GMM: Examples

Econometrics I

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Outline

Motivation

Hansen's contributions

- GMM was developed by Lars Peter Hansen in 1982 as a generalization of the method of moments
- one important advantage of GMM is that it requires less restrictive assumptions than those needed for maximum likelihood estimation
- this is specially interesting in the context of semiparametric models, where the parameter of interest is finite-dimensional, whereas the full shape of the distribution function of the data may not be known
- GMM is currently applied in numerous fields, including labor economics, international finance, finance and macroeconomics
- today we are going to review three applications in finance, macroeconomics, and trade

Non-linear Rational Expectations

Hansen-Singleton (1982)

- Hansen and Singleton (1982) explore the econometric implications of dynamic rational expectations in the context of multi-period asset pricing
- If economic agents solve quadratic optimization problems subject to linear constraints, it is possible to fully characterize the equilibrium dynamic paths
- in more general settings, closed-form solutions for the equilibrium time paths of the variables of interest cannot be obtained
- Hansen-Singleton (1982) avoid this problem by the use of GMM

The Representative Consumer Plan

- the objective of the consumer is to maximize

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_t) \right]$$

  where C_t is consumption at t and β is the discount factor
- in every time period t, the consumer has the choice of investing in 2 assets: Q_t^1 and Q_t^2
- a feasible consumer plan must satisfy

$$C_t + P_t^1 Q_t^1 + P_t^2 Q_t^2 \leq R_t^1 Q_{t-1}^1 + R_t^2 Q_{t-1}^2 + W_t$$

- note that future prices P_τ^j and payoffs R_τ^j are random variables at $t = 0$

First Order Conditions

- first-order necessary conditions for the consumer's problem

$$P_t^1 U'(C_t) = \beta E_t \left[R_{t+1}^1 U'(C_{t+1}) \right]$$
$$P_t^2 U'(C_t) = \beta E_t \left[R_{t+1}^2 U'(C_{t+1}) \right]$$

- Hence

$$E\left[\beta r_t^1 \frac{U'(C_{t+1})}{U'(C_t)} | Z_t \right] = 1$$
$$E\left[\beta r_t^2 \frac{U'(C_{t+1})}{U'(C_t)} | Z_t \right] = 1$$

where $r_t^1 = \frac{R_{t+1}^1}{P_t^1}$ and $r_t^2 = \frac{R_{t+1}^2}{P_t^2}$ and Z_t is composed of lags of all variables

CRRA Utility

if

$$U(C_t) = \frac{(C_t)^{1+\alpha}}{1+\alpha}, \qquad \gamma < 1$$

(Constant Relative Risk Aversion)

- then

$$\frac{U'(C_{t+1})}{U'(C_t)} = \left(\frac{C_{t+1}}{C_t}\right)^{\alpha}$$

- and

$$E\left[\beta r_t^1 \left(\frac{C_{t+1}}{C_t}\right)^{\alpha} | Z_t\right] = 1$$

$$E\left[\beta r_t^2 \left(\frac{C_{t+1}}{C_t}\right)^{\alpha} | Z_t\right] = 1$$

GMM conditions

$$m^1(b,a) = E\left[Z_t\left(br_t^1\left(\frac{C_{t+1}}{C_t}\right)^a - 1\right)\right]$$
$$m^2(b,a) = E\left[Z_t\left(br_t^2\left(\frac{C_{t+1}}{C_t}\right)^a - 1\right)\right]$$

- GMM provides consistent estimates of the time discount β and the relative risk aversion coefficient α
- Since we have over-identifying restrictions, GMM also provides a test for the adequacy of the rational expectations model

The Permanent Income Hypothesis

Motivation

- The permanent income hypothesis predicts that there is a structural and positive relation between current consumption and permanent income
- we let y_i^* represent permanent income
- we observe c_i, current consumption, and y_i, current income
- if we regress c_i on y_i the OLS estimate will not be consistent if $(y_i^* - y_i)$ is correlated with y_i

Structural Equations

- we are interested in the model

$$c_i = \beta y_i^* + v_i$$

  where v_i captures variations in consumption unrelated to permanent income
- we know that

$$y_i = y_i^* + u_i$$

  where u_i represents a transitory shock in income
- to simplify things, let us assume

$$Var\left(\{u_i, v_i, y_i^*\}\right) = \begin{bmatrix} \sigma_u^2 & 0 & 0 \\ 0 & \sigma_v^2 & 0 \\ 0 & 0 & \sigma_*^2 \end{bmatrix}$$

Moment conditions

- since $y_i = y_i^* + u_i$ and $c_i = \beta y_i^* + v_i$
- then

$$E[y_i y_i] = \sigma_*^2 + \sigma_u^2$$
$$E[y_i c_i] = \beta \sigma_*^2$$
$$E[c_i c_i] = \beta \sigma_*^2 + \sigma_u^2$$

- that is, we have four parameters $(\beta, \sigma_u^2, \sigma_v^2, \sigma_*^2)$ but only three moment conditions
- the system is not identified and GMM cannot be implemented

Additional information

- Suppose now that we have additional information on, say, housing expenditure
$$h_i = \gamma y_i^* + \varepsilon_i$$
where ε_i is uncorrelated
- then
$$E[h_i y_i] = \gamma \sigma_*^2$$
$$E[h_i c_i] = \gamma \beta \sigma_*^2$$
$$E[h_i h_i] = \gamma^2 \sigma_*^2 + \sigma_\varepsilon^2$$
- we now have six parameters $\left(\beta, \sigma_u^2, \sigma_v^2, \sigma_*^2, \gamma, \sigma_\varepsilon^2\right)$ and six moment conditions
- the system is just-identified and $\beta = \frac{E[h_i c_i]}{E[h_i y_i]}$

The Log of Gravity

The Traditional Gravity Equation

- many trade theories predict a special pattern in bilateral trade flows between countries that some economists call "gravity laws" because—so they say—these laws are "analogous to Newton's law of universal gravitation"
- any way, regardless of how they call them, the relations are of the form

$$T_{ij} = \alpha_0 Y_i^{\alpha} Y_j^{\beta} D_{ij}^{\gamma}$$

where

T_{ij} : trade flow from country i to country j

Y_i : GDP of country i

Y_j : GDP of country j

D_{ij} : a broad defintion of distance between the two countries

The econometric model

- the gravity laws in economics however do not hold exactly, so empirical economists include a random term

$$T_{ij} = \alpha_0 Y_i^{\alpha_1} Y_j^{\alpha_2} D_{ij}^{\alpha_3} \eta_{ij}$$

- then, assume that

$$E\left[log\left(\eta_{ij}\right)|log\left(Y_i\right), log\left(Y_j\right), log\left(D_{ij}\right)\right] = 0$$

- one can then estimate the equation in logs

The equation in logs

- the equation in logs takes the form

$$\begin{aligned} log\left(T_{ij}\right) = \beta_0 + \alpha_1 log\left(Y_i\right) + \alpha_2 log\left(Y_j\right) \\ + \alpha_3 log\left(D_{ij}\right) + log\left(\eta_{ij}\right) \end{aligned}$$

- there are, however, two problems
- first, many countries do not trade with each other, so $T_{ij} = 0$ and we cannot take the logarithm
  - this can be overcome by nonlinear methods
- second, if the model in levels is heteroskedastic, the assumption

$$E\left[log\left(\eta_{ij}\right) | log\left(Y_i\right), log\left(Y_j\right), log\left(D_{ij}\right)\right] = 0$$

is **NOT** true

Moment Conditions

- Note that

$$T_{ij} = exp\left(\beta_0 + \alpha_1 log\left(Y_i\right) + \alpha_2 log\left(Y_j\right) + \alpha_3 log\left(D_{ij}\right)\right)\eta_{ij}$$

- or, equivalently,

$$T_{ij} = exp\left(x_i\beta\right)\eta_{ij}$$

- such that

$$E\left[T_{ij} - exp\left(x_i\beta\right)|x_i\right] = 0$$

- if we impose these population conditions in the sample we obtain an MM estimator that is equivalent to assuming that the conditional variance is proportional to the conditional mean

Non-linear IV Estimator

Pseudo PML estimation

- the MM estimator is defined using the moment conditions

$$m(b) = \sum_{ij} (T_{ij} - exp(x_{ij}\beta))\, x_{ij}$$

- this estimator is equivalent to assuming that the conditional variance is proportional to the conditional mean
- it is sometimes referred to as the Poisson Maximum Likelihood estimator, although strictly speaking, one does not need to assume the Poisson distribution
- to conduct inference, one has to take into account that the model is heteroskedastic

GMM

- one way to improve the estimation is by adding new exogenous variables so that

$$m(b) = \sum_{ij} (T_{ij} - exp(x_{ij}\beta))z_{ij}$$

- GMM in this context provides consistent estimates for non-linear IV estimation

Summary

- one important advantage of GMM is that it requires less restrictive assumptions than those needed for maximum likelihood estimation
- this is specially interesting in the context of semiparametric models, where the parameter of interest is finite-dimensional, whereas the full shape of the distribution function of the data may not be known
- we have reviewed several examples from the fields of finance, macroeconomics, and trade