sum_k \gamma_k X_{ki}$$

- There is probably some unobserved difference that made some men choose the military and others not, and this difference could be correlated with earning potential.
- If D_i is correlated with unobserved variables that belong to the equation, OLS estimates are inconsistent.
- A possible solution is to find a valid instrumental variable.

An instrument for veteran status

- Concerns about the fairness of the U.S. conscription policy led to the institution of a draft lottery in 1970.
- This lottery was conducted annually during 1970-1972. It assigned random numbers (from 1 to 365) to dates of birth in cohorts of 19-year-olds. Men with the lottery numbers below a cutoff were called to serve (the cutoff was determined every year by the Department of Defense).
- Veteran status was not completely determined by randomized draft eligibility: some volunteered, while others avoided enrollment due to health conditions or other reasons. So, draft eligibility is simply correlated with Vietnam-era veteran status.

Draft eligibility as an instrument 1/2

- Let Z_i indicate draft eligibility (takes the value one if i got a number below the cutoff).
- In order to identify the causal effect of D_i on earnings it is crucial that the only reason for $E(Y_i|Z_i)$ to change when Z_i changes is the variation in $E(D_i|Z_i)$. Draft eligibility affects earnings only through its effect on veteran status.
- A simple check on this is to look for an association between Z_i and personal characteristics that should not be affected by D_i , for example race or sex. Another check is to look for an association between Z_i and Y_i for samples in which there is no relationship between D_i and Z_i .

Draft eligibility as an instrument 2/2

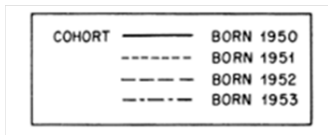
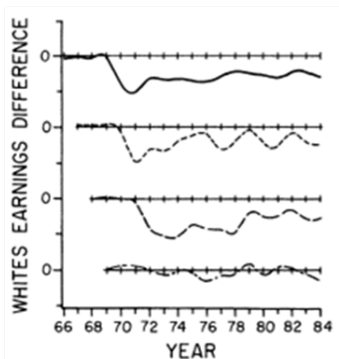
- Angrist looks for instance at 1969 earnings, since 1969 earnings predate the 1970 draft lottery. He finds no effect of the draft eligibility (row 69 in Table 1).
- With the same goal, he also looks at the cohort of men born in 1953. Although there was a lottery drawing that assigned a random number to the 1953 birth cohort in 1972, no one from that cohort was actually drafted. So Z and D are unrelated for this cohort. Angrist finds no significant relationship between earnings and draft eligibility status for men born in 1953 (using the 1972 cutoff).
- These results support the claim that the only reason for draft eligibility to affect earnings is through its impact on veteran status.

Differences in Earnings by Draft Eligibility - Regressions

TABLE 1—DRAFT-ELIGIBILITY TREATMENT EFFECTS FOR EARNINGS

Whites								
Year	FICA Taxable Earnings				Total W-2 Compensation			
	1950	1951	1952	1953	1950	1951	1952	1953
66	-21.8 (14.9)							
67	-8.0 (18.2)	13.1 (16.4)						
68	-14.9 (24.2)	12.3 (19.5)	-8.9 (19.2)					
69	-2.0 (34.5)	18.7 (26.4)	11.4 (22.7)	-4.0 (18.3)				
70	-233.8 (39.7)	-44.8 (36.7)	-5.0 (29.3)	32.9 (24.2)				
71	-325.9 (46.6)	-298.2 (41.7)	-29.4 (40.2)	27.6 (30.3)				
72	-203.5 (55.4)	-197.4 (51.1)	-261.6 (46.8)	2.1 (42.9)				
73	-226.6 (67.8)	-228.8 (61.6)	-357.7 (56.2)	-56.5 (54.8)				
74	-243.0 (81.4)	-155.4 (75.3)	-402.7 (68.3)	-15.0 (68.1)				
75	-295.2 (94.4)	-99.2 (89.7)	-304.5 (85.0)	-28.3 (79.6)				

Differences in Earnings by Draft Eligibility - A Graph



Notes: The figure plots the difference in FICA taxable earnings by draft-eligibility status for the four cohorts born 1950–53. Each tick on the vertical axis represents \$500 real (1978) dollars.

Wald Estimator 1/2

- In a simple model with only D as a control: $Y_i = \beta_0 + \alpha D_i + \varepsilon_i$,
- With Z a valid IV we can write $\alpha = \text{Cov}(Y_i, Z_i) / \text{Cov}(D_i, Z_i)$
- If Z is a dummy variable taking the value one with probability p , for any W we can write:

$$\begin{aligned}\text{Cov}(W_i, Z_i) &= E[W_i Z_i] - E[W_i]E[Z_i] \\ &= \{E[W_i | Z_i = 1] - E[W_i | Z_i = 0]\} p * (1 - p)\end{aligned}$$

- Then:

$$\alpha = \frac{\text{Cov}(Y_i, Z_i)}{\text{Cov}(D_i, Z_i)} = \frac{E[Y_i | Z_i = 1] - E[Y_i | Z_i = 0]}{E[D_i | Z_i = 1] - E[D_i | Z_i = 0]}$$

Wald Estimator 2/2

- If D is also a dummy, for instance representing the treatment group:
 - $E[D_i|Z_i = 1]$ is the probability of $D = 1$ when $Z = 1$, or the proportion of treated among those with $Z = 1$
 - $E[D_i|Z_i = 0]$ is the probability of $D = 1$ when $Z = 0$, or the proportion of treated among those with $Z = 0$
 - The denominator captures the impact of the instrument on the probability of receiving treatment.
- The sample analogue of α is known as the Wald estimator.

The Wald Estimator (conditioning on X):

$$\hat{\alpha}_W(X) = \frac{\bar{Y}(X, Z=1) - \bar{Y}(X, Z=0)}{\bar{P}_{D=1}(X, Z=1) - \bar{P}_{D=1}(X, Z=0)}$$

Wald Estimator in this case

- Numerator:
 - $\bar{Y}(X, Z = 1)$: average earnings for drafted individuals
 - $\bar{Y}(X, Z = 0)$: average earnings for non-drafted individuals
 - Interpret the coefficients in $Y_i = \beta_0 + \beta_1 Z_i + u_i$
- Denominator:
 - $\bar{P}_{D=1}(X, Z = 1)$: participation rate among those drafted: the proportion of veterans ($D = 1$) among those drafted ($Z = 1$)
 - $\bar{P}_{D=1}(X, Z = 0)$: participation rate among those not drafted: the proportion of veterans ($D = 1$) among those not drafted ($Z = 0$)
 - Interpret the coefficients in $D_i = \delta_0 + \delta_1 Z_i + u_i$

Results

Table 2 IV estimates of the effects of military service on US white men born 1950					
Earnings year	Earnings		Veteran status		Wald estimate of veteran effect
	Mean	Eligibility effect	Mean	Eligibility effect	
	(1)	(2)	(3)	(4)	(5)
1981	16,461	- 435.8 (210.5)	0.267	0.159 (.040)	- 2,741 (1,324)
1970	2,758	- 233.8 (39.7)			- 1,470 (250)
1969	2,299	- 2.0 (34.5)			

Notes: Figures are in nominal US dollars. There are about 13,500 observations with earnings in each cohort. Standard errors are shown in parentheses.

Table taken from Angrist and Pischke, Mostly Harmless Econometrics.

- For men born in 1950, there are significant negative effects of eligibility status on earnings in 1970, when these men were beginning their military service and in 1981, ten years later.
- In contrast, there is no evidence of an association between eligibility status and earnings in 1969, the year the lottery drawing for men born in 1950 was held but before anyone born in 1950 was actually drafted.
- Since eligibility status was randomly assigned, estimates in column (2) represent the effect of draft eligibility on earnings.

Wald Estimator

- To go from draft-eligibility effects to veteran-status effects we need the denominator of the Wald estimator, which is the effect of draft-eligibility on the probability of serving in the military:

$$\overline{P}_{D=1}(X, Z=1) - \overline{P}_{D=1}(X, Z=0).$$

- This information is reported in column (4): draft-eligible men were 0.16 more likely to have served in the Vietnam era.
- For earnings in 1981, long after most Vietnam-era servicemen were discharged from the military, the Wald estimate of the effect of military service is about 17 percent of the mean.
- Effects were even larger in percentage terms in 1970, when affected soldiers were still in the army.

Regression Discontinuity: Introduction

- Another approach used in quasi-experiments is called regression discontinuity (RD).
- RD is useful in a case in which:
 - there is a continuous variable W that affects Y .
 - treatment (D) is a discontinuous function of W . In particular, treatment participation depends on W crossing a threshold w_0 .
- Note that W is not a valid instrument because it does not satisfy the exogeneity assumption.
- RD exploits the fact that there is discontinuity in the relation between D and W but continuity in the relation between Y and W .
- Intuitively, considering units within a small interval around the threshold is similar to having a randomized experiment at that point.

Example: The effect of summer schools on grades

- Matsudaira in the Journal of Econometrics (2008) exploits a policy that requires all students in grade levels three and above to attend a summer school program if their final grades were below some threshold.
- One way to estimate the effect of these courses on the following year GPA is to compare students just below the threshold (therefore attending summer school) with students just above the threshold (students that avoided the summer courses by very little).
- As long as the threshold is not used to decide other outcomes it seems reasonable to think that any jump in the outcome around that threshold is due to attending summer courses.

Matsudaira strategy: some quotes

- “...the observed characteristics of students in the neighborhood of the critical pass-fail cutoff scores are nearly identical. This supports the claim that the subsequent differences in mean outcomes of students just below and just above the critical scores are attributable to the causal impact of summer school.”
- the identification strategy ... “is to compare the achievement outcome scores of students just failing the baseline test to those just passing. Under the assumption that all student characteristics affecting achievement vary smoothly with baseline test scores, the difference in outcome scores at the pass-fail cutoff can be used to identify the causal impact of summer school on achievement.”

Idea of regression discontinuity

- The idea of regression discontinuity (RD) is then to estimate the treatment effect by comparing individuals with W just below a threshold w_0 (they will be considered treated) to those individuals with W just above w_0 (untreated).
- If the direct effect of W on the outcome Y is continuous, the treatment effect should show up as a jump in Y around w_0 . The magnitude of this jump estimates the treatment effect.
- Key assumption: individuals right above and below w_0 are comparable. Random variation puts someone above w_0 and someone below, generating differences in treatment. Then, any difference in Y right at w_0 is due to the treatment.

Types of RD

Two types of RD designs:

- In sharp RD design, everyone above (or below) the threshold w_0 gets treatment.
- In fuzzy RD design, crossing the threshold w_0 influences the probability of treatment, but it is not the only determinant.

Sharp RD 1/2

- Sharp RD is when treatment (D_i) is a deterministic and discontinuous function of an observable variable W . For instance:

$$D_i = \begin{cases} 1 & \text{if } W_i < w_0 \\ 0 & \text{if } W_i \geq w_0, \end{cases}$$

where w_0 is a known threshold or cutoff value.

- For instance, all students with $W < w_0$ have to attend summer courses and none of the students with $W \geq w_0$ attend those courses.
- In this case the jump in Y at w_0 is the average effect of the treatment on people at the threshold. It could be a good proxy for the effect on other individuals.

Sharp RD 2/2

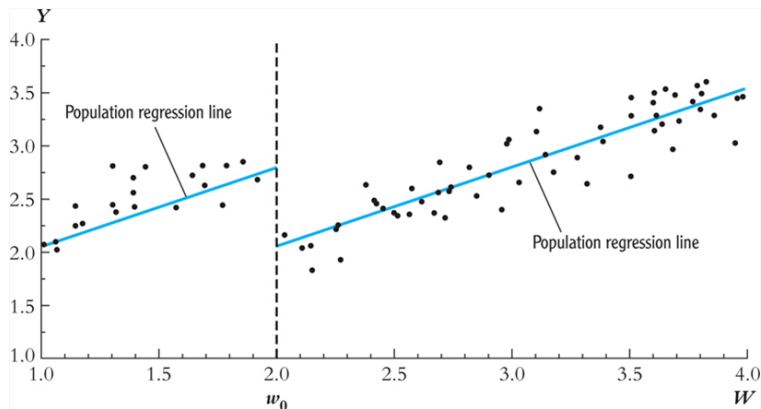
- If the regression model is linear in W , except for the jump due to the treatment, the treatment effect β_1 can be estimated by OLS:

$$Y_i = \beta_0 + \beta_1 D_i + \beta_2 W_i + u_i$$

- If crossing the threshold affects Y_i only through D_i , then OLS estimators are consistent.
- In a sharp RD design D_i is a deterministic function of W_i . The causal effect of D_i is captured by controlling for the relationship between W and Y (represented by W_i).
- It is possible, and common, to include $f(W_i)$ instead of just W_i in the regression, where $f(W_i)$ is continuous in the neighborhood of w_0 . The job of $f(W_i)$ is to capture as good as possible the relationship between W and Y .

Sharp RD - a graph for a linear case

- All individuals with W below w_0 are treated, the treatment effect is the jump or “discontinuity”



From Stock and Watson, chapter 13.

OLS in summer schools example

- Consider the following model: $Y_i = \beta_0 + \beta_1 D_i + \varepsilon_i$, where D_i is a dummy for attending summer school.
- The problem for using OLS to estimate the effect of D_i is that attendance is endogenous.
- The author shows that those attending summer school have lower prior achievement levels, are more likely to be black or hispanic, to qualify for free lunch, and to live in poorer neighborhoods. There may be also unobserved differences between students attending and not attending summer school.
- Therefore we can probably expect a negative bias on estimates of the effect of attending summer school if we just compare the two groups.

Solution with sharp RD 1/2

- If all students getting a score below the cutoff actually attend summer school with probability one we would have a case of sharp RD.
- In that case it can be shown that the effect of summer school on the following year GPA is identified by the difference in grades for students just below and just above the cutoff. This is because students barely below the cutoff should be on average similar in all the relevant factors to those students who scored barely above the cutoff.
- The assumption needed is that the conditional expectations of all characteristics affecting test scores are continuous at the cutoff score.

Solution with sharp RD 2/2

- If students getting a score below 0 attend summer school we should see a jump in summer school attendance at $W = 0$.
- And if summer school has an effect on Y_i , we should see a jump at $W = 0$ also for Y .
- Then, we can estimate the effect of attending summer school using an equation like this:

$$Y_i = \beta_0 + \beta_1 D_i + f(W_i) + u_i,$$

where $f(W_i)$ is a smooth function, for instance a p th-order polynomial.

Fuzzy RD

- In the case of the design called fuzzy RD, crossing the threshold affects the probability of being treated.
- For instance, students with a grade below the cutoff are more likely to attend summer school, but for other reasons they may be exempted. Or students with a grade above the cutoff are less likely to attend but for other reasons they may be asked to attend. This is the case if the rules determining summer school attendance are more complex and do not depend only on previous grades.
- Therefore, crossing the threshold does not force individuals into treatment, other factors are also considered to determine treatment.

Solution with Fuzzy RD

- Defining a dummy variable for crossing the threshold:

$$Z_i = \begin{cases} 1 & \text{if } W_i < w_0 \\ 0 & \text{if } W_i \geq w_0, \end{cases}$$

- Z_i is a relevant instrument for D_i since crossing the threshold affects treatment: $C(Z_i, D_i) \neq 0$.
- If crossing the threshold has no direct effect on Y_i , and only affects Y_i by influencing the probability of treatment, then Z_i is an exogenous instrument for D_i .
- The fuzzy RD approach implies estimating the baseline equation

$$Y_i = \beta_0 + \beta_1 D_i + \beta_2 W_i + u_i,$$

using Z_i as an instrument for D_i .

Matsudaira (2008) example

- Since the rules for attending summer schools depend also on other factors besides final grade, the author uses a fuzzy RD design.
- Let W_i represent student i 's score on the spring math exam in 2001.

Then:

$$M_i = \begin{cases} 1 & \text{if } W_i < 0 \\ 0 & \text{if } W_i \geq 0, \end{cases}$$

M_i indicates that the student is mandated to attend summer school (the cutoff point is normalized to zero).

- He uses M_i (the variable for being below the cutoff point) as an instrument for summer school attendance (D_i) and a cubic polynomial for W .

Discussion

- Appealing approach:
 - Assignment rules with cutoff structure exist in many programs and we take advantage of those rules.
 - Very intuitive: easy to communicate the results.
- Potential problems:
 - The need of very detailed data to have enough observations around the cutoff point.
 - The same cutoff may be used for other programs.
 - Manipulation of the threshold: agents try to choose the assignment variable to be just above or just below the threshold depending on their willingness to be treated or not.
 - Extrapolation to the full population requires more assumptions.
 - Sensitivity to functional form: the relationship between the assignment variable and the outcome variable needs to be correctly captured.