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# ISOLATING CHANGES IN NET RESIDENTIAL SEGREGATION FROM THE EFFECT OF DEMOGRAPHIC FACTORS IN THE U.S., 1989-2005

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## Abstract

This paper investigates racial residential segregation trends net of changes in the racial and the neighborhood marginal distributions of population. It follows two alternative strategies. First, it uses indices which are only invariant to either the race or the neighborhood marginal distribution and emphasize either an evenness or a representativeness segregation concept, respectively. Second, it uses the mutual information, or the *M* index that is not invariant to changes in either of the marginal distributions but admits two decompositions, each of which isolates a term which (a) is invariant to changes in the marginal distribution of one of the variables and the entropy, or diversity, of the other, and (b) reflects changes in either evenness or representativeness. According to the *M* index, net residential segregation in an evenness and in a representative sense considerably decreases for the U.S. public school student population in urban areas in 1989-2005. Invariant indices register a smaller decline in the evenness sense, as well as an increase in residential segregation in the representativeness sense, because of their failure to control, respectively, for changes in the spatial and the racial entropy.

**Keywords**: Multigroup Segregation; Multilevel Segregation; Residential Segregation; Mutual Information; Entropy Indices; Invariance Properties; Econometric Models.

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## I. INTRODUCTION

During the past few decades the U.S. has become increasingly racially and ethnically diverse because minorities have grown much more rapidly than the white population thanks to their greater fertility and/or immigration rates. As a consequence, whites represent less than 50% of the population in many metropolitan areas, and it is no longer the majority group in some others. Moreover, the racial composition in different areas presents wide variations over the country as a whole. For example, just as blacks in the past, Asians or Hispanics today are relatively concentrated in some parts of the country and almost absent in many others.[2] These changes have taken place along a demographic wave that has consistently moved the country's center of population west- and southwards, with California and Texas currently the most populous states. This paper is concerned with the measurement of residential segregation trends under these circumstances.

Multigroup residential segregation usually refers to the tendency of more than two racial groups to have different distributions over locational units within a larger area, say over neighborhoods within a country. This view of residential segregation coincides with the one referred to as "evenness" by Massey and Denton (1988) in the context of occupational segregation by gender. As such, the notion of evenness is compatible with different views about the role that the marginal population distribution by race should play in the measurement of residential segregation and its changes. Some people would argue that a race's contribution to residential segregation should depend on its size, so that if the most segregated races grow faster over time, then the country's segregation should increase. Others would think that, as long as all the racial distributions over neighborhoods remain constant, the measurement of residential segregation should not change. This property is known as composition invariance, scale invariance, or invariance 1, and was originally discussed in the context of intertemporal (or

[2] The terms *white*, *black*, *Asian* and *Native American* are used throughout the paper to refer to non-Hispanic members of these racial groups. Asians include Native Hawaiian and Pacific Islanders; Native Americans include American Indians and Alaska Natives (Eskimo or Aleut). The term *Hispanic* is an ethnic rather than a racial category since Hispanic persons may belong to any race. The term racial group is used throughout to refer to each of these five racial/ethnic categories.

international) comparisons of occupational segregation by gender.[3] An index of segregation that satisfies this property will be said to be invariant 1 or **I1** for short.

Residential segregation admits an alternative interpretation which focuses on the extent neighborhoods become black ghettos, barrios, and Chinatowns, that is, the extent to which neighborhoods have different racial compositions than the population as a whole. This notion of segregation, which is referred to as "representativeness" by Frankel and Volij (2008a), is the multigroup extension of the "isolation" dimension distinguished in Massey and Denton (1988) in the two-group case. There are different views about how the marginal population distribution by neighborhoods should affect the measurement of residential segregation viewed as representativeness. Some people would argue that a neighborhood's contribution to residential segregation should depend on its size, so that if the most segregated neighborhoods grow faster over time, then the country's segregation should increase. Others would think that as long as the neighborhoods' racial mix remains constant, residential segregation should not change. This property is known as invariance 2 or, in the context of occupational segregation, occupational invariance (see Watts, 1998, and Blackburn *et al.*, 1993, 1998). An index of segregation that satisfies this property will be said to be invariant 2, abbreviated as **I2**.

This paper searches for intertemporal comparisons in residential segregation in the U.S. using two strategies. In the first place, in agreement with those that insist that changes in segregation should be invariant to changes in the population marginal distributions, residential segregation is measured with several invariant segregation indices. In practice, the literature has emphasized **I1** indices. In particular, the two indices used in this paper are the general Atkinson, referred to as the *A* index, which is the only **I1** index that satisfies four additional properties, and a general Dissimilarity, or *D* index, which is a multigroup segregation version of the original dissimilarity index due to Duncan and Duncan, 1955. We do not know any **I2** multigroup segregation index or any index that simultaneously satisfies both

[3] See, *inter alia*, James and Taeuber (1985), Watts (1992, 1998), Charles (1992, 1998), Charles and Grusky (1995, 2004), Grusky and Charles (1998), and Hutchens (1991, 2001, 2004).

invariant properties in the multigroup case.[4] However, as pointed out in Mora and Ruiz-Castillo (2008a), by just changing the roles of racial groups and neighborhoods, any **I1** index becomes an **I2** one. Thus, the reciprocal versions of the *A* and the *D* indices, which will be referred to as *A** and *D**, are the **I2** indices used in this paper.

In the second place, one may use a segregation index that is neither **I1** nor **I2** but whose intertemporal changes admit a decomposition that isolates a term that captures segregation changes net of the impact of pure demographic factors, or net segregation changes.[5] This is the case of the mutual information, or *M* index, which was originally proposed by Theil and Finizza (1971) and whose underlying ordering has been recently characterized by Frankel and Volij (2008a).[6] The *M* index simultaneously captures the notions of evenness and representativeness, and it is equal to the demographically weighted average of local segregation indices for each race or each neighborhood. Therefore, changes in residential segregation using this index result from changes in the way races are distributed over neighborhoods and changes in the population marginal distribution over neighborhoods, and/or changes in the neighborhoods' racial composition and changes in the population marginal distributions over races. Hence, the *M* index is neither **I1** nor **I2**. However, Mora and Ruiz-Castillo (2008a) show that changes in the *M* index can be decomposed in two complementary ways: the first decomposition isolates an **I1** term, maintaining constant the marginal distribution over racial groups *and* the entropy or diversity of the population neighborhood distribution, while the second decomposition isolates an **I2** term, maintaining constant the marginal distribution over neighborhoods *and* the entropy of the population racial distribution. This paper contains the first empirical application

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[4] In the two-group case, Charles (1992) and Charles and Grusky (1995) discuss a segregation index that is both **I1** and **I2**.

[5] In the context of occupational segregation by gender, many authors have defended this strategy before (see, *inter alia*, Blau and Hendricks, 1979, Jonung, 1984, Beller, 1985, and Watts, 1992, 1998).

[6] Very few segregation indexes have been similarly characterized. In the two-group case, Chakravarty and Silber (1992) characterize an index of absolute segregation, while Chakravarty and Silber (2007) axiomatically derive a class of numerical indexes of relative segregation which parallel the multidimensional Atkinson inequality indices. Two members of that class are monotonically related to the square root index, independently characterized by Hutchens (2004), and the *M* index. In the multigroup case, Alonso-Villar and Del Río (2008) characterize local segregation indexes in the mutual information context, while Frankel and Volij (2008b) provide an ordinal characterization of the general and the symmetric Atkinson indexes.

of this methodology.[7]

We use data about the student population enrolled in public schools in Core Based Statistical Areas (CBSAs) –urban clusters of 10,000 or more inhabitants, referred to in the sequel as *cities*– during the 1989-90 and 2005-06 academic years. We aggregate the data at school districts that, although they may better correspond to municipalities in many cases, will be referred to as *neighborhoods*. To understand the interest of the empirical application, imagine a situation where school district authorities all over the country are able to implement a policy that reproduces in all schools the racial mix of the district to which they belong. In this scenario, the public student population would still experience the residential segregation studied in this paper, arising from the location decisions about neighborhoods and cities taken by their parents or caretakers among the adult population. Using the same data, Mora and Ruiz-Castillo (2008b) report changes in residential segregation within the student population using the $M$ index. They find that residential segregation declines during the period for all minorities, but since their joint population share increases by almost 13% and the segregation level of all of them is rather high, all minorities taken together contribute to an increase in the $M$ index. At the same time, the segregation index for whites increases, whereas their population share declines; however, since their demographic weight remains high whites also contribute positively to an increase in the $M$ index.[8] Even if one is willing to accept that race and neighborhood sizes should affect the measurement of segregation in a moment in time, the above results provide a good example of the need for disentangling changes in evenness and representativeness from changes due merely to variations in the population marginal distributions over racial groups or spatial units.

[7] Conceptually, Frankel and Volij (2008b) compare the $M$ index (as well as other normalized and un-normalized segregation indices which are neither **I1** nor **I2**) with the $A$ and D indices. These authors, however, do not consider **I2** indices or, above all, decompositions of segregation changes of not invariant indices –like the $M$ index– that isolate invariant terms. On the other hand, although Frankel and Volij (2008b) use the same data set than us, they study U.S. school rather than residential segregation during a slightly different time period.

[8] Analogous, although less dramatic results are found along the spatial dimension: when the spatial units are cities rather than neighborhoods, and cities are grouped into 12 regions, the mutual information index capturing between-cities residential segregation declines in 7 regions representing almost 60% of the population, but the increase in this index in the remaining 5 regions lead to an increase in the mutual information index for the country as a whole.

This paper adopts a multilevel approach where overall residential segregation is due to the fact that people of different races cluster, not only in different neighborhoods within a city, but also in different cities.[9] This poses no problem for the *M* index because it can be expressed as the sum of two terms capturing between- and within-cities residential segregation. Frankel and Volij (2008b) show that the *A* index also satisfies this property. Although the *D* index is not decomposable in that sense, a measure of overall and between-cities segregation can still be legitimately obtained with this index by computing them separately for neighborhoods and cities, respectively. For comparison purposes with the *M* and the *A* indeces, the difference between these two terms can be interpreted as a measure of within-cities segregation. An identical operation can be performed for the $A^*$ and the $D^*$ indices.

The main results of the paper can be summarized as follows. First, the **I1** indices record a reduction in residential segregation in the evenness sense smaller than what it would have been the case if, as the first decomposition of the *M* index does, it could control for the changes in the spatial entropy. Second, the **I2** indices record an increase in residential segregation in the representativeness sense because, contrary to what the second decomposition of the *M* index does, it does not control for the changes in the racial entropy. Third, within the evenness perspective provided by the first decomposition of the M index, it is found that both between- and within-cities net residential segregation decreases for all minorities. The reduction in net between-cities segregation has been greatest for Hispanics, while the reduction in net within-cities segregation has a similar order of magnitude for all minorities. Although less important, whites also experience a reduction in net overall residential segregation.

The remaining of the paper is organized in three Sections. Section II presents some notation, the independence properties and the indices to be used. Section III is devoted to the empirical results, while Section IV contains some concluding comments.

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[9] For other instances of the multilevel approach to residential and school segregation, see Massey and Hajnal (1995), Riardon *et al.* (2000), Fisher *et al.* (2004), and Mora and Ruiz-Castillo (2008b, 2008c).

## II. NOTATION, INVARIANCE PROPERTIES, AND SEGREGATION INDICES

### II.1. Notation

Assume a country partitioned into $N$ geographical units or neighborhoods, indexed by $n$ = 1,…, $N$. Each individual living in a neighborhood belongs to only one of five racial groups, indexed by $g$ = *whites*, *blacks*, *Hispanics*, *Asians*, and *NA* (Native Americans). The data available can be organized into the following 5 x $N$ matrix:

$$X = \left\{ T_g^n \right\}_{g,n} = \begin{bmatrix} T_{whites}^1 & \cdots & T_{whites}^N \\ \vdots & \ddots & \vdots \\ T_{NA}^1 & \cdots & T_{NA}^N \end{bmatrix} \tag{1}$$

where $T_g^n$ is the number of individuals of racial group $g$ living in neighborhood $n$.

The information contained in the joint absolute frequencies of racial groups and neighborhoods, $X$, is usually summarized by means of numerical indices of segregation, $S(X)$. Let $T = \sum_{n=1}^{N} \sum_{g} T_g^n$ be the total population, and denote by $P_{gn} = \left\{ \frac{T_g^n}{T} \right\}_{g,n=1}^{N}$ the joint distribution of racial groups and neighborhoods. In the following, the discussion will be restricted to indices that capture a *relative* view of segregation in which all that matters is the joint distribution, i.e. indices which admit a representation as a function of $P_{gn}$:[10]

$$S(X) = S_I\left(P_{gn}\right).$$

In order to formally introduce the segregation notions of evenness and representativeness, note that any bivariate joint distribution $P_{gn}$ admits two alternative factorizations into a conditional and a marginal:

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[10] This property, satisfied by most segregation indices, is referred to as Size Invariance in James and Taeuber (1985) and as Weak Scale Invariance in Frankel and Volij (2008a). For a study that focuses on translation invariant segregation indices that represent an absolute view of segregation, see Chakravarty and Silber (1992).

$$p_{gn} = p_{n|g} p_g = p_{g|n} p_n, \tag{2}$$

where

$p_{n|g}$ : proportion of individuals in group $g$ who are located in neighborhood $n$,

$p_{g|n}$ : proportion of individuals in neighborhood $n$ who belong to group $g$,

$p_g$ : is the proportion of individuals from racial group $g$ in the population, and

$p_n$ : is the proportion of individuals from neighborhood $n$ in the population.

Let $P_g = \left\{ p_g \right\}_g$ be the marginal distribution by racial groups and $P_{n|g} = \left\{ p_{n|g} \right\}_{n=1}^{N}$ be the conditional distribution by neighborhoods of individuals in group $g$. Since $P_{n|g}$ summarizes all the relevant information regarding how different racial groups are located across neighborhoods, any index of segregation based on the notion of evenness must be a function of $P_{n|g}$ From equation (2) we have:

$$S_I \left( P_{gn} \right) = S_E \left( P_{n|g}, P_g \right). \tag{3}$$

Therefore, any relative index of segregation admits an evenness representation that, without imposing further assumptions, will not be invariant to changes in the marginal distribution by racial groups. Alternatively, denote by $P_{g|n} = \left\{ p_{g|n} \right\}_g$ the conditional distribution by race of individuals in neighborhood $n$ and by $P_n = \left\{ p_n \right\}_{n=1}^{N}$ the marginal distribution by neighborhood size. Since $P_{g|n}$ summarizes all the relevant information regarding how neighborhoods differ in their racial compositions, any index of segregation based on the notion of representativeness must be a function of $P_{g|n}$. From equation (2) we have:

$$S_I \left( P_{gn} \right) = S_R \left( P_{g|n}, P_n \right). \tag{4}$$

Therefore, any relative index of segregation admits a representativeness representation that, without imposing further assumptions, will not be invariant to changes in the marginal distribution by neighborhoods, $P_n$.

As can be seen in expression (1), where the rows are racial groups and the columns are neighborhoods, evenness and representativeness are dual concepts: deviations from evenness (representativeness) correspond to differences in the row (column) percentages. The following observation indicates how close these two views are of each other.

Remark 1. If a segregation index $S$ that captures the notion of evenness when applied to the 5 x $N$ array $X = \{T_g^n\}$ is applied to the $N$ x 5 array $X' = \{T_n^g\}$ where the role of neighborhoods and racial groups are reversed, then what will be called the reciprocal index $S^*$ applied to $X'$ captures equally well the notion of representativeness (and vice versa).

In general, $S(X)$ and $S^*(X')$ will provide a different segregation value for the same data. When this is not the case, the segregation index under consideration is said to be transpose invariant.

**II. 2. Invariance Properties**

In the literature on occupational segregation by gender it has long been noted that both the overall gender composition of employment and the distribution of the population across occupations typically change over time and/or space. Similar phenomena are present in other segregation contexts. As indicated in the Introduction, in the past few decades the U.S. has become increasingly racially diverse and the mean population center of the country has moved consistently west- and southwards. Equations (3) and (4) highlight that those changes in the marginal distributions may result in changes in any index of segregation $S_I(P_{gn})$, independently of the notion of evenness and representativeness that it may capture. However, many people would argue that, as long as the distributions of racial groups over neighborhoods remain constant, the measurement of segregation viewed as evenness should not change. Similarly, as long as the neighborhoods' racial mix remains constant, segregation viewed as representativeness should not change.

The following two axioms have been proposed to capture these ideas. To motivate the first one, consider a situation in which only the size of one or more racial groups vary, so that the marginal distribution $P_g$ changes, but the allocation of racial groups across neighborhoods, $P_{n|g}$, remains constant. Under these circumstances, it has been argued that the segregation index should not change.

**Invariance 1 (I1):** (*Composition Invariance* in James and Taeuber, 1985, and Watts, 1998; *Homogeneity* in Hutchens, 1991; *Scale Independence* in Frankel and Volij, 2008b). If $X = \{T_g^n\}$ and $\widehat{X} = \{\widehat{T}_g^n\}$ are two matrices such that $\widehat{T}_g^n = l_g T_g^n$ for all $n$ and $g$ with $l_g > 0$ for each $g$, then $S(X) = S(\widehat{X})$.

Remark 2. The only relevant magnitudes in the domain of an **I1** index are the conditional neighborhood distributions by ethnic group, $P_{n|g}$; that is, if $S(X)$ satisfies **I1**, then $S_E(P_{n|g}, \overline{P}_g) = S_E(P_{n|g}, \overline{\overline{P}}_g)$ for any two marginal distributions $\overline{P}_g$ and $\overline{\overline{P}}_g$.

For the next property, consider situations in which the school size distribution $P_n$ changes, while the racial mix within each neighborhood $P_{g|n}$ remains constant. It has also been argued that under these conditions segregation should not change.

**Invariance 2 (I2):** (*Occupational Invariance* in Watts 1998, and Blackburn *et al.* 1993, 1995). If $X = \{T_g^n\}$ and $\widehat{X} = \{\widehat{T}_g^n\}$ are two matrices such that $\widehat{T}_g^n = l^n T_g^n$ for all $n$ and $g$ with $l^n > 0$ for each $n$, then $S(X) = S(\widehat{X})$.

Remark 3. The only relevant magnitudes in the domain of **I2** indices are the neighborhoods' racial composition, $P_{g|n}$; that is, if $S(X)$ satisfies **I2**, then $S_R(P_{g|n}, \overline{P}_n) = S_R(P_{g|n}, \overline{\overline{P}}_n)$ for any two marginal distributions $\overline{P}_n$ and $\overline{\overline{P}}_n$.

Remark 4. In view of Remark 1, the reciprocal $S^*(X)$ of any **I1** index $S(X)$ becomes an **I2** index (and vice versa).

### II. 3. Invariant Segregation Indices

The Atkinson segregation indices, which are based on the Atkinson inequality indices, were first introduced by James and Taeuber (1985) for $G$ = 2. Given any scalar $d \in (0,1)$, the Atkinson index in this case equals:

$$A_d(X) = 1 - \left[ \sum_{n=1}^{N} \left( p_{n|g=1} \right)^{d} \left( p_{n|g=2} \right)^{1-d} \right]^{1/(1-d)} .$$

As Frankel and Volij (2008b) argue, this index is difficult to generalize to more than two groups since the outer exponent, $1/(1-d)$, is the reciprocal of the weight on a particular group. Instead, they propose an increasing transformation of the original index which represents the same ordering. Let $p = (p_1, p_2, \ldots, p_G)$ be a vector of $G$ nonnegative weights that sum up to one. The general Atkinson index with weights $p$, $A_p(X)$, is defined by

$$A_p(X) = 1 - \sum_{n=1}^{N} \prod_{g} \left( p_{n|g} \right)^{p_g} .$$

The symmetric Atkinson index, which is obtained when all weights $p_g$ are equal, is **I1**. More generally, whenever the weights $p$ are invariant to the marginal distribution of groups, $P_g$, then $A_p(X)$ will be **I1**. The reciprocal version of $A_p(X)$ is defined by

$$A^*_{p*}(X) = 1 - \sum_{g} \prod_{n=1}^{N} \left( p_{g|n} \right)^{p^*_n},$$

where $p^* = (p^*_1, p^*_2, \ldots, p^*_N)$ are $N$ nonnegative weights that sum up to one. It follows from Remark 4 that $A^*_{p*}(X)$ is **I2** if the neighborhoods' weights $p^*$ are invariant to the marginal distribution of neighborhoods, $P_n$.

A general multigroup version of the Index of Dissimilarity –first proposed by Duncan and Duncan (1955) in the two-group context– can be defined as

$$D_p(X) = \frac{1}{2(G-1)} \sum_{n=1}^{N} \sum_{g} \left| p_{n|g} - \sum_{g'} p_{g'} p_{n|g'} \right| .$$

Frankel and Volij (2008b) define the Unweighted Dissimilarity Index by giving the same weight to each ethnic group, $p_g = 1/G$ for all $g$. Whenever the weights $p$ are invariant to $P_g$, as in the case of the Unweighted Dissimilarity, $D_p(X)$ is **I1**. It follows from Remark 4 that the reciprocal version of the index, i.e.

$$D^*_{p*}(X) = \frac{1}{2(N-1)} \sum_g \sum_{n=1}^{N} \left| p_{g|n} - \sum_{n'=1}^{N} p^*_{n'} p_{g|n'} \right|,$$

is **I2** if the neighborhoods' weights $p^* = (p^*_1, p^*_2, ..., p^*_N)$ are invariant to the marginal distribution of neighborhoods, $P_n$.

**II. 4. The Mutual Information Index of Segregation**

In information theory, the expression

$$M_g = \sum_{n=1}^{N} p_{n|g} \log\left( \frac{p_{n|g}}{p_n} \right),$$

is known as the expected information of the message that transforms the set of proportions $P_n$ to the set of proportions in group $g$, $P_{n|g}$.[11] Since $M_g$ measures the extent to which the conditional distribution $P_{n|g}$ differs from the marginal distribution $P_n$, it can be seen as a local index of segregation for racial group $g$ when segregation is interpreted as a deviation from evenness. The mutual information index $M$ is the weighted average of the local segregation indices for racial groups, with weights equal to the proportion of individuals from each group:

$$M = \sum_g p_g M_g. \tag{2}$$

Thus if the groups that are more segregated grow faster over time, the $M$ index will register an increase in segregation.

Alternatively, the expression

[11] In principle, the logarithm could be computed in any base. In the empirical application we will use the natural logarithm.

$$M^n = \sum_g p_{g|n} \log\left(\frac{p_{g|n}}{p_g}\right) \quad (3)$$

is the expected information of the message that transforms $P_g$ to $P_{g|n}$. Since it measures the extent to which the racial composition in neighborhood $n$ differs from the one for the population as a whole, $M^n$ can be seen as a local index of segregation in neighborhood $n$ when segregation is interpreted as a deviation from representativeness. As Frankel and Volij (2008a) and Mora and Ruiz-Castillo (2005) show, the weighted average of information expectations defined in (3) with weights equal to the proportion of individuals in each neighborhood coincide with the overall index of segregation as defined in equation (2), that is:

$$M = \sum_{n=1}^{N} p_n M^n. \quad (4)$$

Thus if the neighborhoods that are more segregated grow faster over time, the $M$ index will register an increase in segregation.

Equations (2) and (4) indicate that the $M$ index is transpose invariant, it captures the notions of evenness and representativeness in a symmetric fashion, and it is sensitive to variations in the marginal distributions by racial groups and neighborhoods, i.e. it is neither **I1** nor **I2**.

For reasons that will be apparent below, the $M$ index will be motivated using the entropy concept of diversity.[12] For any probability distribution $P = \{p_i\}$ of a discrete variable, say racial groups, the expression

$$E(P) = \sum_i p_i \log\left(\frac{1}{p_i}\right)$$

measures the diversity exhibited by the distribution in the sense that it equals to zero if and only if all individuals are members of a single group, and it is maximized if and only if individuals are evenly

[12] See Theil and Finizza (1971) Theil (1972), Reardon and Firebaugh (2002), and Frankel and Volij (2008a). The $M$ index can also be motivated as a statistical measure of association as in Zoloth (1974, 1976), Flückiger and Silber (1999), and Mora and Ruiz-Castillo (2008c).

distributed among all the groups. Theil and Finizza (1971) show that the mutual information index is the weighted average of the differences between the neighborhood and overall measures of racial diversity, with weights equal to the proportion of individuals in each neighborhood:

$$M = \sum_{n=1}^{N} p_n \left( E\left(P_g\right) - E\left(P_{g|n}\right) \right).$$

Finally, the index $M$ can also be expressed as the weighted average of the differences between the racial-specific and the overall measures of neighborhood diversity, with weights equal to the proportion of individuals in each racial group:

$$M = \sum_{g} p_g \left( E\left(P_n\right) - E\left(P_{n|g}\right) \right).$$

### II.5 Decompositions of Pairwise Comparisons of the M Index[13]

Differences in the index of segregation between any two situations may result from differences in the marginal distributions, $P_g$ and $P_n$, as well as from differences in the conditional distributions, $P_{n|g}$ and $P_{g|n}$. As indicated in the Introduction, there are reasons to argue that pairwise comparisons of segregation should net out the effect of differences in the marginal distributions. Such comparisons can be accomplished in at least two ways. First, if the index is invariant, then the comparison of the index will also be invariant. Second, if the index is not invariant, then observed differences may be decomposed so that one of the terms in the decomposition reflects changes in segregation which are due only to changes in one of the conditional distributions. This is the strategy applied for the $M$ index in the rest of this Section.

For the sake of concreteness, assume that there is data on a country in two periods, $X(t)$, $t = \left\{t^1, t^2\right\}$ and that we are concerned with the intertemporal comparison of segregation, $\Delta M = M\left(X(t^2)\right) - M\left(X(t^1)\right)$. Mora and Ruiz-Castillo (2008a) show that in pairwise comparisons the

[13] This Section draws substantially from Mora and Ruiz-Castillo (2008a).

$M$ index admits two decompositions into three terms. Given the set $\boldsymbol{p} = (\boldsymbol{p}_1, \boldsymbol{p}_2, ..., \boldsymbol{p}_G)$ of $G$ nonnegative weights that sum up to one, the first decomposition is the following:

$$\Delta M = \Delta P_{n|g}(\boldsymbol{p}) + \Delta E^n + \Delta P_g(\boldsymbol{p}), \tag{5}$$

where

$$\Delta P_{n|g}(\boldsymbol{p}) = \sum_{g=1}^{G} \boldsymbol{p}_g \sum_{n=1}^{N} \left\{ p_{n|g}(t^2) \log\left(p_{n|g}(t^2)\right) - p_{n|g}(t^1) \log\left(p_{n|g}(t^1)\right)\right\},$$
$$\Delta E^n = E\left(P_n(t^2)\right) - E\left(P_n(t^1)\right),$$
$$\Delta P_g(\boldsymbol{p}) = \sum_{t=t^1,t^2} (-1)^{1(t=t^1)} \left\{ \sum_{g=1}^{G} \left(p_g(t) - \boldsymbol{p}_g\right) \sum_{n=1}^{N} p_{n|g}(t) \log\left(p_{n|g}(t)\right)\right\},$$

and $1(\bullet)$ is the indicator function. The three terms in equation (5) can be interpreted as follows. First, $\Delta E^n$ captures segregation changes due to intertemporal changes in the entropy of neighborhoods. Second, $\Delta P_g(\boldsymbol{p})$ captures segregation changes due to differences between the marginal distribution by groups and the groups' weights, i.e. differences between $P_g(t)$ and $\boldsymbol{p}$. Third, $\Delta P_{n|g}(\boldsymbol{p})$ gives the difference in segregation that arises from changes in $P_{n|g}(t)$ when both the overall entropy of neighborhoods and the marginal distribution $P_g(t)$ remain constant, and the latter equals $\boldsymbol{p}$. Notice that $\Delta P_{n|g}(\boldsymbol{p})$ is ***I1*** in the sense that it equals zero if $T_g^n(t^2) = \boldsymbol{l}_g T_g^n(t^1)$.

Given the set $\boldsymbol{p}^* = (\boldsymbol{p}_1^*, \boldsymbol{p}_2^*, ..., \boldsymbol{p}_N^*)$ of $N$ non-negative weights that sum up to one, the second decomposition is the following:

$$\Delta M = \Delta P_{g|n}(\boldsymbol{p}^*) + \Delta E_g + \Delta P_n(\boldsymbol{p}^*), \tag{6}$$

where

$$\Delta P_{g|n}(\boldsymbol{p}^*) = \sum_{n=1}^{N} \boldsymbol{p}_n \sum_{g=1}^{G} \left\{ p_{g|n}(t^2) \log\left(p_{g|n}(t^2)\right) - p_{g|n}(t^1) \log\left(p_{g|n}(t^1)\right)\right\},$$
$$\Delta E_g = E\left(P_g(t^2)\right) - E\left(P_g(t^1)\right),$$
$$\Delta P_n(\boldsymbol{p}^*) = \sum_{t=t^1,t^2} (-1)^{1(t=t^1)} \left\{ \sum_{n=1}^{N} \left(p_n(t) - \boldsymbol{p}_n^*\right) \sum_{g=1}^{G} p_{g|n}(t) \log\left(p_{g|n}(t)\right)\right\}.$$

The three terms in equation (9) can be interpreted as follows. First, $\Delta E_g$ captures segregation changes due to changes in the entropy of racial groups. Second, $\Delta P_n(\mathbf{p}^*)$ measures segregation changes due to differences between $P_n(t)$ and $\mathbf{p}^*$. Third, $\Delta P_{g|n}(\mathbf{p}^*)$ gives the difference in segregation that arises from changes in $P_{g|n}(t)$ when both the overall entropy of racial groups and $P_n(t)$ remain constant, and the latter equals $\mathbf{p}^*$. Note that $\Delta P_{g|n}(\mathbf{p}^*)$ is ***I2*** in the sense that it equals zero if $T_g^n(t^2) = \lambda_n T_g^n(t^1)$.

To understand the differences between these decompositions and what can be accomplished with **I1** and **I2** indices, think of a situation in which the marginal distribution by neighborhoods changes from $P_n$ to $\hat{P}_n$ while neither the marginal distributions by race nor the conditional distribution by groups at neighborhood $n$ change, i.e. $\hat{P}_g = P_g$ and $\hat{P}_{g|n} = P_{g|n}$. In this case, the change in the marginal distribution by neighborhoods will result in proportional changes in the conditional distribution by neighborhoods of students in group $g$, $\hat{P}_{n|g} = P_{n|g}\frac{\hat{P}_n}{P_n}$. Consider first the comparison between using decomposition (6) or using any **I2** index. For any given set $\mathbf{p}^*$ of weights, the change in segregation will be entirely attributed to changes in the marginal distribution by neighborhoods and the terms $\Delta P_{g|n}(\mathbf{p}^*)$ and $\Delta E_g$ will be equal to zero. Thus, according to decomposition (6), segregation viewed as representativeness will not change. Since $\hat{P}_{g|n} = P_{g|n}$, the same conclusion will be reached when using any **I2** index. Consider now the comparison between using decomposition (5) or using any **I1** index which fails to be **I2**. For any given set $\mathbf{p}$ of weights, equation (5) would decompose the change in segregation partly as a result of the change in the entropy by neighborhoods, $\Delta E^n$, and partly as a result of a change in net segregation viewed as evenness, $\Delta P_{n|g}(\mathbf{p}) = \Delta M - \Delta E^n$. In contrast, the change in segregation reported by the **I2** index will not discount the effect on evenness of the change in the entropy of neighborhoods. Similarly, an example can be given to show that while any **I2** index which fails to be **I1** will not discount the effect on representativeness on the change in the entropy of racial groups, the **I2** term in decomposition (6) will.

Decompositions (5) and (6) can only be computed after specific values for $p$ and $p*$ are chosen. Naïve candidates are $P_g(t^1)$ and $P_g(t^2)$ for $p$ and $P_n(t^1)$ and $P_n(t^2)$ for $p*$. However, as Karmel and MacLachlan (1988) argue, the decompositions obtained using these weights separately are generally inappropriate in the sense that each of their terms cannot be thought of as arising from the comparisons of two actual datasets. Following the approach advocated in Deutsch *et al.* (2006), Mora and Ruiz-Castillo (2008a) show, however, that the average of the two decompositions obtained using $P_g(t^1)$ and $P_g(t^2)$ as $p$ and the average of the two decompositions obtained using $P_n(t^1)$ and $P_n(t^2)$ as $p*$ are free of this criticism.

## III. EMPIRICAL RESULTS

### III.1. Data and Demographic Trends

We use the Common Core of Data (CCD) compiled by the National Center for Educational Statistics (NCES).[14] This dataset contains school enrolment records by racial groups from all public schools in the United States. Results are reported for the academic years 1989-90 (the first year for which complete enrolment data is available) and 2005-06. As defined by the U.S. Office of Management and Budget (OMB), the term Core Based Statistical Area (CBSA) is a collective term for both metropolitan and micropolitan statistical areas. A metropolitan area contains a core urban area of 50,000 or more population, and a micro area contains an urban core of at least 10,000 (but less than 50,000) population. Schools districts are retrospectively assigned CBSA codes based on 2005 ZIP codes so that comparisons over time can be made without changes in city boundary definitions affecting the results. As indicated in the Introduction, cities and neighborhoods are identified with CBSAs and school districts within them, respectively, while the student population enrolled in public schools in such cities and neighborhoods will be simply referred to in the sequel as the population.

[14] Other versions of this data set have been used in studies of residential and school segregation by Reardon *et al.* (2000), Frankel and Volij (2008a, 2008b), and Mora and Ruiz-Castillo (2008b, 2008c).

Since the decomposition methods for changes in residential segregation according to the *M* index are valid for the homogeneous case in which the two situations under comparison share a common structure, a sample has been constructed with a common set of 834 cities and 5,429 neighborhoods.[15] Basic statistics about this population, including its racial and spatial entropy as a measure of diversity, are offered in Table 1.

**Table 1 around here**

As can be seen in Panel A, minorities (namely, Native Americans, blacks, Asians, and Hispanics) already represent 34.8% of the 24.8 million people of the total population in 1989; furthermore, since all of them grow rather rapidly while whites actually decrease during this period, they represent as much as 48.1% of the 25.5 million people of the total population in 2005. Among minorities, the black, Native American and Asian populations increase by 10%, 30% and 32%, respectively, while Hispanics increase by 72%. As a result, there is a considerable change in the marginal distribution by race: in 2005 whites represent 13.3 percentage points less than in 1989, and Hispanics, blacks, and Asians 10.1, 1.7 and 1.3 points more, respectively. Not surprisingly, the racial diversity of the population, measured by its entropy, considerably increases by 17.6% during this period (see Panel C in Table 1).

The spatial distribution of the population across neighborhoods and cities is markedly skewed right, as shown by the large difference between the average and the median number of individuals per neighborhood and per city in both years (see Panel B). At the neighborhood level, average and median sizes slightly increase; changes in the marginal distribution by neighborhoods are hard to summarize, but the entropy –a measure of diversity or concentration– remains essentially stable (see Panel C). At the city level, the mean slightly increases, while the median and the entropy slightly decrease.

### III. 2. Changes In Segregation According to the Mutual Segregation Index

[15] The main difference between the common sample used in this paper and the one in Ruiz-Castillo (2008b) is twofold: the latter includes 869 cities and 5,834 neighborhoods in 1989, 1.3% less whites, and 1% and 0.3% more blacks and Hispanics, respectively; on the other hand, in 2005 there are 999 cities and 7,707 neighborhoods, 0.8% and 0.7% less whites and Hispanics and 1.4% more blacks. However, as will be seen below, residential segregation changes according to the *M* index in the two papers are of the same order of magnitude.

What is the impact on residential segregation of such rather important demographic changes? In order to understand the interplay between changes in segregation and demographic changes when segregation is measured according to the *M* index, recall that, from the evenness point of view, overall segregation in 1989 is $M = \sum_g p_g M_g$, where $\left(p_g M_g\right)$ is the contribution to this magnitude by racial group *g*. Therefore, the change in overall segregation, $\Delta M$, can be expressed as

$$\Delta M = \sum_g \Delta\left(p_g M_g\right).$$

On the other hand, overall segregation can be decomposed as the sum of a between- and a within-cities segregation term, $M = BC + WC$. Therefore,

$$\Delta M = \Delta BC + \Delta WC;$$

that is, changes in overall segregation result from changes in between- and within-cities segregation, $\Delta BC$ and $\Delta WC$. Similarly, for each $g$ we have that $M_g = BC_g + WC_g$, so that $BC = \sum_g p_g BC_g$ and $WC = \sum_g p_g WC_g$, where $\left(p_g BC_g\right)$ and $\left(p_g WC_g\right)$ are the contributions to $BC$ and $WC$ by racial group *g*, respectively. Therefore, we have

$$\Delta BC = \sum_g \Delta\left(p_g BC_g\right),$$

$$\Delta WC = \sum_g \Delta\left(p_g WC_g\right).$$

The information of total changes ($\Delta M$, $\Delta BC$, $\Delta WC$), as well as the details by racial group ($\Delta M_g$, $\Delta BC_g$, $\Delta WC_g$) is in Panel A in Table 2. Panel B reports the changes in the contributions to each of the indices by each racial group, $\left(\Delta\left(p_g M_g\right), \Delta\left(p_g BC_g\right), \Delta\left(p_g WC_g\right)\right)$.

**Table 2 around here**

Regarding changes in racial groups' segregation indices, $\Delta M_g$, the most remarkable feature is perhaps the increase in whites' index of overall segregation and the decrease in that index for all minorities. But behind this general pattern, the following three points should be emphasized. First, in 2005 blacks, Hispanics, and Native Americans have simultaneously less between- and within-cities segregation than in 1989. Second, between-cities segregation strongly declines for Asians, but their within-cities segregation term is the only one increasing among the minorities. Third, whites register an important increase of both between- and within-cities segregation. This is consistent with the idea that some whites migrate to whitier areas, thereby causing an increase in their own segregation indices.

To understand the changes in the contributions to residential segregation by racial group, and hence in overall segregation, it is necessary to take into account variations in the population marginal distribution by race (see column 3 in Table 1). As can be seen in the last column of Panel A in Table 2, the change in overall residential segregation for the country, $\Delta M$, is equal to 4.2, or 11.6% of the level reached in 1989. In the mutual information framework, this means that a randomly selected individual's neighborhood in 2005 conveys more information about its race than in 1989. To see how this is possible, note that

$$\Delta M = \sum_g p_g{}^* \Delta M + \Delta p_g M_g \tag{7}$$

where $p_g{}^*$ is the marginal distribution in 2005. To begin with, consider the trends for whites. Whites' contribution to segregation increases by 3.0 index points –which accounts for 71.4% of the increase in overall segregation for the country as a whole– because the increase in their segregation index in the first term of equation (7) more than compensates the negative effect of the decrease in its population share in the second term of that equation. On the contrary, changes in the contributions to overall segregation in the case of Hispanics, Asians, and Native Americans are positive because their population shares increase. For blacks, in contrast, changes in the contribution to overall segregation are negative because the change in the contribution to within-cities segregation turns out to be negative in spite of the fact that their population share also increases.

Although useful for understanding the role of changes in the local indices of segregation in segregation changes for the country as a whole, decompositions like (7) fail to properly isolate changes in residential segregation net of changes in the marginal distributions. Sub-sections III.3 and III.4 will present two strategies to achieve this end.

### III.3. Changes in Residential Segregation According to Invariant Indices

A potential drawback of measuring segregation using the symmetric Atkinson index and the Unweighted Dissimilarity index, both of which employ equal weights for all racial groups, is that their values might be unduly sensitive to the inclusion of a relatively small group, say Native Americans in the U.S. Indeed, when Native Americans, which account for less than one percentage point of the total population, are included in the sample, the value of the symmetric Atkinson index increases by 12.5% in 1989 and by 11.3% in 2005. The corresponding values for the Unweighted Dissimilarity index are 3.6% and 2.3%.[16] An alternative strategy is to compute the general Atkinson and Dissimilarity indeces, $A_{\mathbf{p}}(X)$ and $D_{\mathbf{p}}(X)$, using as weights $\mathbf{p}$ the demographic importance of each of the racial groups in a given reference year. A similar strategy can be implemented for the reciprocal indices using as weights the demographic importance of the neighborhoods in a reference year. For comparability with the results regarding the mutual information index decompositions, in the following $P_g(1989)$ and $P_g(2005)$ are used as $\mathbf{p}$ for computation of the general Atkinson and Dissimilarity indices and $P_n(1989)$ and $P_n(2005)$ are used as $\mathbf{p}^*$ for the computation of the reciprocal Atkinson and reciprocal Dissimilarity indices. We focus on the averages of the resulting indices, which will be

[16] Similarly, reciprocal indices which place the same weight to each neighborhood may be very sensitive to the exclusion of small neighborhoods. For example, if the smallest neighborhoods which account for one percentage point of the student population are excluded from the sample, then the symmetric Atkinson index increases by 15.3% in 1989 and by 6.9% in 2005. The corresponding figures for the Unweighted Dissimilarity are 3% and 2.4%.

denoted by $\overline{A}, \overline{A}^*, \overline{D}$, and $\overline{D}^*$.[17] Frankel and Volij (2008b) show that the $A$ index is decomposable into a between-cities and a within-cities term as the mutual information $M$ index. As pointed out in the Introduction, although the $D$ index is not decomposable in that sense, the difference between a measure of overall segregation and a measure of between-cities segregation can be interpreted, for comparison purposes, as a measure of within-cities segregation. An identical operation can be performed for the $A*$ and the $D*$ indices. Panel A in Table 3 contains the changes in $\overline{A}, \overline{A}^*, \overline{D}$, and $\overline{D}^*$, and their between- and within-cities decompositions.

**Table 3 around here**

The message is clear: the sign of the change in residential segregation during 1989-2005 depends on the notion of segregation advocated. In the first place, both **I1** indices agree on two points. First, taking into account that according to the $M$ index residential segregation for all minorities goes down by a considerable amount, one would expect any **I1** index to register a decrease in residential segregation from an evenness perspective. Indeed, both $\overline{A}$ and $\overline{D}$ report an overall residential segregation reduction by 16.6% and 4.9%, respectively.[18] Second, most of this decrease is attributable to a reduction in between-cities segregation. In the second place, from a representativeness perspective both **I2** indices also agree on two points. First, according to $\overline{A}^*$ and $\overline{D}^*$ overall residential segregation increases by 24.9% and 26.9%, respectively. Second, this is the result of increases in both between- and within-cities segregation. However, according to the reciprocal Atkinson $\overline{A}^*$ index both terms are equally responsible for the overall increase, while according to the reciprocal Dissimilarity $\overline{D}^*$ index about 2/3 of the overall increase is attributable to the increase in the between-cities component.

[17] Detailed results are available upon request.

[18] These results are compatible with the changes in school segregation reported in Frankel and Volij (2008b). The reduction in school segregation according to the Atkinson index using as weights the marginal distribution by race in 1987-88 and 2005-06 are 23.1% and 29.1%. Using the Unweighted Dissimilarity index, this reduction is 6.3%.

Finally, the pattern of change by racial group is also quite different (see Panel B in Table 3). According to $\overline{A}^*$, due to a reduction in their contribution to between-cities residential segregation, the more important minorities –blacks, Hispanics, and Asians– contribute negatively to the overall change; consequently, the increase in overall segregation is accounted for by a positive and strong white contribution. However, according to $\overline{D}^*$ not only whites, but also the remaining groups, positively contribute to the general increase in residential segregation.

### III. 4. Changes In Net Segregation According to the M Index

In this Section, we present results for decompositions (5) and (6) using the procedure advocated by Mora and Ruiz-Castillo (2008a). In particular, we report the average of the two decompositions obtained using $P_g$ (1989) and $P_g$ (2005) as $\boldsymbol{p}$ and the average of the two decompositions obtained using $P_n$ (1989) and $P_n$ (2005) as $\boldsymbol{p}$ *.[19] Results are shown in Panel A of Table 4.[20] Panel B contains the changes in the conditional distributions for each $g$ within the evenness perspective

$$\Delta\widetilde{M}_g = \sum_{n=1}^{N}\left\{ p_{n|g}(t^2)\log\left(p_{n|g}(t^2)\right) - p_{n|g}(t^1)\log\left(p_{n|g}(t^1)\right)\right\}. \quad (8)$$

**Table 4 around here**

This information deserves the following 4 comments. First, the more remarkable result is perhaps the dramatic decrease in net overall segregation according to both the evenness and the representativeness views: 22.9 index points, or 63.2% of the overall residential segregation reached in 1989 according to evenness, and 13.9 points or 38.4% according to representativeness. Within the mutual information framework, both views complement each other by emphasizing a different

[19] Detailed results are available upon request.

[20] As was observed in Section III.2, the change in between- and within-cities segregation in this paper's common sample is equal to 2.5 and 1.7. These values do not differ greatly from 1.0 and 2.5, which are the corresponding values reported by Mora and Ruiz-Castillo (2008b) for the entire unbalanced sample; therefore, the change in overall segregation in both samples is equal to 4.2 versus 3.5 index points. Given these differences, we feel comfortable that the following results obtained in this paper are equally applicable to the segregation changes observed in the former paper.

diagnostic. In the first place, there is a reduction in net residential segregation according to an evenness perspective, i.e. $\Delta\overline{P}_{n|g} < 0$, so that when we choose an individual of a given race, in 2005 we know from her race less about the neighborhood she lives in than in 1989. Both in absolute and relative terms, the more dramatic change is the reduction in within-cities net segregation, that is, when the selection of an individual of a given race takes place once a certain city has been chosen. In the second place, the second decomposition shows a reduction in net residential segregation in a representativeness perspective, i.e. $\Delta\overline{P}_{g|n} < 0$, so that when we choose any individual in a given neighborhood we learn less in 2005 about her race than in 1989. This negative change in net segregation according to a representativeness view actually takes place at the city level: once a city has been chosen, if we select an individual in a given neighborhood, we know more about her race in 2005 than in 1989. In other words, within-cities net segregation slightly increases during the period.

Second, these results differ markedly from those obtained with the **I2** indices, $\overline{A}^*$ and $\overline{D}^*$, according to which when a representativeness perspective is adopted residential segregation increases during 1989-2005. Moreover, from an evenness perspective the reduction in residential segregation according to the **I1** indices $\overline{A}$ and $\overline{D}$ is of a smaller order of magnitude than the reduction according to the term $\Delta\overline{P}_{n|g}$ in the first decomposition of *DM*. How can this be explained?

Invariant indices only neutralize changes in one of the marginals. First, **I2** indices convey changes in residential segregation according to a representativeness view independently of changes in the marginal distribution over neighborhoods, $\Delta P_n$. But in so doing they mix up the effect of changes in the conditional distribution that focus on the racial composition by neighborhood, $\Delta P_{g|n}$, with the impact of changes in the marginal distribution by race, $\Delta P_g$. As can be seen in the third term of the first decomposition, the impact of the latter on residential segregation is positive and strong (particularly for between-cities segregation). By holding constant the change in racial entropy, the second decomposition of the $M$ index partially controls for this effect, allowing the reduction in residential segregation in a representativeness view to reveal itself through the term $\Delta\overline{P}_{g|n}$ at the between-cities level. In the absence of such control, the **I2** indices $\overline{A}^*$ and $\overline{D}^*$ indicate an increase in residential segregation in the representative sense (particularly at the between-cities segregation level, as expected).

Second, **I1** indices convey changes in residential segregation according to an evenness view independently of changes in the marginal distribution over races, $\Delta P_g$. But in so doing they mix up the effect of changes in the conditional distribution that focus on the spatial distribution by race, , $\Delta P_{n|g}$, with the impact of changes in the marginal distribution by neighborhood, $\Delta P_n$. As can be seen in the third term of the second decomposition, the impact of the latter on residential segregation is slightly positive (as a result of a small positive effect in within-cities segregation, almost offset by a negative effect in between-cities segregation). By holding constant the change in spatial entropy, the first decomposition of the $M$ index partially controls for this effect, allowing the reduction in residential segregation in an evenness view to reveal itself through the term $\Delta\overline{P}_{n|g}$, particularly at the within-cities level. In the absence of such control, the **I1** indices $\overline{A}$ and $\overline{D}$ indicate an increase in residential segregation in the evenness sense of a smaller order of magnitude (particularly at the within-cities segregation level, as expected).

It could be argued that, knowing in advance the sign and strength of the racial and spatial entropy changes, either **I1** or **I2** indices could then be selected accordingly. Even in this case, the analysis of residential segregation changes would be restricted to either an evenness or a representativeness view. An obvious alternative is to use the two decompositions of the M index that provide in all cases an integrated version of both views.

Finally, it is very important to know about changes in net segregation by race group at different spatial levels, a question that becomes possible in the evenness view (see Panel B in Table 4). First, as expected from the detailed geographical analysis in Mora and Ruiz-Castillo (2008b), the reallocation of Hispanics and, to a lower extent, the remaining minorities over the U.S. cities during 1989-2005 causes strong decreases in between-cities net segregation for all these groups. Second, the previous literature on residential segregation based on pairwise comparisons of within-cities segregation tends to emphasize the decrease of residential segregation between whites and blacks. Here, not only blacks, but also Asians and Hispanics experience strong decreases of a similar order of magnitude in within-cities net segregation. Third, as a consequence, all minorities and, above all, Hispanics, exhibit very strong reductions in overall net segregation. Although less important, whites also experience a reduction in net overall residential segregation.

## IV. CONCLUSIONS

Whether the measurement of segregation according to an evenness or a representativeness view should depend on race and/or neighborhood sizes is a debatable question. In either case, most people would agree that it is convenient to isolate changes in some notion of net segregation from pure demographic changes in the marginal distributions over races and/or neighborhoods. This paper has confronted for the first time different strategies to achieve that end with data for the U.S. student population enrolled in urban public schools in 1989-2005.

First, two **I1** indices invariant only to changes in the marginal distribution over races –the Generalized Atkinson and a generalized Dissimilarity index– have registered a decrease in residential segregation in an evenness sense. Second, their reciprocal indices, invariant only to the marginal distribution over neighborhoods, have registered an increase in residential segregation in a representativeness sense. Third, residential segregation changes have been measured using the *M* index that simultaneously captures both evenness and representativeness segregation notions but it is neither **I1** nor **I2**. During 1989-2005, the interplay between residential segregation changes and changes in the population marginal distributions has given rise to an increase in residential segregation according to the

*M* index. However, as shown originally in Mora and Ruiz-Castillo (2008a), such changes are decomposable in two ways. In the first decomposition, changes in the conditional distribution by race – net of changes in the marginal distribution by race and in the spatial entropy– have registered a decrease in residential segregation in an evenness sense greater than the one recorded by **I1** indices. This decomposition permits studying changes in net segregation by group and geographical level. In particular, it has been found that the reduction in net between-cities segregation has been greatest for Hispanics, while the reduction in net within-cities segregation has a similar order of magnitude for all minorities. Finally, in the second decomposition, changes in the conditional distribution by neighborhood –net of changes in the marginal distribution by neighborhood and in the racial entropy– have registered a decrease in residential segregation in a representative sense, contrary to the increase recorded by **I2** indices.

These results illustrate that in practical situations simultaneous changes in both marginal distributions cloud the measurement of net segregation. More fundamentally, they emphatically exemplify that **I1** or **I2** multigroup indices may provide a misleading view as they only control for the changes in one of the two marginals. It then appears always advisable to use index decompositions to isolate changes of segregation from pure demographic changes. To our knowledge, the two decompositions of the *M* index used in this paper are the more complete in the sense that they isolate changes in segregation from changes in the marginal distribution of one of the variables and the entropy changes in the other variable. However, future research may consider decompositions of changes in **I1** and **I2** indices which isolate terms invariant to changes in the entropy by neighborhoods and changes in the entropy by racial groups, respectively.

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**Table 1. Descriptive Statistics for the U.S. Urban Student Population Enrolled in Public Schools**

**PANEL A: Race Frequencies**

| | 1989 (1) | | 2005 (2) | | Change (3) = (2) – (1) | | Change in %: (4) = (3)/(1)x100 | |
|---|---|---|---|---|---|---|---|---|
| | millions | In % | millions | In % | millions | In % | millions | In % |
| **Minorities** | 8.6 | 34.8 | 12.2 | 48.1 | 3.6 | 13.3 | 42.1 | 38.1 |
| **Native Americans** | 0.2 | 0.7 | 0.2 | 0.9 | 0.1 | 0.2 | 33.8 | 30.0 |
| **Asians** | 1.0 | 4.2 | 1.4 | 5.5 | 0.4 | 1.3 | 36.1 | 32.3 |
| **Blacks** | 4.0 | 16.1 | 4.5 | 17.8 | 0.5 | 1.7 | 13.7 | 10.5 |
| **Hispanics** | 3.4 | 13.8 | 6.1 | 23.9 | 2.7 | 10.0 | 77.3 | 72.4 |
| **Whites** | 16.1 | 65.2 | 13.2 | 51.9 | -2.9 | -13.3 | -18.1 | -20.3 |
| **All** | **24.8** | **100.0** | **25.5** | **100.0** | **0.7** | **0.0** | **2.9** | **0.0** |

**PANEL B: No. of Students per Organizational Units**

| | Per District | | | | Per city | | | |
|---|---|---|---|---|---|---|---|---|
| | 1989 | 2005 | Change | % Change | 1989 | 2005 | Change | % Change |
| **Average** | 4,560 | 4,691 | 130.7 | 2.9 | 29,683 | 30,534 | 850.6 | 2.9 |
| **Median** | 1,967 | 2,112 | 145.0 | 7.4 | 6,132 | 5,823 | -308.5 | -5.0 |

**PANEL C: Population Diversity Measured by its Entropy**

| | 1989 (1) (% max) | 2005 (2) (% max) | Change (3)=(2)-(1) | Change in % (4) = (3)/(1)x100 |
|---|---|---|---|---|
| **Racial Entropy** | | | | |
| **Racial Groups** ($E_g$) | 101.3 (62.9) | 119.1 (74.0) | 17.8 | 17.6 |
| **Spacial Entropy** | | | | |
| **Neighborhoods** ($E^n$) | 760.2 (88.4) | 764.6 (88.9) | 4.5 | 0.6 |
| **Cities** ($E^{Cities}$) | 524.2 (77.9) | 517.4 (76.9) | -6.8 | -1.3 |

**Notes:** The data source for this and the following tables is the Common Core of Data (CCD) compiled by the National Center for Educational Statistics (NCES). See footnote 2 in the text for the definitions of the race categories. Cities refers to Core Based Statistical Areas and neighborhoods refers to school districts. The population refers to the population within the common set of 834 cities and 5,429 neighborhoods, accounting for 93.2 and 70.6% of the urban student population enrolled in public schools in 1989 and 2005, respectively. **% max** reports the value of the entropy as a percentage of its maximum value.

**Table 2. Changes in Segregation According to the Mutual Information Index. 1989-2005**

**Panel A: Changes in Indices**

| | | Details by Racial Groups | | | | | All | |
|---|---|---|---|---|---|---|---|---|
| | | **Whites** | **Blacks** | **Hispanics** | **Asians** | **Native Americans** | | |
| **Total** | $\Delta M_g =$ | 8.9 | -8.5 | -32.6 | -11.0 | -20.3 | $\Delta M =$ | 4.2 |
| **Between-Cities** | $\Delta BC_g =$ | 4.5 | -1.2 | -26.8 | -15.2 | -9.0 | $\Delta BC =$ | 2.5 |
| **Within-Cities** | $\Delta WC_g =$ | 4.4 | -7.3 | -5.8 | 4.2 | -11.3 | $\Delta WC =$ | 1.7 |

**Panel B: Changes in Contributions by Racial Group**

| | | **Whites** | **Blacks** | **Hispanics** | **Asians** | **Native Americans** |
|---|---|---|---|---|---|---|
| **Total** | $\Delta\left(p_g M_g\right) =$ | 3.0 | -0.3 | 0.9 | 0.5 | 0.1 |
| **Between-Cities** | $\Delta\left(p_g BC_g\right) =$ | 1.7 | 0.3 | 0.3 | 0.0 | 0.2 |
| **Within-Cities** | $\Delta\left(p_g WC_g\right) =$ | 1.3 | -0.7 | 0.6 | 0.5 | 0.0 |

**Notes:** See Section II.4 in the main text for the definition of the indeces $M_g$ and $M$. See Section III.2 in the main text for the definitions of $BC_g, WC_g, BC$, and $WC$, and the definitions of the racial contributions, $p_g M_g, p_g BC_g, p_g WC_g$.

**Table 3. Changes in Segregation for Invariant Indices. 1989-2005**

**Panel A: Changes in Indices**

| | **I1 Indices (Evenness)** | | **I2 Indices (Representativeness)** | |
|---|---|---|---|---|
| | **Change** | **% Change** | **Change** | **% Change** |
| **Generalized Atkinson** | $\Delta\overline{A}$ | $\Delta\overline{A}/\overline{A}x100$ | $\Delta\overline{A}^*$ | $\Delta\overline{A}^* / \overline{A}^* x100$ |
| **Total** | -7.6 | -16.6 | 8.4 | 24.9 |
| **Between Cities** | -5.6 | -21.2 | 3.8 | 17.7 |
| **Within Cities** | -1.9 | -10.2 | 4.6 | 37.2 |
| **Generalized Dissimilarity** | $\Delta\overline{D}$ | $\Delta\overline{D}/\overline{D}x100$ | $\Delta\overline{D}^*$ | $\Delta\overline{D}^* / \overline{D}^* x100$ |
| **Total** | -2.9 | -4.9 | 7.8 | 26.9 |
| **Between Cities** | -2.4 | -5.2 | 5.9 | 21.4 |
| **Within Cities** | -0.4 | -3.8 | 2.0 | 119.2 |

**Panel B: Racial Contributions to Changes in *I2* Indices (Representativeness)**

| | **Whites** | **Blacks** | **Hispanics** | **Asians** | **Native Americans** |
|---|---|---|---|---|---|
| **Generalized Atkinson** | | | | | |
| **Total** | 18.2 | -2.3 | -6.8 | -0.7 | 0.0 |
| **Between Cities** | 15.1 | -1.9 | -8.3 | -1.1 | -0.1 |
| **Within Cities** | 3.1 | -0.4 | 1.5 | 0.4 | 0.1 |
| **Generalized Dissimilarity** | | | | | |
| **Total** | 2.9 | 1.0 | 3.3 | 0.5 | 0.2 |
| **Between Cities** | 1.7 | 0.8 | 2.8 | 0.4 | 0.2 |
| **Within Cities** | 1.2 | 0.2 | 0.5 | 0.1 | -0.1 |

**Notes:** See Section II.2 in the main text for the definition of the indeces. For each index, **Change** is the change in the year-specific averages across the resulting values obtained when using as weights the sample marginal distributions in both 1989 and 2005. Racial contributions reports the racial contributions to each **I2** index, i.e. $1/G - \prod_{n=1}^{N}\left(p_{g|n}\right)^{p_n^*}$ for the Reciprocal Atkinson and $\frac{1}{2(N-1)}\sum_{n=1}^{N}\left|p_{g|n} - \sum_{n'=1}^{N} p^*_{n'} p_{g|n'}\right|$ for the Reciprocal Dissimilarity. The Within-Cities Gereralized Dissimilarity is defined as the difference between the Total and the Between-Cities indices.

**Table 4. Net Segregation Changes from the Mutual Information Index. 1989-2005**

| | PANEL A: Decompositions of Gross Changes | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Change | | I1 Decomposition (Evenness) | | | I2 Decomposition (Representativeness) | | |
| | $\Delta M$ | (in %) | $\Delta \overline{P}_{n\mid g}$ | $\Delta E^{n}$ | $\Delta \overline{P}_{g}$ | $\Delta \overline{P}_{g\mid n}$ | $\Delta E_{g}$ | $\Delta \overline{P}_{n}$ |
| **Total** | 4.2 | 11.6 | -22.9 | 4.5 | 22.6 | -13.8 | 17.8 | 0.2 |
| **Between-Cities** | 2.5 | 11.7 | -7.7 | -6.8 | 17.0 | -14.5 | 17.8 | -0.8 |
| **Within-Cities** | 1.7 | 11.5 | -15.2 | 11.3 | 5.6 | 0.7 | 0.0 | 1.0 |

| | PANEL B: Changes in Evenness by Racial Group | | | | |
|---|---|---|---|---|---|
| | Whites | Blacks | Hispanics | Asians | Native Americans |
| **Total** | -8.5 | -34.8 | -53.0 | -37.0 | -29.9 |
| **Between-Cities** | 0.4 | -7.8 | -31.5 | -11.3 | -13.7 |
| **Within-Cities** | -8.9 | -27.1 | -21.5 | -25.8 | -16.1 |

**Notes:** See equations (5) and (6) in the main text for the definition s of the two decompositions. $\Delta \overline{P}_{n\mid g}$ denotes the average of the terms $\Delta P_{n\mid g}(\boldsymbol{p})$ in decomposition (5) when taking as $\boldsymbol{p}$ weights $P_g(1989)$ and $P_g(2005)$, while $\Delta \overline{P}_{g}$ denotes the corresponding average of the terms $\Delta P_g(\boldsymbol{p})$. $\Delta \overline{P}_{g\mid n}$ denotes the average of the terms $\Delta P_{g\mid n}(\boldsymbol{p}^*)$ in decomposition (6) when taking as $\boldsymbol{p}^*$ weights $P_n(1989)$ and $P_n(2005)$, while $\Delta \overline{P}_{n}$ denotes the corresponding average of the terms $\Delta P_n(\boldsymbol{p})$. See equation (8) in the main text for the definition of the changes in evenness by racial group.