Online Appendix

Unobserved Heterogeneity, Exit Rates and Re-employment Wages

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1 Alternative Cost Schemes

Consider the alternative economy in which firms incur cost c per machine quality unit when posting the vacancy, and $c_0 = 0$. In addition to the assumptions made in the paper, let us impose that $\lim_{k\to 0} F_k(k,s)k = 0$. This property is satisfied for example by a Cobb-Douglas function. The value functions in this alternative setting are:

$$rU_s = \max_k \nu(Q(k))\alpha\mathcal{S}_s(k) \tag{1}$$

$$(r+\lambda)\mathcal{S}_s(k) = F(k,s) - rU_s \tag{2}$$

$$rV = -ck + \eta(Q(k))\sum \gamma(k,s)(1-\alpha)\mathcal{S}_s(k) \tag{3}$$

The definition of the symmetric steady-state equilibrium does not change. We first claim that Lemma 3.1 also holds in this economy.[1] That is, if skilled workers are willing to apply to submarket k, then their surplus outweighs the surplus produced by the unskilled, $\mathcal{S}_h(k) > \mathcal{S}_\ell(k)$. The following proposition states that there is no equilibrium in which both types of workers apply to the same submarket.

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[1]The proof does not change from the one for the benchmark economy analyzed in the paper. Therefore, it is omitted here.

Proposition 1.1 *There is no equilibrium in which workers of different types search for jobs in the same submarket. That is, for all active submarket $k \in K$, either $\gamma(k,\ell) = 1$ and $\gamma(k,h) = 0$ or vice versa.*

Proof The proof is analogous to the one in the paper for Proposition 3.2, and it is by contradiction. Suppose there is an equilibrium in which a submarket k is active and $\gamma(k,\ell), \gamma(k,h) > 0$. Therefore, $\mathcal{S}_\ell(k) < \mathcal{S}_h(k)$. Since both types of workers enter submarket k, we have

$$rU_s = \alpha\nu(Q(k))\mathcal{S}_s(k), \text{ for } s \in \{\ell, h\} \tag{4}$$

Consider now the alternative submarket k', with $k' \equiv k+\epsilon$ and ϵ arbitrarily small (not necessarily positive). By differentiating expression (4) with respect to k, we obtain

$$\frac{\partial q_s}{\partial k} = \frac{\nu(Q(k))}{-\nu'(Q(k))}\frac{F_k(k,s)}{\mathcal{S}_s(k)(r+\lambda)} > 0, \text{ for } s \in \{\ell, k\} \tag{5}$$

where $\frac{\partial q_s}{\partial k}$ measures how much the queue length would change to keep the worker of type s indifferent between submarkets k and k'. Recall that ν is a decreasing function. The above expression takes into account that

$$(r+\lambda)\frac{d\mathcal{S}_s(k)}{dk} = F_k(k,s) \tag{6}$$

What determines the difference of the queue change across skills is the difference in the elasticity of the surplus function with respect to machine quality, $\frac{d(\ln(\mathcal{S}_s(k)))}{dk}$. We now analyze two possible cases depending on whether this difference is positive or not.

<u>Case 1:</u> Suppose that $\frac{d(\ln(\mathcal{S}_h(k)))}{dk} > \frac{d(\ln(\mathcal{S}_\ell(k)))}{dk}$. Then, expression (5) implies that $\frac{\partial q_h}{\partial k} > \frac{\partial q_\ell}{\partial k} > 0$. Therefore, firms deviating to k' arbitrarily close to, but higher than k would only attract skilled workers. Since $\mathcal{S}_\ell(k) < \mathcal{S}_h(k)$, the deviating firms would see a discrete jump in profits.

<u>Case 2:</u> Otherwise, then $\frac{\partial q_\ell}{\partial k} > \frac{\partial q_h}{\partial k} > 0$. In this case, firms deviating to k' arbitrarily close to, but lower than k would only attract skilled workers. Since $\mathcal{S}_\ell(k) < \mathcal{S}_h(k)$, the deviating firms would see a discrete jump in profits.

In both cases, we reach a contradiction; hence, submarket k cannot be active in equilibrium.‖

As a result, if there exists a symmetric steady-state equilibrium, it must be separating. We now argue about how a separating equilibrium would look like in this economy.

Regardless of whether exit rates from unemployment are higher for the skilled or not, and conditional on existence of a separating equilibrium, the variation of the composition of the unemployed over time ensures that the average exit rate falls with unemployment duration. Therefore, we conclude that the mechanism we propose in the paper generates declining exit rates from unemployment independently of the assumption on the cost scheme. Now, for this decline to be accompanied by a fall in average wages over unemployment duration, the exit rates and wages must be higher for the same worker type.

Remark: These results would not change if firms fully financed the machine quality costs at the hiring time instead.

A separating equilibrium. In a separating allocation, firms are indifferent to place their vacancies in either submarket. Using expressions (1) and (2), the equilibrium zero-profit condition implies

$$\eta(q_\ell)\frac{\mathcal{S}_\ell(k_\ell)}{k_\ell} = \eta(q_h)\frac{\mathcal{S}_h(k_h)}{k_h} \Leftrightarrow \frac{\eta(q_\ell)}{r+\lambda+\alpha\nu(q_\ell)}\frac{F(k_\ell,\ell)}{k_\ell} = \frac{\eta(q_h)}{r+\lambda+\alpha\nu(q_h)}\frac{F(k_h,h)}{k_h} \tag{7}$$

Factor $\frac{\eta(q)}{r+\lambda+\alpha\nu(q)}$ is increasing in q. Fraction $\frac{F(k,s)}{k}$ is increasing in s, and decreasing in k because of the concavity of function F and the limit assumption made above.

Given that workers obtain a share of the surplus, which is increasing in the machine quality, conditional on being employed, all types are better off at the highest machine quality offered in equilibrium. It follows that there cannot be a separating equilibrium in which the machine quality and the exit rate for type i workers are both lower than for type $-i$ workers because then the former would be better off applying to type $-i$ jobs. There are two possible cases left:

Case 1: $q_h < q_\ell$ and $k_h < k_\ell$. Type h jobs are associated with a lower machine quality and a higher job-finding rate. According to expression (7), this requires $\frac{F(k_\ell,\ell)}{k_\ell} < \frac{F(k_h,h)}{k_h}$. This inequality always holds because this fraction is a decreasing function in machine quality together with the assumption of capital-skill complementarity.

Case 2: $q_\ell < q_h$ and $k_\ell < k_h$. Type h jobs are associated with a higher machine quality and a lower job-finding rate. According to expression (7), this requires $\frac{F(k_h,h)}{k_h} < \frac{F(k_\ell,\ell)}{k_\ell}$. This will hold when capital-skill complementarities and the skill difference are not sufficiently big.

Solving out the counterparts of problems (P_h) and (P_ℓ) for this setting does not help to theoretically discard either of these two cases. Notice that, in the first case, wages fall with duration if capital-skill complementarities and/or skill gap were big enough. In contrast, wages increase with duration in the second case.

1.1 Example

We now consider the particular case of a Cobb-Douglas production technology: $F(k, a) = k^\sigma a^{1-\sigma}$, where $\sigma \in (0, 1)$ and $a \in \{a_\ell, a_h\}$. We start looking at the economy with perfect information about workers types and contracts can be type contingent. To determine the equilibrium allocation for type s workers, we solve the following problem:

$$\max_{k,q} \quad \nu(q)\alpha\mathcal{S}_s(k)$$
$$ck \leq \eta(q)(1-\alpha)\mathcal{S}_s(k)$$

The FOC -for interior solutions- with respect to q and k are, respectively:

$$\begin{aligned} -\nu'(q)\alpha &= \xi\eta'(q)(1-\alpha) \\ \big(\nu(q)\alpha + \xi\eta(q)(1-\alpha)\big)\frac{F_k(k,s)}{r+\lambda} &= \xi c \end{aligned}$$

where ξ is the Lagrangian multiplier. Isolating ξ from the first equation and replacing it into the second one, we obtain

$$\frac{kF_k(k,s)}{F(k,s)} = \zeta(q)\frac{r+\lambda}{r+\lambda+\nu(q)\alpha} \Leftrightarrow \sigma = \zeta(q)\frac{r+\lambda}{r+\lambda+\nu(q)\alpha}$$

Since the right hand side is increasing in q, under standard conditions, there exists a unique solution to this equation. Notice that the solution of the FOC does not depend on the worker's type. The equilibrium k is determined by the zero-profit condition

$$ck = \eta(q)(1-\alpha)\frac{F(k,s)}{r+\lambda+\nu(q)\alpha}$$

Recall that $\frac{F(k,s)}{k}$ is decreasing in q. Therefore, in equilibrium with perfect information and type-contingent contracts, skilled workers obtain higher wages as apply to jobs with higher machine qualities, but face the same exit rates from unemployment as their unskilled counterparts.

Let us now turn to the setting with unobserved types. In this setting, the allocation above cannot be an equilibrium because unskilled workers are better off applying to skilled jobs. The allocation in the skilled submarket solves the following problem:

$$\max_{k,q} \quad \nu(q)\alpha\mathcal{S}_h(k)$$
$$ck \leq \eta(q)(1-\alpha)\mathcal{S}_h(k)$$
$$\nu(q)\alpha\mathcal{S}_\ell(k) \leq U_\ell$$

The two constraints hold with equality.[2] Therefore, the following two conditions determine the equilibrium pair (q_h, k_h)

$$\begin{aligned} \nu(q_h)\mathcal{S}_\ell(k_h) &= \nu(q_\ell)\mathcal{S}_\ell(k_\ell) \\ \frac{\eta(q_h)}{r+\lambda+\alpha\nu(q_h)}\frac{F(k_h,h)}{k_h} &= \frac{c}{1-\alpha} \end{aligned}$$

The first equation ensures that unskilled workers are indifferent between applying to skilled and unskilled jobs, whereas the second equation is the zero-profit condition. It appears from the first condition that exit rates are no longer equal to one another as it would follow that both types apply to the same contract. However, without making additional assumptions, we cannot conclude what is the order of queue lengths, machine qualities and wages across types.

Figure 1 shows exit rates from unemployment, re-employment wages, and machine qualities for a range of capital elasticities using the same parameter values and functional forms that we used for the example in the paper. Exit rates are always higher for skilled workers, which is compensated by smaller capital levels for all elasticity values. A lower machine quality does not necessarily imply a lower wage because of the skill difference. Indeed, wages are higher for skilled workers for almost all elasticities below 0.5.

2 Alternative Contracting Spaces

We now argue that the equilibrium allocation is not constrained efficient if there is over-investment by analyzing a wider contracting environment. As the following analysis shows, constrained effi-

[2] Let ξ_1 and ξ_2 denote the Lagrangian multipliers of the two constraints. After some manipulations, the FOC become

$$\begin{aligned} \xi_1 q &= \frac{\zeta(q)}{1-\zeta(q)}\frac{\alpha}{1-\alpha}\left(1-\xi_2\frac{\mathcal{S}_\ell(k)}{\mathcal{S}_h(k)}\right) \\ \frac{F_k(k,h)}{r+\lambda} &= \zeta(q)\left(\frac{\mathcal{S}_h(k)}{k}-\xi_2\frac{\mathcal{S}_\ell(k)}{k}\right)+\xi_2\mathcal{S}_\ell(k)\left((1-\zeta(q))\frac{\frac{\partial\mathcal{S}_\ell(k)}{\partial k}}{\mathcal{S}_\ell(k)}+\zeta(q)\frac{\frac{\partial\mathcal{S}_h(k)}{\partial k}}{\mathcal{S}_h(k)}\right) \end{aligned}$$

ciency also imposes conditions on the wage setting.

The contracting space analyzed in the benchmark economy is far from being complete as firms can only commit to a machine quality k. In the competitive search literature, firms attract workers by posting a wage contract. As mentioned in the Introduction, in (Michelacci and Suarez 2006), firms are allowed to commit to either single-wage offers or surplus-bargaining jobs, with the labor share being an exogenous parameter. In this section, we briefly discuss a wider contracting environment in which contracts may stipulate any combination of a wage and a labor share of the surplus. The details are described below in subsection 2.1.

Let $\mathcal{X}$ denote the space of contracts $x \equiv (k, w, \alpha)$. A worker matched with a machine quality k under such a contract obtains a wage w plus a share α of the joint surplus. Let $X \subset \mathcal{X}$ denote the subset of active submarkets. The following proposition characterizes the equilibrium allocation.

Proposition 2.1 *There does not exist an equilibrium with both types of workers searching for the same jobs. There does exist a separating equilibrium,* $\{G, X, Q, \Gamma, \{U_s, \mathcal{S}_s\}_s\}$*, where* $X = \{x_\ell, x_h\}$*,* $q_s \equiv Q(x_s)$*,* $\gamma(x_s, s) = 1$*,* $k_s = \overline{k}_s$*, and* $w_s + \alpha_s \mathcal{S}_s(\overline{k}_s) = (1 - \zeta(q_s))\mathcal{S}_s(\overline{k}_s)$*, for* $s \in \{\ell, h\}$*. Furthermore, the separating equilibrium is constrained efficient.*

First, as an extension of Proposition 1.1, we claim that no pooling submarket can be active in equilibrium. There exists a separating equilibrium. Firms commit to the machine quality that maximizes the surplus created by a match with a given worker type. Firms targeting skilled workers have now other instruments to discourage unskilled workers from applying. They may commit to a sufficiently high labor share and pay a negative wage if necessary. Indeed, a continuum of contracts deliver the equilibrium allocation. This result has been shown for a different environment in the personnel literature. See e.g. (Lazear 2004). In equilibrium, firms obtain a share $\zeta(q_s)$ of the surplus they generate. Similarly to the benchmark economy, the employment prospects of skilled workers are better in all dimensions. Therefore, the patterns of exit rates and re-employment wages over the unemployment duration stated in the paper carry over to this wider contracting environment.

Now, we inspect the efficiency properties of this equilibrium allocation. We find that a social planner who could observe the worker types and make type-contingent offers would determine the same allocation as we have in our market economy with adverse selection. Therefore, we conclude that the equilibrium allocation in the benchmark economy is not constrained efficient if there is over-investment. However, not having over-investment in equilibrium does not ensure the efficient job creation level as firm entry also depends on the surplus-splitting terms.

2.1 Further details.

We first solve the social planner's problem, and then analyze the market economy with the space of contracts defined in Section 2.

Consider a social planner who can perfectly observe worker types and makes type-contingent offers. Given that different workers can be assigned to different locations, it is optimal to think of a single location for each type of worker. As is standard in the search literature with risk-neutral agents, the planner's objective is to maximize the net output. The planner chooses the stream of queue lengths and machine qualities to maximize the expected discounted output net of vacancy creation costs in each location. The planner cannot directly match workers to vacancies. That is, it is also subject to the matching technology described in the model section of the paper. We can write the planner's problem in type s location as

$$\begin{aligned} \max \int_0^\infty e^{-rt}\left((1-u_t)\big(F(k_t,s)-ck_t\big)-c_0\frac{u_t}{q_t}\right)dt \\ \text{s. to} \qquad \dot{u}_t = \lambda(1-u_t)-u_t\nu(q_t) \end{aligned} \tag{8}$$

The current value Hamiltonian is

$$H = (1-u_t)\big(F(k_t,s)-ck_t\big)-c_0\frac{u_t}{q_t}+\xi_t\big(\lambda(1-u_t)-u_t\nu(q_t)\big)$$

The necessary conditions are:

$$\begin{aligned} \frac{\partial H}{\partial k} = 0 \quad &\Rightarrow \quad F_k(k_t,s)=c \\ \frac{\partial H}{\partial q} = 0 \quad &\Rightarrow \quad \frac{c_0}{q_t^2} = \xi_t\nu'(q_t) \\ \dot{\xi}_t = r\xi_t - \frac{\partial H}{\partial u} \quad &\Rightarrow \quad \dot{\xi}_t = r\xi_t + F(k_t,s)-ck_t+\frac{c_0}{q_t}+\xi_t(\lambda+\nu(q_t)) \end{aligned}$$

In the steady state, after manipulating the above equations, we obtain

$$F_k(k,s) = c \quad \Rightarrow \quad k = \overline{k}_s \tag{9}$$

$$\eta(q)\zeta(q)\frac{F(k,s)-ck}{r+\lambda+\nu(q)(1-\zeta(q))} = c_0 \tag{10}$$

There exists a unique solution of this system of equations. We refer to the efficient queue length as $\overline{q}_s$. As a result, $(\overline{k}_s,\overline{q}_s)_s$ takes part of the constrained efficient allocation in the perfect-information economy.

Now, we turn to analyze the market economy.

Proof of Propostion 2.1.

Let (P_s') denote the generalization of problem (P_s). That is,

$$\begin{array}{lll} & \sup\limits_{q,x} & \nu(q)\big(w+\alpha\mathcal{S}_\ell(x)(r+\lambda)-rU_\ell\big) \\ (P_\ell') & \text{s. to} & \eta(q)\big(F(k,\ell)-ck-w-\alpha\mathcal{S}_\ell(x)(r+\lambda)\big)\geq c_0(r+\lambda) \end{array}$$

$$\begin{array}{lll} & \sup\limits_{q,x} & \nu(q)\big(w+\alpha\mathcal{S}_h(x)(r+\lambda)-rU_h\big) \\ (P_h') & \text{s. to} & \eta(q)\big(F(k,h)-ck-w-\alpha\mathcal{S}_h(x)(r+\lambda)\big)\geq c_0(r+\lambda) \\ & & \nu(q)\big(w+\alpha\mathcal{S}_\ell(x)(r+\lambda)-rU_\ell\big)\leq rU_\ell(r+\lambda) \end{array}$$

First, we solve the problem (P_ℓ'). From the first order conditions as well as the zero-profit condition, we directly obtain

$$\begin{aligned} F_k(k,\ell) &= c \\ \eta(q)\zeta(q)\frac{F(k,\ell)-ck}{r+\lambda+\nu(q)(1-\zeta(q))} &= c_0 \\ w+\alpha\mathcal{S}_\ell(k)(r+\lambda) &= (1-\zeta(q))\big(F(k,\ell)-ck\big)+\zeta(q)rU_\ell \end{aligned}$$

Notice that the first two equations coincide with conditions (9) and (10). That is, the equilibrium allocation in the type ℓ submarket is constrained efficient. The last condition says that firms targeting unskilled workers have infinitely many ways to reward the labor inputs. One possibility is to commit to a wage offer.

We proceed analogously to solve problem (P_h'). Manipulating the necessary first order conditions, we obtain that the multiplier of the second constraint must be zero. That is, suppose that the Lagrange multiplier of the second constraint is strictly positive in the steady state, $\xi_2>0$. The first order conditions with respect to w and α lead to $\mathcal{S}_h(k)=\mathcal{S}_\ell(k)$. As the second constraint must bind, we have that $U_\ell=U_h$. This implies that $F(k,\ell)=F(k,h)$, which contradicts the assumptions made on the production technology. Then, the problem is equivalent to its type ℓ counterpart, and

$$\begin{aligned} F_k(k,h) &= c \\ \eta(q)\zeta(q)\frac{F(k,h)-ck}{r+\lambda+\nu(q)(1-\zeta(q))} &= c_0 \\ w+\alpha\mathcal{S}_h(k)(r+\lambda) &= (1-\zeta(q))\big(F(k,h)-ck\big)+\zeta(q)rU_h \end{aligned}$$

As a result, the equilibrium allocation in the skilled market is also constrained efficient. The combination of a wage w_h and a labor share α_h must satisfy the non-participation condition for the unskilled.‖

References

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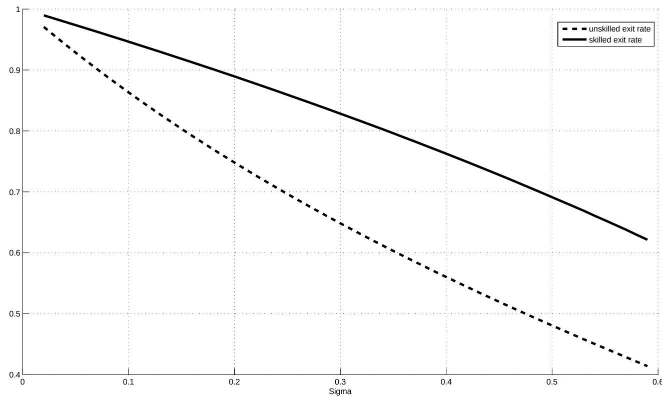
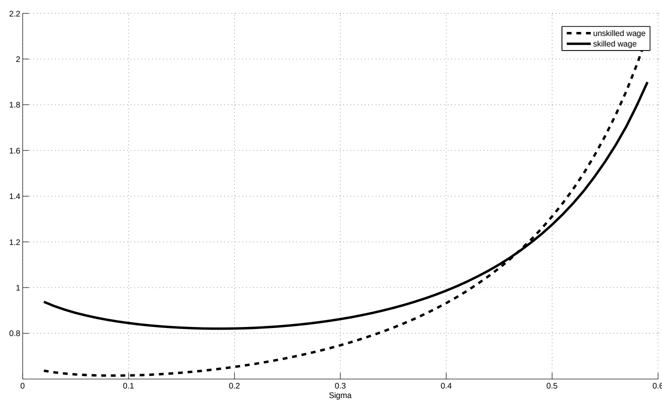
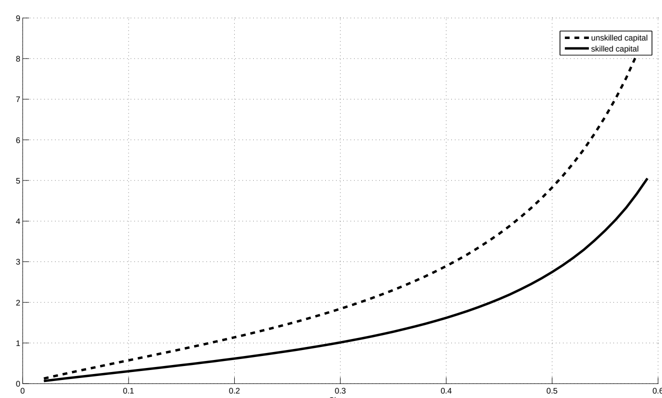
Michelacci, C., and J. Suarez (2006): "Incomplete Wage Posting," *Journal of Political Economy*, 114(6), 1098–1123.



(a) Exit rates from unemployment



(b) Re-employment wages



(c) Machine quality

Figure 1: Comparative Statics