## Problem Set 4:

## $\operatorname{GMM}$

1. Suppose that you want to study why some women choose to participate in the labor market while other only do homework. You want to test the hypothesis that the level of education affects the probability of participation. To do so, you consider the following parametric nonlinear model:

$$Prob\left(work = 1 | educ\right) = \frac{1}{1 + exp\left(-\left(\beta_0 + \beta_1 educ\right)\right)}$$

where work is a binary variable that takes value 1 if the woman participates in the labor market and 0 otherwise, and educ represents the years of education and can take values between 5 and 15.

- (a) Suppose that you have consistent and asymptotically normal estimates for  $\beta_0$  and  $\beta_1$ . Explain in detail how you would investigate whether education affects the probability of participation.
- (b) Using the fact that work is a binary variable, prove that the expectation of work for a given value of education  $educ^0$  is

$$E\left(work|educ^{0}\right) = \frac{1}{1 + exp\left(-\left(\beta_{0} + \beta_{1}educ^{0}\right)\right)}$$

- (c) Explain why the estimator computed by Stata command line
  .gmm (work-{bx:educ}-{b0}),instruments(educ)
  is not consistent.
- (d) Based on the result in (b), propose a consistent and asymptotically normal estimator of  $\beta_1$  and write down the Stata command that you would use. Explain your answer.
- (e) Suppose that education and work are both influenced directly by the woman's innate ability, but that you have in your data the income of the woman's parents at the time that she was born, *income*, and the number of children that the woman has, *children*. Explain how you would change your modeling strategy to obtain a consistent estimate of  $\beta_1$ .
- 2. Suppose that your dataset contains monthly, seasonally adjusted, observations on US real per capita consumption of nondurables, *consumption*. This measure of consumption is paired with five stock returns: the equally-weighted average return on all stocks listed on the New York Stock Exchange (ewr), the value-weighted average of returns on the New York Stock Exchange (vwr), and equally-weighted average real returns on the stocks of three two-digit SEC industries. The industries chosen were chemicals (sec28), transportation and equipment (sec37), and other retail trade (sec50).
  - (a) Explain in detail how you would estimate Hansen and Singleton (1982) model with Stata command gmm.
  - (b) Which command would you use to test for rational expectations? Explain.