4.4. Vertical Differentiation

The Hotelling model studies situations of horizontal differentiation since for equal prices there are always consumers that prefer A to B and others B to A.

Let’s modify the Hotelling model to incorporate quality differences between the goods (i.e. vertical differentiation)
4.4. Vertical Differentiation

Firms and consumers are located in the interval 
\([0,1]\). All consumers prefer a good close to 1.
Consumers are uniformly distributed along \([0,1]\).
2 firms A and B located in \(a\) and \(b\) respectively.
W.l.o.g \(0 \leq a \leq b \leq 1\)

Utility of consumer \(x\) is:

Two-stage game:
1st stage: Firms select their locations (i.e. their product quality)
2nd stage: Firms compete in prices simultaneously)
4.4. Vertical Differentiation

We solve the game backwards.

2nd stage: Suppose there is an indifferent consumer between A and B:

\[ U_\hat{x}(A) = a\hat{x} - p_A = b\hat{x} - p_B = U_\hat{x}(B) \]

\[ \Leftrightarrow \hat{x} = \frac{p_B - p_A}{b - a} \]

If prices are equal all consumers buy from B (i.e. the indifferent consumer is located at zero).

Consumers to the right of \( \hat{x} \) buy from B and to the left buy from A:

Therefore demand for A is \( 1 - \hat{x} \) and for B is \( \hat{x} \).
4.4. Vertical Differentiation

Note that if \( p_A > p_B \) all consumers would buy from B, (higher quality and lower prices), the indifference curves would not cross because there would not be an indifferent consumer.

\[ U_A^*(B) \]

\[ U_A^*(A) \]

\[ -p_B \]

\[ -p_A \]

4.4. Vertical Differentiation

Let’s suppose \( c = 0 \), the problem of firm A is:

Maximize \( \pi_A(a, b, p_A, p_B) = p_A \hat{x} = p_A \left( \frac{p_B - p_A}{b - a} \right) \)

FOC: \( \frac{\partial \pi_A}{\partial p_A} = 0 \iff \frac{p_B - p_A}{b - a} - \frac{1}{b - a} p_A = 0 \)

\( \iff \frac{p_B}{b - a} - \frac{2}{b - a} p_A = 0 \iff p_A = \frac{p_B}{2} \) for \( b \neq a \)

Firm A’s reaction function
El problema de la empresa B es:

\[ \text{Max } \pi_B(a, b, p_A, p_B) = p_B(1 - \hat{x}) = p_B \left(1 - \frac{p_B - p_A}{b - a}\right) \]

FOC: \[ \frac{\partial \pi_B}{\partial p_B} = 0 \Leftrightarrow 1 - \frac{p_B - p_A}{b - a} - \frac{1}{b - a} p_B = 0 \]

\[ \Leftrightarrow 1 - \frac{2p_B}{b - a} + \frac{1}{b - a} p_A = 0 \Leftrightarrow \frac{2p_B}{b - a} = \frac{b - a + p_A}{b - a} \]

\[ \Leftrightarrow p_B = \frac{b - a + p_A}{2} \text{ for } b \neq a \]

Firm B’s reaction function

The equilibrium is the solution of the system:

\[
\begin{cases}
    p_A = \frac{p_B}{2} \\
    p_B = \frac{b - a + p_A}{2} = \frac{b - a + \frac{p_B}{2}}{2} \\
    \Leftrightarrow \frac{3}{4} p_B = \frac{b - a}{2} \Leftrightarrow p_B = \frac{2(b - a)}{3} > 0 \text{ and } p_A = \frac{b - a}{3} > 0 = MC
\end{cases}
\]

The firm with the highest quality charges a higher price but both firms charge above marginal cost. The higher is the difference in quality (i.e. the higher is the distance (b-a)) the higher are both prices.
4.4. Vertical Differentiation

Demands in the 2nd stage are:

\[
\begin{align*}
D_A(a, b) &= \hat{x} = \left( \frac{p_B(a, b) - p_A(a, b)}{b - a} \right) = \frac{1}{3} \\
D_B(a, b) &= (1 - \hat{x}) = \left( 1 - \frac{p_B(a, b) - p_A(a, b)}{b - a} \right) = \frac{2}{3}
\end{align*}
\]

Profits are:

\[
\begin{align*}
\pi_A(a, b) &= p_A \hat{x} = \frac{b - a}{3} \left( \frac{p_B - p_A}{b - a} \right) = \frac{1}{3} (p_B - p_A) \\
\pi_B(a, b) &= p_B (1 - \hat{x}) = \frac{2(b - a)}{3} \left( 1 - \frac{p_B - p_A}{b - a} \right) \\
\pi_A(a, b) &= \frac{1}{3} \left( \frac{2(b - a)}{3} - \frac{b - a}{3} \right) = \frac{b - a}{9} \\
\pi_B(a, b) &= \frac{2}{3} \left( (b - a) - p_B + p_A \right) = \frac{4(b - a)}{9} > \pi_A(a, b)
\end{align*}
\]
4.4. Vertical Differentiation

1st stage:

\[
\begin{align*}
\text{Max } \pi_A(a, b) &= \frac{b-a}{9} \\
\text{FOC: } \frac{\partial \pi_A}{\partial a} &= -\frac{1}{9} < 0 \Rightarrow a^* = 0 \\
\text{Max } \pi_B(a, b) &= \frac{4(b-a)}{9} \\
\text{FOC: } \frac{\partial \pi_B}{\partial b} &= \frac{4}{9} > 0 \Rightarrow b^* = 1
\end{align*}
\]

**Principle of maximum differentiation.**

Intuition: With vertical differentiation, firms specialize in a given quality niche (high valuation consumers and low valuation consumers). The higher is the difference in quality the higher is the market power of each firm.

Because \( a^* = 0, b^* = 1 \),

\[
\begin{align*}
\pi_A^* &= \frac{1}{9}, \pi_B^* = \frac{4}{9} \\
p_A^* &= \frac{1}{3}, p_B^* = \frac{2}{3} \\
x^* &= \frac{1}{3}
\end{align*}
\]

**Conclusion:** Firms look for maximum differentiation from their rivals. Although qualities here have the same cost (\( c=0 \)), still firm A prefers to produce an inferior good in order to differentiate from the rival. If firms choose to locate sequentially, then the first one to enter would select \( b=1 \) where profits are higher.
4.4. Vertical Differentiation

If consumers may choose not to buy

\[ \text{Demand for } B \text{ between } 0 \text{ and } z = p_B / a \text{ would be better off if they do not buy. The indifferent consumer is the same as before.} \]

\[ D_A(p_A, p_B) = \hat{x} - z = \frac{p_B - p_A}{b - a} \]

\[ D_B(p_A, p_B) = 1 - \hat{x} = 1 - \frac{p_B - p_A}{b - a} \]