Too Much Information Sharing? Welfare Effects of Sharing Acquired Cost Information.*

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Abstract

We study how information acquisition choices of firms in a Cournot duopoly may change the effects of information sharing on the consumer surplus.

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*Naturally, all errors are ours.

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1 Introduction

Should Cournot duopolists be allowed to share information about their costs of production? The answer from the literature on information sharing in oligopoly seems to be an unambiguous "no" (see e.g. Vives and Kühn, 1995, and Vives, 1999 for surveys). In the first place, information sharing among competing firms decreases the consumer surplus when the two firms compete in quantities (Shapiro, 1986, Sakai and Yamato, 1989). Moreover, information sharing may facilitate collusion between firms, which also hurts consumer surplus. Hence, a policy maker, who maximizes expected consumer surplus, should prohibit agreements among Cournot duopolists to share information about their costs.

This conclusion can be drawn in settings where firms receive information exogenously. In this paper we show that the policy conclusion may become ambiguous when information is endogenous, i.e. firms acquire information themselves. This counterintuitive result is based on the following effects. Consumer surplus increases with the amount of information held by firms. The incentives of acquiring information are larger when firms are allowed to share information. As previous literature pointed out, if we take the level of information exogenously, allowing firms to share information has a negative effect (positive effect) on the consumer surplus (the profits of the firms).

In the next section we describe the model. Section 3 defines the concept of Integral Precision for signals. Section 4 briefly describes the equilibrium strategies. Section 5 compares expected consumer surplus levels in equilibrium. Section 6 presents a simple example in which the indirect effect of information sharing dominates the direct effect. Finally, section 7 concludes the paper. All proofs are relegated to the Appendix.

2 The Model

2.1 Preferences and Technology

Consider an industry where two risk-neutral firms (firms 1 and 2) compete in quantities of a homogeneous good. The representative consumer's surplus from consuming quantity X is $U(X) - PX$, with $X \equiv x_1 + x_2$, and:

$$U(X) \equiv \alpha X - \frac{1}{2}\beta X^2. \tag{1}$$

Hence, the inverse demand function is linear in total output, i.e. $P(X) = \alpha - \beta X$. The demand intercept α is sufficiently high. Firms have constant marginal costs of production. The cost of firm i, which is unknown ex-ante, is distributed according to $F_i : \left[0, \theta^h\right] \to [0, 1]$ with mean $\overline{\theta}_i$. Firm i's profit of producing quantity x_i is simply (for $i = 1, 2$):

$$\pi_i(\mathbf{x}; \theta_i) = \left[P(X) - \theta_i\right] x_i, \tag{2}$$

with $\mathbf{x} = (x_1, x_2)$. The consumer surplus, given total consumption X, equals:

$$S(X) = U(X) - P(X)X = \frac{1}{2}\beta X^2 \tag{3}$$

In the remainder of the paper we make the normalization $\beta = 1$ to save on notation.

2.2 Firms' Information Structures

Firms' marginal costs are initially uncertain. Firm i can acquire a costly signal S_i^δ about θ_i. Signal S_i^δ is characterized by the family of distributions $\{H_\delta(s|\theta_i)\}_{\theta_i}$. The parameter δ orders the signals according to their accuracy in the sense of Integral Precision (see section 3). The cost of acquiring a signal S_i^δ of accuracy δ is denoted by $c(\delta)$, where c is increasing in δ.

Given the true marginal cost θ_i, which is a realization of random variable Θ_i, S_i^δ is represented by the conditional distribution $H_\delta(s|\theta_i) = \Pr(S_i^\delta \leq s|\Theta_i = \theta_i)$. The prior distribution $F_i(\theta)$ and the signal distribution, $\{H_\delta(s|\theta_i)\}_{\theta_i}$, define the information structure, i.e. the joint distribution of (Θ_i, S_i^δ).

We assume that $H_\delta(s|\theta_i)$ admits a density $h_\delta(s|\theta_i)$. The marginal distribution of S_i^δ is denoted by $H_i^\delta(s)$ and satisfies:

$$H_i^\delta(s) = \int^s \int_0^{\theta^h} h_\delta(y|\theta) dF(\theta) dy.$$

Let $F_i^\delta(\theta_i|s_i^\delta)$ and $E_i[\theta|s_i^\delta]$denote the posterior distributions and the conditional expectation of Θ_i conditional on $S_i^\delta = s_i^\delta$

2.3 Firms' Information Sharing Policies

If the antitrust authority allows information sharing between firms, the firms simultaneously choose their information sharing policy with their competitor before they acquire the signal. We focus on a parametric family of information-sharing policies. Firm i chooses $\rho_i \in [0, 1]$, which implies that firm j will receive the informative message, $m_i = s_i^\delta$ (the private realization of the signal S_i^δ), with probability ρ_i and with the complementary probability, firm j will receive the non-informative message, $m_i = \varnothing$.

2.4 Timing

1. Initially, an antitrust authority chooses whether to allow or prohibit information sharing between the firms in the industry. The authority maximizes the expected consumer surplus.

2. In the second stage, firms simultaneously choose their information-sharing policy with their competitor, $\rho_i \in [0, 1]$, taking into account the decision of the antitrust authority.[1]

[1]In other words, firms unilaterally choose whether to precommit to information sharing. Alternative assumptions could be to allow the firms to precommit cooperatively to share information (through a *quid pro quo* agreement), or

3. The marginal costs of firms 1 and 2 are determined by two independent draws from their corresponding distributions F_1 and F_2, respectively.

4. Firms simultaneously choose information acquisition investments: $c(\delta_i)$, with c increasing in δ_i for $i = 1, 2$. Firm i's investment δ_i determines the precision of the firm's cost signal S_i^δ. Signal S_i^δ is characterized by the family of distributions $\{H_\delta(s|\theta_i)\}_{\theta_i}$.

5. Firms send messages about their signal in accordance with their information-sharing policies in stage 2. If firm i precommitted to share its information in accordance with ρ_i, then firm j will receive an informative message $m_i = s_i^\delta$ (the private realization of the signal S_i^δ) with probability ρ_i and an uninformative message $m_i = \varnothing$, with probability $1 - \rho_i$.

6. In the final stage firms simultaneously choose their output levels, $x_i \geq 0$ for firm i, to maximize the expected value of (2), i.e., firms are Cournot competitors.

We solve the game backwards, and restrict the analysis to perfect Bayesian equilibria. Before solving the model, we want to discuss how the choice of information acquisition investment δ_i determines the information structure.

3 Information Criteria: Integral Precision

In this paper we assume that the parameter δ_i rank signals according to Integral Precision. Precision criteria (introduced by Ganuza and Penalva, 2009) are based on the principle that an information structure, i.e., the joint distribution of the state of the world and the signal, is more informative (*more precise*) than another if it generates more dispersed conditional expectations. This dispersion effect arises because the sensitivity of conditional expectations to the realized value of the signal depends on the informational content of the signal. If the informational content of the signal is low, conditional expectations are concentrated around the expected value of the prior. When the informational content is high, conditional expectations depend to a large extent on the realization of the signal which increases their variability.

In our context, given the prior distribution $F(\theta)$, we assume that if $\delta_i > \delta_i'$ then $E_i[\theta|S_i^\delta]$ is "more spread out" than $E_i[\theta|S_i^{\delta'}]$. Ganuza and Penalva (2009) introduce a different *precision* criteria which is defined by combining this approach with different variability orders. In the present paper, we will use *Integral Precision* which is based on the convex order:

Definition 1 (Convex Order) *Let Y and Z be two real-valued random variables with distribution F and G respectively. Then Y is greater than Z in the* convex order *($Y \geq_{cx} Z$) if for all convex real-valued functions ϕ, $E[\phi(Y)] \geq E[\phi(Z)]$ provided the expectation exists.*

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to assume that firms make strategic information sharing choices (i.e. each firm chooses whether to share information after it learns its signal). As it turns out, in equilibrium the information sharing choices are not affected by these changes of assumptions.

Using the convex order, Ganuza and Penalva define Integral Precision to order signals in terms of their informativeness:

Definition 2 (Integral Precision) *Given a prior $F_i(\theta)$ and two signals S_1 and S_2: S_1 is more* integral precise *than S_2 if $E_i[\theta|S_1]$ is greater than $E_i[\theta|S_2]$ in the convex order.*

Ganuza and Penalva (2009) show that *Integral Precision* is weaker than (is implied by) all common informativeness orders based on the value of information for a decision maker (Blackwell, 1951, Lehmann, 1988, and Athey and Levin, 2001). In other words, if S_1 is more valuable for a decision maker than S_2, then S_1 is more integral precise than S_2. The following information models are consistent with *Integral Precision.*

Normal Experiments: Let $F_i(\theta) \sim \mathcal{N}(\mu, \sigma_v^2)$ and $S_i^\delta = \theta_i + \epsilon_\delta$, where $\epsilon_\delta \sim \mathcal{N}(0, \sigma_\delta^2)$ and is independent of θ_i. The variance of the noise, σ_δ^2, orders signals in the usual way: we assume that $\delta > \delta' \iff \sigma_\delta^2 < \sigma_{\delta'}^2$ and the signal with a noise term that has lower variance is more informative in terms of Integral Precision.

Linear Experiments: Let $F_i(\theta)$ have mean μ. With probability δ the signal is perfectly informative, $S_i^\delta = \theta_i$, and with probability $1-\delta$ the signal is pure noise, $S_i^\delta = \epsilon$ where $\epsilon \sim F_i(\theta)$ and is independent of θ_i. Let S_i^δ and $S_i^{\delta'}$ be two such signals. If $\delta > \delta'$, i.e. S_i^δ reveals the truth with a higher probability than $S_i^{\delta'}$, then S_i^δ is more informative than $S_i^{\delta'}$ in terms of Integral Precision.

Binary Experiments: Let θ_i be equal to θ^h with probability q and θ^l with probability $1-q$. The signal, S_i^δ, can take two values h or l, where $\Pr[S_i^\delta = k|\theta_i = \theta^k] = \frac{1}{2}(1+\delta_i)$ for $i \in \{1,2\}$ and $k \in \{l,h\}$, where $0 \leq \delta_i \leq 1$. The parameter δ_i orders signals in the usual way: higher δ implies greater Integral Precision.

Uniform Experiments: Let $F(\theta)$ be the uniform distribution on $[0,1]$ and let $H_\delta(s|\theta_i)$ be uniform on $[\theta_i - 1/2\delta, \theta_i + 1/2\delta]$. For any δ, δ' with $\delta > \delta'$, S_i^δ is more informative than $S_i^{\delta'}$ in terms of Integral Precision.

Partitions: Let $F(\theta)$ have support equal to $[0,1]$. Consider two signals generated by two partitions of $[0,1]$, $\mathcal{A}$ and $\mathcal{B}$, where $\mathcal{B}$ is finer than $\mathcal{A}$.[2] Using these partitions, one can define signals S_i^δ and $S_i^{\delta'}$ in the usual way: signal S_i^δ $[S_i^{\delta'}]$ tells you which set in the partition $\mathcal{A}$ $[\mathcal{B}]$ contains θ_i.[3] Assuming that a larger δ means a finer partition, δ orders signals according to Integral Precision.

4 Solving the Model: Equilibrium Strategies

First we characterize the equilibrium output levels. Second, we analyze the information acquisition choices of firms. Finally, we analyze the information sharing choices of the firms.

[2] A partition, $\mathcal{A}$, is obtained by dividing $[0,1]$ into disjoint subsets, $\mathcal{A} = \{A_1, .., A_k\}$, i.e., $\cup_{j=1}^k A_j = [0,1]$ and $A_i \cap A_j = \emptyset$ for all $i, j = 1, .., k$ with $i \neq j$. Partition $\mathcal{B}$ is finer than $\mathcal{A}$, that is for all $B \in \mathcal{B}$, there exists $A \in \mathcal{A}$ such that $B \subseteq A$.

[3] However, observing A_j $[B_j]$ does not allow you to distinguish between different states of the world within that set.

4.1 Output Levels

Each firm chooses its output level on the basis of its own information, s_i, and the information received from its competitor, $m_j \in \{s_j, \varnothing\}$. In order to save notation we do not make explicit the dependence of s_i over δ_i. The expected cost given the uninformative message $m_j = \varnothing$ is: $E\{\theta_j|\varnothing\} = \overline{\theta}_j$.

For any combination of messages m_i and m_j firm i with signal s_i maximizes its expected profit, which yields the following first-order condition:

$$x_i(s_i) = \frac{1}{2}\Big(\alpha - E\{\theta_i|s_i\} - E\{x_j(s_j)|m_j\}\Big) \tag{4}$$

for $i, j = 1, 2$ with $i \neq j$. Solving the system of equations (4) for $i = 1, 2$ gives the following equilibrium output level of firm i (for $i, j = 1, 2$ with $i \neq j$):

$$x_i^*(s_i; m_i, m_j) = \frac{1}{3}\left(\alpha - 2E\{\theta_i|s_i\} + E\{\theta_j|m_j\} + \frac{1}{2}\left[E\{\theta_i|s_i\} - E\{\theta_i|m_i\}\right]\right) \tag{5}$$

where $E\{\theta_i|m_i\} = E_{s_i}\{E(\theta_i|s_i)|m_i\}$. We can define the ex-ante output level given that $m_i = s_i$ with probability ρ_i for $i = 1, 2$.

$$X_i^*(s_i; \rho_i, \rho_j) = \frac{1}{3}\left(\alpha - 2E\{\theta_i|s_i\} + \rho_j E\{\theta_j|s_j\} + (1-\rho_j)\overline{\theta}_j + \frac{(1-\rho_i)}{2}\left[E\{\theta_i|s_i\} - \overline{\theta}_i\right]\right)$$

The expected equilibrium market profits of firm i with signal s_i, and messages m_i and m_j equals: $\pi_i^*(s_i; m_i, m_j) = x_i^*(s_i; m_i, m_j)^2$. Hence

$$\Pi_i(\delta_i, \rho_i, \rho_j) \equiv E_{s_i, m_i}\left\{E_{s_j, m_j}[x_i^*(s_i; m_i, m_j)^2]\right\} - c(\delta_i) \tag{6}$$

where

$$E_{s_i, m_i}\left\{E_{s_j, m_j}[x_i^*(s_i; m_i, m_j)^2]\right\} = \rho_i E_{s_i}\left\{E_{s_j, m_j}[x_i^*(s_i; s_i, m_j)^2]\right\} + (1-\rho_i) E_{s_i}\left\{E_{s_j, m_j}[x_i^*(s_i; \varnothing, m_j)^2]\right\}$$

and $E_{s_j, m_j}[.]$ is defined likewise.

4.2 Information Acquisition

In this subsection we analyze the relationship between the information acquisition incentives and the information sharing policy.

Lemma 1 $\Pi_i(\delta_i, \rho_i, \rho_j)$ *is supermodular in* (δ_i, ρ_i).

In other words, for $\rho_i > \rho_i'$, $\Pi_i(\delta_i, \rho_i, \rho_j) - \Pi_i(\delta_i, \rho_i', \rho_j)$ is weakly increasing in δ_i for all ρ_j, which implies that information-sharing firms have a greater incentive to acquire information than concealing firms.

Proposition 1 *(i) The information acquisition investment of a firm is independent of the information sharing choice* ρ_j *of its competitor. (ii) The equilibrium information acquisition investment of a firm* δ_i *is increasing in the information sharing parameter* ρ_i.

The next section analyzes incentives of firms to share information.

4.3 Information Sharing

For any given precision, information sharing is a dominant strategy for each firm (Gal-Or, 1986, Shapiro, 1986). We cannot directly apply this result since in our model the precision is not exogenously given, but it depends on the information sharing choices of the firms. However, it is easy to verify that sharing information is also a dominant strategy in our framework.

Proposition 2 $\Pi_i(\delta_i, \rho_i, \rho_j)$ *is increasing in* ρ_i *for all* δ_i *and* ρ_j*.*

In the next section we analyze the effects of information sharing and acquisition on the expected consumer surplus.

5 Expected Consumer Surplus

First, we characterize the expected consumer surplus levels for exogenously given information acquisition levels. Second, we characterize the consumer surplus levels in equilibrium.

Given symmetric information acquisition investments, and the symmetric information sharing choices, the expected consumer surplus from information sharing is as follows: First we establish the following basic property of the consumer surplus.

$$S(\delta_i, \delta_j, \rho_i, \rho_j) = \frac{1}{2} E_{s_i, m_i} \left\{ E_{s_j, m_j} [x_i^*(s_i, m_i, m_j) + x_j^*(s_j, m_j, m_i)]^2 \right\}$$

Lemma 2 $S(\delta_i, \delta_j, \rho_i, \rho_j)$ *is decreasing in* ρ_k *and increasing in* δ_k *for any* $k \in \{i, j\}$*.*

Proposition 1 and Proposition 2 give the following interesting trade-off. On the one hand, for any exogenously given signals' precision (δ_i, δ_j) the expected surplus is decreasing in the information sharing parameters (ρ_i, ρ_j). Therefore, if the precision were exogenously given, then information sharing should be prohibited. On the other hand, information sharing increases the incentives to invest in information acquisition (Proposition 1). Higher investment levels increase the expected consumer surplus. Hence, when the signal's precision is not exogenous, but determined endogenously by information acquisition investments, the antitrust authority's choice (between allowing and disallowing information sharing), will depend on the trade-off between these two conflicting effects. In fact, it is possible that the second effect outweighs the first effect, as we illustrate in the next section.

6 Information Sharing May Increase Consumer Surplus

We set up our model in a binary framework with linear information acquisition costs: (i) Firms' marginal costs have distribution $F_i(\theta)$, for $i = 1, 2$. (ii) The information structures are binary: $s_i \in \{\theta_i, \varnothing\}$, where $\Pr[s_i = \theta_i | \Theta_i = \theta_i] = \delta_i$ and $\Pr[s_i = \varnothing | \Theta_i = \theta_i] = 1 - \delta_i$ for

$i \in \{1, 2\}$, where $\delta_i \in \{0, 1\}$.[4] (iii) The information-sharing policies are also binary $\rho_i \in \{0, 1\}$, and $m_i \in \{\theta_i, \varnothing\}$. (iv) Finally, the cost of information acquisition is linear, i.e. $c(\delta) = \lambda\delta$.

Then, the expected cost of firm i given signal s_i and investment δ_i is:

$$E\{\theta_i|s_i\} = \begin{cases} \theta_i, & \text{if } s_i = \theta_i \\ \overline{\theta}_i, & \text{if } s_i = \varnothing \end{cases} \tag{7}$$

The expected cost given the uninformative message $m_j = \varnothing$ is: $E\{\theta_j|\varnothing\} = \overline{\theta}_j$.

Now, we can reproduce our previous results for this specific framework. We first compare the information acquisition incentives under information sharing $\rho_i = 1$ and information concealment $\rho_i = 0$.

In this example, the expected equilibrium profit (6) reduces to:

$$\begin{aligned} \Pi_i(\delta_i; \rho_i, \rho_j) &= \delta_i \left[\rho_i E_{\theta_i}\left\{E_{s_j,m_j}[x_i^*(\theta_i; \theta_i, m_j)^2]\right\} + (1-\rho_i)E_{\theta_i}\left\{E_{s_j,m_j}[x_i^*(\theta_i; \varnothing, m_j)^2]\right\}\right] \\ &\quad +(1-\delta_i)E_{s_j,m_j}\{x_i^*(\varnothing; \varnothing, m_j)^2\} - \lambda\delta_i \end{aligned}$$

with

$$E_{s_j,m_j}[x_i^*(s_i; m_i, m_j)^2] = \delta_j\rho_j E_{\theta_j}\{x_i^*(s_i; m_i, \theta_j)^2\} + (1-\delta_j\rho_j)x_i^*(s_i; m_i, \varnothing)^2$$

First, we consider firm i's marginal profit from information acquisition when the firm shares information (for $\rho_j \in \{0, 1\}$):

$$\begin{aligned} \Pi_i(1; 1, \rho_j) - \Pi_i(0; 1, \rho_j) &= E_{\theta_i}\left\{E_{s_j,m_j}[x_i^*(\theta_i; \theta_i, m_j)^2 - x_i^*(\varnothing; \varnothing, m_j)^2]\right\} - \lambda \\ &= -\frac{2}{3}E_{\theta_i}\left\{\left[\theta_i - \overline{\theta}_i\right] E_{s_j,m_j}[x_i^*(\theta_i; \theta_i, m_j) + x_i^*(\varnothing; \varnothing, m_j)]\right\} - \lambda \\ &= \frac{4}{9}E_{\theta_i}\left\{\left[\theta_i - \overline{\theta}_i\right]\theta_i\right\} - \lambda = \frac{4}{9}var(\theta_i) - \lambda \end{aligned} \tag{8}$$

The profit-maximizing choice of information acquisition depends on the trade-off between the marginal revenue, $\frac{4}{9}var(\theta_i)$, and marginal cost of information acquisition, λ. The comparison of marginal revenue and cost gives the following information acquisition choice for firm i in equilibrium (for any ρ_j, with $i, j = 1, 2$ and $i \neq j$):

$$\delta_i^*(1, \rho_j) = \begin{cases} 1, & \text{if } \lambda \leq \frac{4}{9}var(\theta_i) \\ 0, & \text{otherwise} \end{cases} \tag{9}$$

Second, firm i's marginal profit from information acquisition when the firm conceals information is (for $\rho_j \in \{0, 1\}$):

$$\begin{aligned} \Pi_i(1; 0, \rho_j) - \Pi_i(0; 0, \rho_j) &= E_{\theta_i}\left\{E_{s_j,m_j}[x_i^*(\theta_i; \varnothing, m_j)^2 - x_i^*(\varnothing; \varnothing, m_j)^2]\right\} - \lambda \\ &= -\frac{1}{2}E_{\theta_i}\left\{\left[\theta_i - \overline{\theta}_i\right] E_{s_j,m_j}[x_i^*(\theta_i; \varnothing, m_j) + x_i^*(\varnothing; \varnothing, m_j)]\right\} - \lambda \\ &= \frac{1}{4}E_{\theta_i}\left\{\left[\theta_i - \overline{\theta}_i\right]\theta_i\right\} - \lambda = \frac{1}{4}var(\theta_i) - \lambda \end{aligned} \tag{10}$$

[4] That is, the information structure is a special case of: (1) the linear experiment with $\delta_i \in \{0, 1\}$, or (2) partitions, where $\delta_i = 0$ gives the degenerate partition $[0, \theta^h]$, and $\delta_i = 1$ gives an infinitely fine partition of the interval $[0, \theta^h]$.

The evaluation of (10) gives the following information acquisition choice in equilibrium (for any ρ_j, with $i, j = 1, 2$ and $i \neq j$):

$$\delta_i^*(0, \rho_j) = \begin{cases} 1, & \text{if } \lambda \leq \frac{1}{4} var(\theta_i) \\ 0, & \text{otherwise} \end{cases} \tag{11}$$

Information acquisition choices (9) and (11) compare as follows. First, the equilibrium investments are always extreme (i.e., $\delta_i^*(0, \rho_j), \delta_i^*(1, \rho_j) \in \{0, 1\}$). Second, information sharing gives firm i a greater or equal incentive to acquire information as information concealment (i.e., $\delta_i^*(1, \rho_j) \geq \delta_i^*(0, \rho_j)$ for any i, ρ_j, and λ). In particular, if $\lambda < var(\theta_i)/4$ of $\lambda > 4var(\theta_i)/9$, then the information acquisition choice is not affected by information sharing, i.e., $\delta_i^*(1, \rho_j) = \delta_i^*(0, \rho_j) \in \{0, 1\}$. By contrast, if $var(\theta_i)/4 < \lambda < 4var(\theta_i)/9$, then an information-sharing firm acquires information whereas an information-concealing firm acquires no information, i.e., $\delta_i^*(1, \rho_j) = 1 > 0 = \delta_i^*(0, \rho_j)$.

Lemma 1 and Proposition 1 follow immediately from the comparison of (8) and (10). Hence, information-sharing firms have a greater incentive to acquire information than concealing firms. As we will see below, this ranking of information equilibrium investments plays a major role in the ranking of the expected consumer surpluses.

Following Proposition 2 we know that it is a dominant strategy for the firms to share information (i.e., $\rho_i = 1$ for $i = 1, 2$), if they are allowed to do so. Now, we have to compare the consumer surplus levels in equilibrium when we allow to share information (i.e., $\rho_i = \rho_j = 1$), and when it is forbiden to do so (i.e., $\rho_i = \rho_j = 0$).

As we show above, the expected consumer surplus from information sharing is as follows:

$$\begin{aligned} S(\delta_1, \delta_2; 1, 1) &\equiv \frac{1}{2} E_{s_1} E_{s_2} \left\{ \left(\sum_{i=1}^{2} x_i^*(s_i; s_i, s_j) \right)^2 \right\} \\ &= \frac{1}{2} \left[\delta_1 \delta_2 E_{\theta_1} E_{\theta_2} \left\{ \left(\sum_{i=1}^{2} x_i^*(\theta_i; \theta_i, \theta_j) \right)^2 \right\} \right. \\ &\quad + \sum_{i \neq j} \delta_i (1 - \delta_j) E_{\theta_i} \left\{ \left(x_i^*(\theta_i; \theta_i, \varnothing) + x_j^*(\varnothing; \varnothing, \theta_i) \right)^2 \right\} \\ &\quad \left. + (1 - \delta_1)(1 - \delta_2) \left(\sum_{i=1}^{2} x_i^*(\varnothing; \varnothing, \varnothing) \right)^2 \right] \end{aligned} \tag{12}$$

If sharing information is not allowed, then the expected consumer surplus from information concealment is:

$$\begin{aligned} S(\delta_1, \delta_2; 0, 0) &\equiv \frac{1}{2} E_{s_i} E_{s_2} \left\{ \left(\sum_{i=1}^{2} x_i^*(s_i; \varnothing, \varnothing) \right)^2 \right\} \\ &= \frac{1}{2} \left[\delta_1 \delta_2 E_{\theta_1} E_{\theta_2} \left\{ \left(\sum_{i=1}^{2} x_i^*(\theta_i; \varnothing, \varnothing) \right)^2 \right\} \right. \\ &\quad + \sum_{i \neq j} \delta_i (1 - \delta_j) E_{\theta_i} \left\{ \left(x_i^*(\theta_i; \varnothing, \varnothing) + x_j^*(\varnothing; \varnothing, \varnothing) \right)^2 \right\} \end{aligned} \tag{13}$$

$$+ (1-\delta_1)(1-\delta_2)\left(\sum_{i=1}^{2} x_i^*(\varnothing;\varnothing,\varnothing)\right)^2\Bigg]$$

Following Lemma 2, we can show that both expected consumer surplus functions are increasing in the information acquisition investment δ_i, i.e. $\partial S/\partial \delta_i > 0$.

$$\begin{aligned}
\frac{\partial S(\delta_1,\delta_2;1,1)}{\partial \delta_i} &= \frac{1}{2}\Bigg[\delta_j E_{\theta_1}E_{\theta_2}\left\{\left(\sum_{k=1}^{2} x_k^*(\theta_k;\theta_k,\theta_l)\right)^2 - \left(x_i^*(\varnothing;\varnothing,\theta_j)+x_j^*(\theta_j;\theta_j,\varnothing)\right)^2\right\} \\
&\quad + (1-\delta_j)E_{\theta_i}\left\{\left(x_i^*(\theta_i;\theta_i,\varnothing)+x_j^*(\varnothing;\varnothing,\theta_i)\right)^2 - \left(\sum_{k=1}^{2} x_k^*(\varnothing;\varnothing,\varnothing)\right)^2\right\}\Bigg] \\
&= \frac{1}{18}var(\theta_i) > 0
\end{aligned}$$

Similarly,

$$\begin{aligned}
\frac{\partial S(\delta_1,\delta_2;0,0)}{\partial \delta_i} &= \frac{1}{2}\Bigg[\delta_j E_{\theta_1}E_{\theta_2}\left\{\left(\sum_{k=1}^{2} x_k^*(\theta_k;\varnothing,\varnothing)\right)^2 - \left(x_i^*(\varnothing;\varnothing,\varnothing)+x_j^*(\theta_j;\varnothing,\varnothing)\right)^2\right\} \\
&\quad + (1-\delta_j)E_{\theta_i}\left\{\left(x_i^*(\theta_i;\varnothing,\varnothing)+x_j^*(\varnothing;\varnothing,\varnothing)\right)^2 - \left(\sum_{k=1}^{2} x_k^*(\varnothing;\varnothing,\varnothing)\right)^2\right\}\Bigg] \\
&= \frac{1}{8}var(\theta_i) > 0
\end{aligned}$$

Now we come back to the equilibrium investments in information acquisition. The equilibrium investments are either identical (i.e., $\delta_i^*(1,1) = \delta_i^*(0,0)$ for all i), or at opposite extreme points of the investment domain (i.e., $\delta_i^*(1,1) = 1 > 0 = \delta_i^*(0,0)$ for all i). In the former case the consumer surpluses are equal. In the latter case the consumer surplus is greater under information sharing, since the surplus comparison reduces to the following:

$$\begin{aligned}
S(1,1;1,1) &= \frac{1}{2}E_{\theta_1}E_{\theta_2}\left\{\left(\sum_{i=1}^{2} x_i^*(\theta_i;\theta_i,\theta_j)\right)^2\right\} \\
&> \frac{1}{2}\left(\sum_{i=1}^{2} x_i^*(\varnothing;\varnothing,\varnothing)\right)^2 = S(0,0;0,0)
\end{aligned}$$

where the inequality follows from the convexity of the expected consumer surplus function.

7 Conclusion

We have shown that given that the incentives of acquiring information are larger when firms are allowed to share information. This is a important remark regarding the antitrust authority's decision of whether or not to allow to share information. Because contrary to conventional wisdon, consumer surplus could be larger when firms are allowed to share information.

Hwang (1995) makes a related observation about the importance of information acquisition incentives for the welfare comparison between perfect competition, oligopoly, and monopoly. Although perfect competition yields the highest expected welfare for any exogenously given precision of information, it may fail to do so when the precision is determined endogenously, since firms in perfectly competitive markets may have a lower incentive to acquire information. Whereas Hwang changes the mode of competition while keeping information sharing constant, we do the opposite. Persico (2000) makes a related observation for auction models with affiliated values. For a given information structure the second price auction yields a higher expected revenue to an auctioneer than the first price auction. But the first price auction gives a greater incentive to acquire information, which may reverse the expected revenue ranking.

A Appendix

We make repeated use of the following result.

Lemma 3 *If δ ranks signals according to Integral Precision, then the variance of $E_i[\theta|S_i^{\delta}]$ is increasing on δ.*

Proof of Lemma 3:

$$var(E_i[\theta|S_i^{\delta}]) = E\{(E_i[\theta|S_i^{\delta}] - \overline{\theta_i})^2\} = E\{E_i[\theta|S_i^{\delta}]^2\} - \overline{\theta_i}^2.$$

Given that $(E_i[\theta|S_i^{\delta}] - \overline{\theta_i})^2$ is a convex function, the result is a direct implication of the definitions of the convex order and integral presion. ■

Notice that

$$var(E_i[\theta|S_i^{\delta}]) = E\{(E_i[\theta|S_i^{\delta}] - \overline{\theta_i})^2\} = E\{E_i[\theta|S_i^{\delta}]^2\} - \overline{\theta_i}^2.$$

Then by Lemma 3, $E\{E_i[\theta|S_i^{\delta}]^2\}$ is increasing on δ.

Proof of Lemma 1:

$$\Pi_i(\delta_i, \rho_i, \rho_j) - \Pi_i(\delta_i, \rho_i', \rho_j) = (\rho_i - \rho_i')\left[E_{s_i}\left\{E_{s_j,m_j}[x_i^*(s_i, s_i, m_j)^2]\right\} - E_{s_i}\{E_{s_j,m_j}[x_i^*(s_i, \varnothing, m_j)^2]\}\right]$$

Remenber that

$$x_i^*(s_i, m_i, m_j) = \frac{1}{3}\left(\alpha - 2E\{\theta_i|s_i\} + E\{\theta_j|m_j\} + \frac{1}{2}\left[E\{\theta_i|s_i\} - E\{\theta_i|m_i\}\right]\right)$$

We can factorize $E_{s_i}\left\{E_{s_j,m_j}[x_i^*(s_i, s_i, m_j)^2]\right\}$ and $E_{s_i}\left\{E_{s_j,m_j}[x_i^*(s_i, s_i, m_j)^2]\right\}$ in the following way:

$$E_{s_i}\left\{E_{s_j,m_j}[x_i^*(s_i, s_i, m_j)^2]\right\} = E_{s_i}\left\{E_{s_j,m_j}[(a-b)^2]\right\} = E_{s_i}\left\{E_{s_j,m_j}[(a^2 - 2ab + b^2)]\right\}$$

and,

$$E_{s_i}\left\{E_{s_j,m_j}[x_i^*(s_i, \varnothing, m_j)^2]\right\} = E_{s_i}\left\{E_{s_j,m_j}[(a-c)^2]\right\} = E_{s_i}\left\{E_{s_j,m_j}[(a^2 - 2ac + c^2)]\right\}$$

where,

$$a = \frac{1}{3}(\alpha - 2E\{\theta_i|s_i\} + E\{\theta_j|m_j\}) + \frac{1}{2}E\{\theta_i|s_i\}, b = \frac{1}{6}E\{\theta_i|s_i\}, \text{and } c = \frac{1}{6}\overline{\theta_i}.$$

Then

$$\begin{aligned}
& E_{s_i}\left\{E_{s_j,m_j}[x_i^*(s_i, s_i, m_j)^2]\right\} - E_{s_i}\left\{E_{s_j,m_j}[x_i^*(s_i, \varnothing, m_j)^2]\right\} \\
= \; & E_{s_i}\left\{E_{s_j,m_j}[(-2ab + b^2 + 2ac - c^2)]\right\} = E_{s_i}\left\{E_{s_j,m_j}[(2a(c-b) + b^2 - c^2)]\right\} \\
= \; & E_{s_i}\left\{E_{s_j,m_j}[(-2a + b + c)(b-c)]\right\} = E_{s_i}\left\{E_{s_j,m_j}[(-2a + b)(b-c)]\right\}
\end{aligned}$$

In this last simplification, we use that $E_{s_i}\left\{E_{s_j,m_j}[(b-c)]\right\} = E_{s_i}\left\{E_{s_j,m_j}[\frac{1}{6}(E\{\theta_i|s_i\}-\overline{\theta_i})]\right\} = 0$. Then, any constant multiply by $E_{s_i}\left\{E_{s_j,m_j}[(b-c)]\right\}$ will be also equal to 0. Using this fact

$$\begin{aligned}
& E_{s_i}\left\{E_{s_j,m_j}[x_i^*(s_i,s_i,m_j)^2]\right\} - E_{s_i}\left\{E_{s_j,m_j}[x_i^*(s_i,\varnothing,m_j)^2]\right\} \\
= \; & E_{s_i}\left\{E_{s_j,m_j}[(\frac{7}{6}E\{\theta_i|s_i\})\cdot(\frac{1}{6}(E\{\theta_i|s_i\}-\overline{\theta_i}))\right\} \\
= \; & \frac{7}{36}(E_{s_i}\{E\{\theta_i|s_i\}^2\}-\overline{\theta_i}^2)
\end{aligned}$$

Therefore

$$\begin{aligned}
\Pi_i(\delta_i,\rho_i,\rho_j)-\Pi_i(\delta_i,\rho_i',\rho_j) &= (\rho_i-\rho_i^{'})\frac{7}{36}(E_{s_i}\{E\{\theta_i|s_i\}^2\}-\overline{\theta_i}^2) \\
&= (\rho_i-\rho_i^{'})\frac{7}{36}var(E_i[\theta|S_i^\delta])
\end{aligned}$$

This expresion is increasing in δ by Lemma 1.

■

Proof of Proposition 1: The result follows from Theorem 4 of Milgrom and Shanon (1994) and Lemma 1. ■

Proof of Proposition 2:

Direct from the proof of Lemma 1.

$$\begin{aligned}
\frac{\partial\Pi_i(\delta_i,\rho_i,\rho_j)}{\partial\rho_i} &= [E_{s_i}\left\{E_{s_j,m_j}[x_i^*(s_i,s_i,m_j)^2]\right\} - E_{s_i}\{E_{s_j,m_j}[x_i^*(s_i,\varnothing,m_j)^2] \\
&= \frac{7}{36}(E_{s_i}\{E\{\theta_i|s_i\}^2\}-\overline{\theta_i}^2) = \frac{7}{36}var(E_i[\theta|S_i^\delta]) \geq 0.
\end{aligned}$$

■

Proof of Proposition 2:

$$\begin{aligned}
& S(\delta_i,\delta_j,\rho_i,\rho_j) = \frac{1}{18}[\rho_i\rho_j E_{s_i,m_j}\left\{E_{s_j,m_j}[2\alpha - E\{\theta_i|s_i\} - E\{\theta_j|s_j\}]^2\right\} \\
& +(1-\rho_i)\rho_j E_{s_i,m_j}\left\{E_{s_j,m_j}[2\alpha - 2E\{\theta_i|s_i\} - E\{\theta_j|s_j\} + E\{\theta_i\} + \frac{1}{2}\left[E\{\theta_i|s_i\} - E\{\theta_i\}\right])]^2\right\} \\
& +\rho_i(1-\rho_j) E_{s_i,m_j}\left\{E_{s_j,m_j}[2\alpha - E\{\theta_i|s_i\} - 2E\{\theta_j|s_j\} + E\{\theta_j\} + \frac{1}{2}\left[E\{\theta_j|s_j\} - E\{\theta_j\}\right])]^2\right\} \\
& +(1-\rho_i)(1-\rho_j) E_{s_i,m_j}\left\{E_{s_j,m_j}[2\alpha - 2E\{\theta_i|s_i\} - 2E\{\theta_j|s_j\} + E\{\theta_i\} + E\{\theta_j\} + \frac{1}{2}\left[E\{\theta_j|s_j\} - E\{\theta_j\}\right]\right. \\
& \left.+\frac{1}{2}\left[E\{\theta_i|s_i\} - E\{\theta_i\}\right])]^2]\right\} \\
= \; & \frac{1}{18}[\rho_i\rho_j E_{s_i,m_j}\left\{E_{s_j,m_j}[2\alpha - E\{\theta_i|s_i\} - E\{\theta_j|s_j\}]^2\right\} \\
& +(1-\rho_i)\rho_j E_{s_i,m_j}\left\{E_{s_j,m_j}[(2\alpha - E\{\theta_i|s_i\} - E\{\theta_j|s_j\}) - \frac{1}{2}\left[E\{\theta_i|s_i\} - E\{\theta_i\}\right]]^2\right\} \\
& +\rho_i(1-\rho_j) E_{s_i,m_j}\left\{E_{s_j,m_j}[2\alpha - E\{\theta_i|s_i\} - E\{\theta_j|s_j\} - \frac{1}{2}\left[E\{\theta_j|s_j\} - E\{\theta_j\}\right])]^2\right\}
\end{aligned}$$

$$+(1-\rho_i)(1-\rho_j)E_{s_i,m_j}\left\{E_{s_j,m_j}[\left(2\alpha - E\{\theta_i|s_i\} - E\{\theta_j|s_j\} - \frac{1}{2}\left[E\{\theta_j|s_j\} - E\{\theta_j\}\right]\right)\right.$$
$$\left.-\frac{1}{2}\left[E\{\theta_i|s_i\} - E\{\theta_i\}\right]]^2\right\}$$

For obtaining the result, it is enough to show that the second term is larger than the first, and the four term is larger than the third. We start by comparing the first and second.

$$\begin{aligned}
& E_{s_i,m_j}\left\{E_{s_j,m_j}[2\alpha - E\{\theta_i|s_i\} - E\{\theta_j|s_j\}]^2\right\} \\
& -E_{s_i,m_j}\left\{E_{s_j,m_j}[(2\alpha - E\{\theta_i|s_i\} - E\{\theta_j|s_j\}) - \frac{1}{2}\left[E\{\theta_i|s_i\} - E\{\theta_i\}\right]]^2\right\} \\
= \;& E_{s_i,m_j}\left\{E_{s_j,m_j}[+2[(2\alpha - E\{\theta_i|s_i\} - E\{\theta_j|s_j\})][\frac{1}{2}\left[E\{\theta_i|s_i\} - E\{\theta_i\}\right]] - \frac{1}{4}[E\{\theta_i|s_i\} - E\{\theta_i\}]^2\right\} \\
= \;& E_{s_i,m_j}\left\{E_{s_j,m_j}[[-E\{\theta_i|s_i\}^2 + E\{\theta_i\}^2] - \frac{1}{4}[E\{\theta_i|s_i\} - E\{\theta_i\}]^2\right\} < 0.
\end{aligned}$$

Now, we follows by comparing the fourth and the third.

$$\begin{aligned}
& E_{s_i,m_j}\left\{E_{s_j,m_j}[2\alpha - E\{\theta_i|s_i\} - E\{\theta_j|s_j\} - \frac{1}{2}\left[E\{\theta_j|s_j\} - E\{\theta_j\}\right])]^2\right\} \\
& -E_{s_i,m_j}\left\{E_{s_j,m_j}[\left(2\alpha - E\{\theta_i|s_i\} - E\{\theta_j|s_j\} - \frac{1}{2}\left[E\{\theta_j|s_j\} - E\{\theta_j\}\right]\right) - \frac{1}{2}\left[E\{\theta_i|s_i\} - E\{\theta_i\}\right]]^2\right\} \\
= \;& E_{s_i,m_j}\left\{E_{s_j,m_j}[[2\alpha - E\{\theta_i|s_i\} - E\{\theta_j|s_j\} - \frac{1}{2}\left[E\{\theta_j|s_j\} - E\{\theta_j\}\right]\left[E\{\theta_i|s_i\} - E\{\theta_i\}\right]\right. \\
& \left.-\frac{1}{4}\left[E\{\theta_i|s_i\} - E\{\theta_i\}\right]^2]\right\} \\
= \;& E_{s_i,m_j}\left\{E_{s_j,m_j}[[-E\{\theta_i|s_i\}^2 + E\{\theta_i\}^2] - \frac{1}{4}[E\{\theta_i|s_i\} - E\{\theta_i\}]^2\right\} < 0.
\end{aligned}$$

This concludes the proof of point (i). To prove point (ii) we have to show that all terms are increasing on δ. We start by showing the first one.

$$\begin{aligned}
& E_{s_i,m_j}\left\{E_{s_j,m_j}[2\alpha - E\{\theta_i|s_i\} - E\{\theta_j|s_j\}]^2\right\} \\
= \;& E_{s_i,m_j}\left\{E_{s_j,m_j}[[4\alpha^2 + E\{\theta_j|s_j\}^2 + E\{\theta_i|s_i\}^2 - 2[2\alpha E\{\theta_i|s_i\} + 2\alpha E\{\theta_j|s_j\}] - E\{\theta_i|s_i\}E\{\theta_j|s_j\}]^2\right\}
\end{aligned}$$

Notice that everything is independent of δ_i but $E_{s_i,m_j}\left\{E_{s_j,m_j}[E\{\theta_i|s_i\}^2]\right\}$that is increasing.

The second term

$$\begin{aligned}
& E_{s_i,m_j}\left\{E_{s_j,m_j}[(2\alpha - E\{\theta_i|s_i\} - E\{\theta_j|s_j\}) - \frac{1}{2}\left[E\{\theta_i|s_i\} - E\{\theta_i\}\right]]^2\right\}) \\
= \;& E_{s_i,m_j}\{E_{s_j,m_j}[[(2\alpha - E\{\theta_i|s_i\} - E\{\theta_j|s_j\})]^2 - [(2\alpha - E\{\theta_i|s_i\} - E\{\theta_j|s_j\})]\left[E\{\theta_i|s_i\} - E\{\theta_i\}\right]] \\
& +\frac{1}{4}\left[E\{\theta_i|s_i\} - E\{\theta_i\}\right]^2])\}
\end{aligned}$$

we know that $E_{s_j,m_j}[[(2\alpha - E\{\theta_i|s_i\} - E\{\theta_j|s_j\})]$ is increasing on δ_i, and $E_{s_i,m_j}\left\{E_{s_j,m_j}[-[(2\alpha - E\{\theta_i|s_i\} - E\{\right.$ simplifies to $E_{s_i,m_j}\left\{E_{s_j,m_j}[E\{\theta_i|s_i\}^2 - E\{\theta_i\}^2]\right\}$ which is increasing on δ_i. Finally, $E_{s_i,m_j}\left\{E_{s_j,m_j}[\frac{1}{4}\left[E\{\theta_i|s_i\} - E\{\right.\right.$ simplifies also to $\frac{1}{4}E_{s_i,m_j}\left\{E_{s_j,m_j}[E\{\theta_i|s_i\}^2 - E\{\theta_i\}^2]\right\}$.

For the third (fourth) term we can make a similar decomposition as we did for the first (second) term to show that it is increasing in δ_i, since $\frac{1}{2}\left[E\{\theta_j|s_j\} - E\{\theta_j\}\right]$ is independent of δ_i. ■

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