ADVANCED MICROECONOMICS - 2013/2014

Sheet 2: Choice under uncertainty

Instructions: The exercises you have to deliver are marked in green. The cover sheet can be downloaded from the course webpage (and it is compulsory). Fill the data and staple this cover page, together with the rest of the exercises, in the upper left-hand corner. This material will not be returned, so make photocopies and keep the originals for yourself.

1 Suppose there are three events.

(a) An individual has a preference relation $\succeq$ over lotteries that can be represented by an expected utility function. This relationship satisfies

$$(1,0,0) \succ (0,1,0) \succ (0,0,1), \qquad \left(\frac{1}{4},\frac{1}{2},\frac{1}{4}\right) \sim \left(\frac{5}{12},\frac{1}{4},\frac{1}{3}\right)$$

Determine an utility function that represents the preferences $\succeq$.

(b) Another individual has a preference relation $\succeq$ over lotteries verifying

$$(1,0,0) \sim \left(0,\frac{1}{2},\frac{1}{2}\right), \qquad (0,0,1) \succ (0,1,0), \qquad \left(\frac{1}{2},\frac{1}{2},0\right) \succ \left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$$

Is it possible to represent the above preference relation by an expected utility function?

2 The government considers four possibilities

- Possibility A: No evacuation is necessary, and none is performed.
- Possibility B: An evacuation is performed that is unnecessary.
- Possibility C: An evacuation is performed that is necessary.
- Possibility D: No evacuation is performed, and a flood causes a disaster.

The government is indifferent between the sure outcome B and the lottery of A, with probability p, and D, with probability $1-p$. It is also indifferent between the sure outcome C and the lottery of A, with probability q, and D, with probability $1-q$. Suppose it prefers A to D and that the Expected Utility Theorem conditions are satisfied. Construct an expected utility function that represents these preferences.

3 An individual has an utility function over monetary payoffs defined by $v(x) = \ln x$. His initial wealth is w and his subjective probability that his favorite team wins the league is p. If he wins he gets the amount he bets, but if his team is beaten he loses his bet. Knowing that the optimal bet is x_0, determine the subjective probability p.

4 A risk-averse agent has an initial wealth w, but he can lose D m.u. with probability π. He can buy an insurance. Its price is q, per monetary unit insured. Prove that if the insurance is not actuarially ($q > \pi$), then the agent chooses an amount of insurance $\alpha^* < D$

5 Consider a risk averse agent with utility function $v(x)$ on monetary units and a utility function

$$U(F) = \int v(z)\, dF(z)$$

defined over lotteries. Suppose that, in the future, there are two possible states, that occur with probabilities π and $1-\pi$. Suppose that the agent can choose between two assets , $r_1 = (1,1)$ and $r_2 = (0,3)$ with prices (respectively) $q_1 = 1$ and $q_2 = 1$. (For example, the asset r_2 pays 0 units of money if (with probability π) the estate 1 happens and it pays 3 units of money if (with probability $1-\pi$) estate 2 occurs. The initial wealth of the agent is w. Let α bet the amount of units of asset r_2 acquired by the agent.

(a) Determine for which values of π the agent chosses $\alpha = 0$ or $\alpha = w$.

(b) Assuming that the utility of the agent is

$$v(x) = \sqrt{x}$$

compute the amount, α, of units of asset r_2 purchased by the agent. How does α/w change as the initial rent w of the agent varies? Compute the coefficients of absolute and relative risk aversion of the agent. Explain the results obtained above in view of these coefficients.

$\boxed{6}$ Suppose that the set of events is finite $C = \{c_0, c_1, \ldots, c_n\}$, where events are sorted in descending order $c_0 \succeq c_1 \succeq \ldots \succeq c_n$ according to some preference relation $\succeq$ that satisfies the continuity and independence axioms. Let u be an utility function representing the preference relation $\succeq$. We identify the set $\mathcal{L}$ of lotteries on C with the simplex of dimension n. Prove that the solutions of the problems

$$\max_{p \in \mathcal{L}} u(p) \quad \text{y} \quad \min_{p \in \mathcal{L}} u(p)$$

is attained at one of the points $c_0, c_1, \ldots, c_n$.

$\boxed{7}$ An agent owns a car and an initial wealth of ω euros. He faces three alternatives over his future: (i) with probability p_1 he will suffer a minor accident which will cost him L_1 euros; (ii) with probability p_2 he will suffer a major accident which will cost him $L_2 > L_1$ euros; and (iii) with probability $1 - p_1 - p_2$ he will not suffer any accident and he will keep his initial wealth ω. The agent is risk averse and maximizes his expected utility. There are two possible insurance policies

- Policy 1: It includes an amount of L m.u. deductible. In this policy, the agent pays r monetary units to the insurance company. And the insurance company pays $L_i - D$ to the agent, in case an accident occurs.
- Policy 2: It offers partial coverage. It costs r and the company would pay him $(1-\alpha)L_i$ in case an accident occurs

Suppose that $r = p_1(L_1 - D) + p_2(L_2 - D) = p_1(1-\alpha)L_1 + p_2(1-\alpha)L_2$. Prove that the agent prefers the insurance with a deductible.

$\boxed{8}$ Consider an agent with an initial wealth of $m = 10$ and a utility function $v(x) = \ln x$ on money. The agent consumes an amount c of today's wealth and saves the rest for tomorrow's consumption. The interest rate is $r = 5/100$. In addition, the agent receives tomorrow an uncertain rent (due, for example to uncertainty about his employment status): with probability $\pi = 1/2$ he gets $y + \alpha$ and with probability $1 - \pi = 1/2$ he gets $y - \alpha$, where $y = 5$ and $0 \leq \alpha \leq 5$.

(a) Write the consumption choice problem today of the agent.

(b) Express the consumption today $c(\alpha)$ and the savings of the agent as functions of α.

(c) Prove that $c'(\alpha) < 0$, that is, an increase in the future risk, induces the agent to consume less today and save more for the future.

$\boxed{9}$ Consider an economy with two assets. A bond which pays 1 in every state and a risky asset that pays a with probability π and b ($b \neq a$) with probability $1 - \pi$. Suppose that the demands of the assets are, respectively, x_1 and x_2. Suppose that the agent has preferences of the von Neumann–Morgenstern type and that he is risk averse. The initial wealth is 1 and these are also the prices of both assets.

(a) Give a necessary condition, in terms of a and b, such that the demand of the bond is positive.

(b) Give a necessary condition, in terms of a and b, such that the demand of the risky asset is positive.

$\boxed{10}$ Suppose that an agent has preferences given by the utility function $u(x_1, x_2) = \pi_1 v(x_1) + \pi_2 v(x_2)$ with $\pi_1, \pi_2 \geq 0$, $\pi_1 + \pi_2 = 1$. Suppose that $v' > 0$ and that $v'' < 0$. Prove that, if the agent is indifferent among the points (x_1, x_2) and (x_1', x_2'), then he will strictly prefer

$$\left(\lambda x_1 + (1-\lambda)x_1', \lambda x_2 + (1-\lambda)x_2'\right) \qquad 0 < \lambda < 1$$

to any one of the above points.

$\boxed{11}$ Regret Theory: Suppose two lotteries $x = (x_1, x_2, \ldots, x_s)$ and $y = (y_1, y_2, \ldots, y_s)$. Each of the events occurs with probability $\pi_1, \pi_2, \ldots, \pi_s$. The expected regret of the lottery x with respect to lottery y is

$$A(x, y) = \sum_{s=1}^{S} \pi_s h\left(\max\{0, y_s - x_s\}\right)$$

where h is an increasing function. That is, h measures the regret experienced by the agent for having chosen x once he knows the resulting state of the world. wonders what he would have obtained, had he chosen y. We say that x is as good as y, in the presence of regret, if $A(x, y) \leq A(y, x)$. Suppose that there are three states, with $\pi_1 = \pi_2 = \pi_3 = 1/3$ and that $h(x) = \sqrt{x}$. Consider the lotteries

$$\begin{aligned} x &= (0, -2, 1) \\ y &= (0, 2, -2) \\ z &= (2, -3, -1) \end{aligned}$$

Prove that the preference order induced on these three lotteries is not transitive.

$\boxed{12}$ Suppose that an agent has a utility function on money $v(x) = x^2 + \alpha x$ with $\alpha < 0$, defined for $x \leq \frac{-1}{2\alpha}$, with von Neumann-Morgenstern preferences on money. Prove that for every distribution function F,

$$U(F) = (\sigma^2(F) + \mu(F)^2) + \alpha\mu(F)$$

where

$$\mu(F) = \int x \, dF, \qquad \sigma^2(F) = \int (x - \mu(F))^2 \, dF$$

That is, the utility on a distribution function is determined by the mean and the variance of the distribution.

$\boxed{13}$ Compute the following Riemann–Stieltjes integrals.

(a) $\int_{-\infty}^{\infty} x^2 \, dF(x)$, where

$$F(x) = \begin{cases} 0 & \text{si } x < 1 \\ 1/3 & \text{si } 1 \leq x < 6 \\ 5/6 & \text{si } 6 \leq x < 10 \\ 1 & \text{si } 10 \leq x. \end{cases}$$

(b) $\int_0^{\infty} e^{-x} \, d(x^2)$.

$\boxed{14}$ Suppose that $u(x)$ is a continuous function on the interval $[a, b]$ and that $F(x)$ has a continuous derivative $F'(x) = f(x)$ on that interval. Prove that

$$\int_a^b v(x) \, dF(x) = \int_a^b v(x) f(x) \, dx.$$

Use this result to compute, again, $\int_0^{\infty} e^{-x} \, d(x^2)$.

$\boxed{15}$ Consider an agent with a utility function on money

$$v(x) = -e^{-rx}$$

where r is the coefficient of absolute risk aversion of the agent. The preferences on lotteries F of the agent satisfy the assumptions of the Expected Uitlity Theorem:

$$U(F) = \int_{\mathbb{R}} v(x)\, dF(x)$$

Compute $U(F)$ when the distribution function of a lottery F is a normal of mean μ and variance σ^2. That is, the density function of F is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

$\boxed{16}$ Consider the set of lotteries with three prizes: 100, 200 and 300. Fix a lottery $\pi = (\pi_1, \pi_2, \pi_3)$. In the simplex of probabilities, draw the lotteries that dominates π in the sense first-order stochastic dominance.

$\boxed{17}$ Suppose that there are two agents whose utility function on money is $v(x) = \sqrt{x}$. Each agent has a house whose value is m. And each faces (independently of the other) a probability 1/5 that a fire destroys the house. In country A the law stipulates that, in case of fire, each citizen has to assume his own losses. In country B the law forces that the loss incurred in a fire be shared by all the citizens.

(a) What is the expected monetary loss in each country?

(b) In which country do the agents prefer to live?