Applied Economics

Growth and Convergence[1]

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[1] Based on Acemoglu (2008) and Barro y Sala-i-Martin (2004)

Outline

Cross-Country Income Differences

- There are very large differences in income per capita and output per worker across countries.
- Some examples, GDP per capita in 2010, Norway $54600, USA $47200, Spain $29400, Botswana $14000, Uruguay $13700, Brazil $10800, Nigeria $2500 (numbers in 2010 U.S. dollars and are adjusted for purchasing power parity (PPP), CIA World Factbook).
- The figures show the distribution of countries by income per capita, and income per worker.

Cross-Country Income Differences

(Source: Acemoglu 2008)

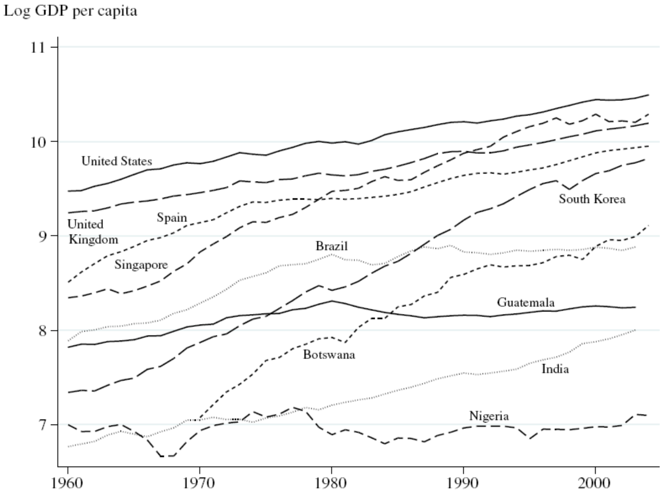
Cross-Country Income Differences

Fuente: Acemoglu(2008)

Economic Growth and Income Differences

- There are big differences in growth rates too. The US and the UK have similar growth patterns which are different from the rest.
- Another group of countries, as Japan, Singapore, South Korea, show high growth rates, and although they started with very low income levels, today they are close to the income in the rich countries.
- There are countries with similar income per capita levels in 1960 but quite different 40 years later (Botswana and Nigeria).
- Spain grows relatively rapidly between 1960 and the mid-1970s, but not so fast afterwards.

Economic Growth and Income Differences



Fuente: Acemoglu(2008)

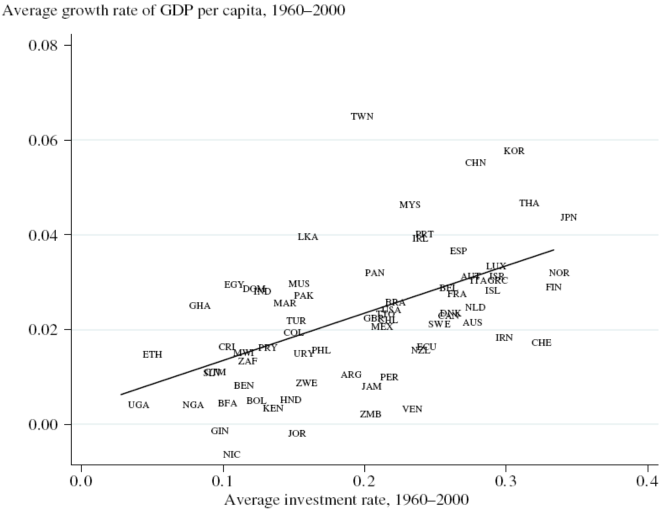
Economic Growth and Income Differences

- Inequality in income per capita and income per worker across countries shown by the highly dispersed distributions.
- Slight increase in inequality across nations (though not necessarily across individuals in the world economy).
- Differences in growth rates across countries.
- Should we care about these differences in income and growth across countries?

Growth and other variables

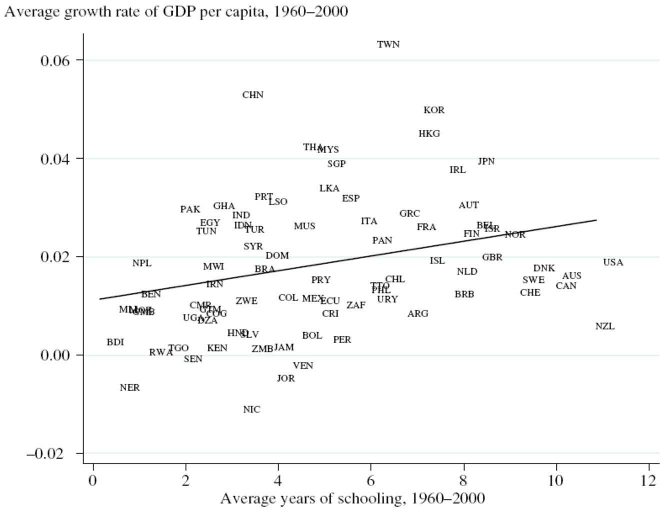
- We would like to know which specific characteristics of a country (including policies and institutions) have a causal effect on growth.
- We would like to estimate this causal effect.
- We start by looking at the relationship between growth and other variables that we think may be important for growth, as investment and education.

Growth and other variables



Fuente: Acemoglu(2008)

Growth and other variables



Fuente: Acemoglu(2008)

Growth and other variables

- Positive correlation between growth and investment rate and between average years of schooling and growth.
- This suggests that countries that have grown faster are typically those that have invested more in physical and human capital.
- DO NOT imply that physical or human capital investments are the causes of economic growth (only positive correlation).
- Potential fundamental causes:
  - institutional differences
  - geographic differences
  - cultural differences
  - luck...
- We need a model to illustrate the mechanics of economic growth and cross-country income differences. And a model that we can estimate...

The Solow Model

- Easy model for the proximate causes of economic growth and cross-country income differences.
- Let's start with a production function:

$$Y_t = F[L_t, K_t, A_t]$$

- The output depends on labor (L_t), capital (K_t) and the level of technology A_t (productivity).
- The potential sources of output growth are three: labor, capital and the level of technology (productivity)

The Solow Model and the data: regressions

- One way to bring the Solow model to the data is by regressions (Barro (1991), Mankiw, Romer y Weil (1992), Levine y Renelt (1992), Durlauf and Johnson (1995)).
- We need to formulate an econometric model based on the Solow model.
- We assume a Cobb-Douglas production function to simplify the econometric model

$$Y_t = (A_t L_t)^{1-\alpha} K_t^{\alpha}$$

The Solow Model

- The model assumes that a constant fraction of output, s, is invested. The evolution of K in the model is:

$$
\begin{aligned}
K_t &= (1-\delta)K_{t-1} + I_t \\
&= (1-\delta)K_{t-1} + sY_{t-1} \\
&= (1-\delta)K_{t-1} + s[(A_{t-1}L_{t-1})^{1-\alpha}K_{t-1}^{\alpha}]
\end{aligned}
$$

  where δ is the rate of depreciation.

- Moreover it assumes that L_t grows at a fixed rate n and A_t grows at a fixed rate g.

The Solow Model

- Let's define k as the stock of capital per effective unit of labor $(k_t = K_t/(A_t L_t))$, and y as the level of output per effective unit of labor, $(y_t = Y_t/(A_t L_t))$.
- Using the variables in terms of effective unit of labor in the production function, we get:

$$
\begin{aligned}
y_t &= \frac{Y_t}{A_t L_t} \\
&= \frac{(A_t L_t)^{1-\alpha} K_t^{\alpha}}{A_t L_t} = \frac{(A_t L_t)(A_t L_t)^{-\alpha} K_t^{\alpha}}{A_t L_t} \\
&= k_t^{\alpha}
\end{aligned}
$$

The Solow Model

- From the evolution of K_t, A_t y L_t we get the evolution of k_t:

$$
\begin{aligned}
k_t &= \frac{K_t}{A_t L_t} \\
&= (1-\delta)k_{t-1}\frac{A_{t-1}L_{t-1}}{A_t L_t} + s k_{t-1}^{\alpha}\frac{A_{t-1}L_{t-1}}{A_t L_t} \\
&= \frac{(1-\delta)k_{t-1} + s k_{t-1}^{\alpha}}{(1+g+n)}
\end{aligned}
$$

- If we divide by k_{t-1} we get the growth rate of the stock of capital per effective unit of labor:

$$
\frac{k_t}{k_{t-1}} = \frac{1-\delta + s k_{t-1}^{\alpha-1}}{(1+g+n)}
$$

The Solow Model

- The steady-state value, defined by $k_t = k_{t-1}$, is

$$k^* = \left[\frac{s}{(n+g+\delta)} \right]^{1/(1-\alpha)}$$

- And the equation for the income per capita in the steady-state (taking logs) is:

$$\begin{aligned} ln\left(\frac{Y_t}{L_t}\right) &= ln(A_t) + ln(y_t) = ln(A_t) + ln(k_t^{\alpha}) \\ &= ln(A_t) + ln\left(\left[\frac{s}{(n+g+\delta)} \right]^{\alpha/(1-\alpha)} \right) \end{aligned}$$

The Solow Model and the data: regressions

- Assuming $ln(A_t) = a + gt + \varepsilon$ the income per capita in any moment $(t = 0)$:

$$ln\left(\frac{Y_t}{L_t}\right) = a + \frac{\alpha}{1-\alpha} ln(s) - \frac{\alpha}{1-\alpha} ln(n+g+\delta) + \varepsilon \qquad (1)$$

- A lot of empirical research is based on this equation.
- What do we need to assume to estimate the equation using OLS?
- We will estimate this model following Mankiw, Romer and Weil in The Quarterly Journal of Economics, 1992 (MRW).

The Augmented Solow Model

- Mankiw, Romer and Weil also analyze an "Augmented" Solow model.
- In the augmented model they include the stock of human capital to the Solow growth model.
- We should expect changes in the estimation results if we think that human capital is an omitted variable in the previous equation.
- First we will see the theoretical augmented model.

The Augmented Solow Model

- We will use the following Cobb-Douglas production function, where H is the stock of human capital :

$$Y_t = H_t^{\beta}(A_t L_t)^{1-\alpha-\beta} K_t^{\alpha}$$

- Let $h = H/AL$ be the stock of human capital per effective unit of labor, s_k the fraction of income invested in physical capital and s_h the fraction invested in human capital:

$$\dot{k}_t = s_k y_t - (n+g+\delta)k_t$$
$$\dot{h}_t = s_h y_t - (n+g+\delta)h_t$$

The Augmented Solow Model

- We assume that $\alpha + \beta < 1$: decreasing returns to all capital.
- The previous equations imply that the economy converges to the steady-state defined by

$$k_t^* = \left(\frac{s_k^{1-\beta} s_h^{\beta}}{n+g+\delta} \right)^{1/(1-\alpha-\beta)}$$

$$h_t^* = \left(\frac{s_k^{\alpha} s_h^{1-\alpha}}{n+g+\delta} \right)^{1/(1-\alpha-\beta)}$$

The Augmented Solow Model

- Substituting these equations into the production function and taking logs gives an equation for the income per capita:

$$ln\left(\frac{Y_t}{L_t}\right) = a - \frac{\alpha+\beta}{1-\alpha-\beta} ln(n+g+\delta) \quad (2)$$

$$+ \frac{\alpha}{1-\alpha-\beta} ln(s_k) + \frac{\beta}{1-\alpha-\beta} ln(s_h) + \varepsilon \quad (3)$$

- This equation shows how income per capita depends on population growth and accumulation of physical and human capital.
- We will also estimate this equation as in MRW.

The Solow Model and Convergence

- We could also use the Solow model to analyze convergence. We will need to use the model outside the steady-state.
- Approximating around the steady state, the speed of convergence is given by:

$$\frac{dlny_t}{dt} = (1-\alpha)(n+g+\delta)[lny^* - lny_t]$$

- If we "calibrate" the speed of convergence with figures for advanced economies: $g \approx 0.02$, $n \approx 0.01$, $\delta \approx 0.05$, $\alpha \approx 1/3$
  - Then the convergence rate will be around 0.053 (5.3% of the gap between y^* and y_t disappear in one year).
  - This implies that the economy moves halfway the steady state in a little bit more than 13 years.

The Solow Model and Convergence

- Using the convergence equation we can obtain a growth regression similar to those estimated by Barro (1991).

$$g_{i,t,t-1} = \beta_0 + \beta_1 logy_{i,t-1} + \varepsilon_{i,t}$$

- $g_{i,t,t-1}$ is the growth rate between dates $t-1$ and t in country i
- $\varepsilon_{i,t}$ is a stochastic term capturing all omitted influences
- Barro and Sala-i-Martin refer to this equation as *unconditional convergence*..

The Solow Model and Convergence

- Unconditional convergence may be too demanding:
- Requires income gap between any two countries to decline, irrespective of what types of technological opportunities, investment behavior, policies, and institutions these countries have.
- If countries differ according to certain observable characteristics, a more appropriate regression equation may be:

$$g_{i,t,0} = \beta_0 + \beta_1 logy_{i,0} + \theta X_i + \varepsilon_{i,t}$$

where g is the growth rate and X_i are relevant observable characteristics.

- This is called *conditional convergence*
- Based on the Solow model, these variables are the investment rate and the growth rate of effective labor.

The Solow Model and Convergence

- The convergence models we usually find in applied economics are based on the idea of conditional convergence.
- X may include: schooling rate by gender, fertility rate, investment rate, inflation rate, openness, institutional variables.
- This kind of regressions tend to show a negative estimate of β_1.
- We will also estimate convergence equations using MRW data.

Regression Analysis - Problems

- This kind of models have not only been used to support conditional convergence, but also to estimate the “determinants of economic growth”.
- In this cases θ: information about causal effects of certain variables on economic growth.
- Several problems with regressions of this form:
  - Many variables in X_i, and $logy_i$ are endogenous: jointly determined with g_i .
  - Measurement error or other transitory shocks to y_i.
  - The Solow model is based on a closed economy