3.6. The Spence Dixit model

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3.6. The Spence Dixit model

- In many situations we have firms already established in the market that must face entrants or potential entrants in the market.
- The strategic behavior of incumbents may constitute a barrier to entry.
- We are going to use a model like Stackelberg but in capacities (instead of quantities), which allows a better understanding of two issues :

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1. Why there may be a firm that chooses first? In quantities it did not make much sense but if we think in capacities it could be that one of the firms obtained the technology first
2. Why did quantity represent commitment i.e. it could not be changed? In quantities this does not make much sense but in capacities, it makes all the sense because capacities are sunk.

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In this model, firms compete in quantities in the short-run and in capacities in the long-run.

Game:

Stage 1: Firm 1 (the incumbent) chooses capacity level K_1 at a cost c_0K_1; Firm 2 (the potential entrant) observes the decision of firm 1

Stage 2: Both firms choose (q_1,q_2) simultaneously as well as their capacities (K'1,K2) where K'1≥K1 (note firm 1 may increase but not decrease its capacity)

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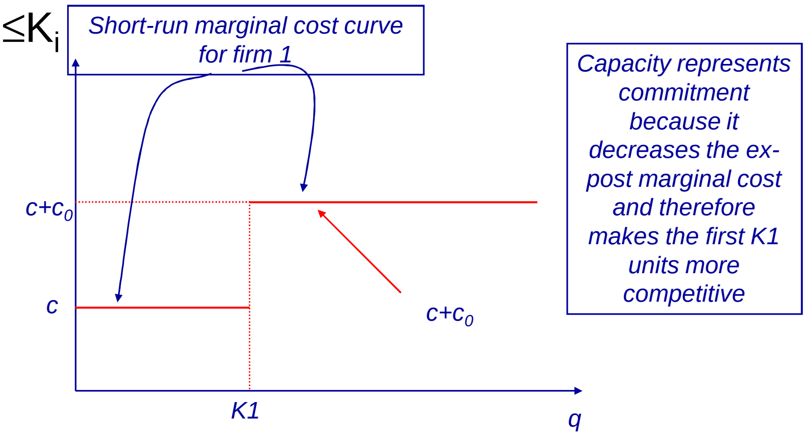
Mc=c (marginal cost of q)

$q_i \leq K_i$



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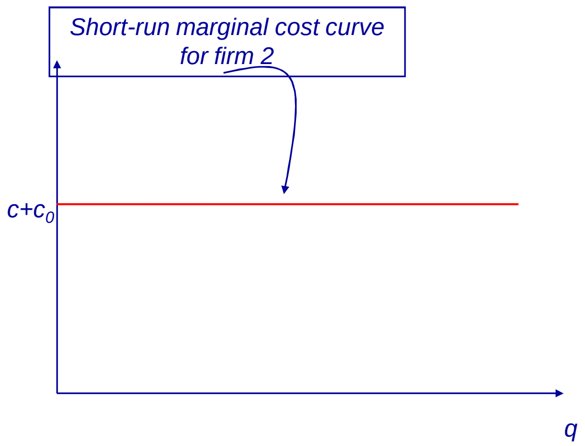

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P(Q)=a-bQ

The reaction function of firm 2 is the same as before:

$$R_2(q_1) = \frac{a - bq_1 - c_0 - c}{2b}$$

The reaction function of firm 1is:

$$\bar{R}_1(q_2) = \frac{a - bq_2 - c}{2b} > R_1(q_2) \;\text{ for } q_1 \le K_1$$
$$R_1(q_2) = \frac{a - bq_2 - c - c_0}{2b} \;\text{ for } q_1 > K_1$$

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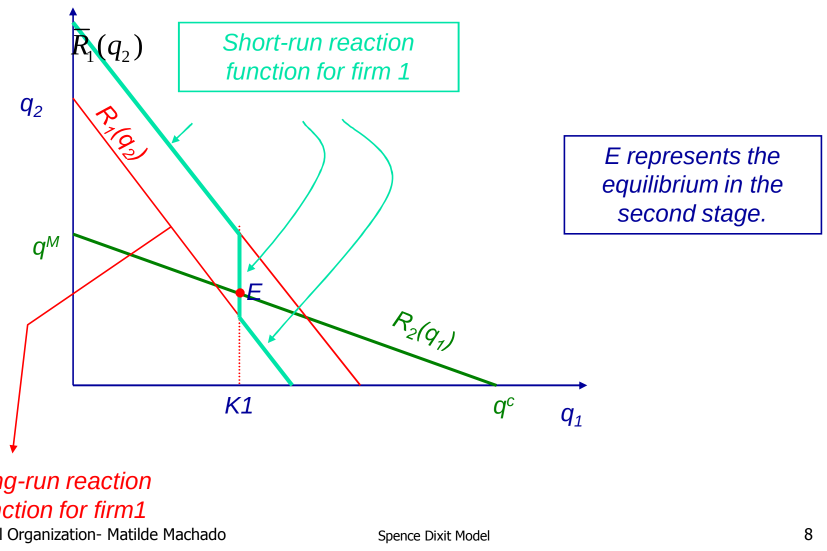

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Firm 1 chooses $K'_1=0$ and firm 2 $K_2=q_2$ or equivalently $q_1=K_1$ (note that firm 1 will always want to use all its capacity, in other words will never choose a capacity level that would remain idle) and $q_2=K_2$ (since if $q_2<K_2$ could produce the same quantity at a lower cost). Therefore, we may rewrite the inverted demand as:

$P=a-b(q_1+q_2)=a-b(K_1+K_2)$

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Assume now that b=1 and that $a-c-c_0=1$

Then firm 1's profit function in the 1st stage (knowing that $q_1=K_1$ in the second stage) would be:

$\Pi=(a-b(K_1+K_2)-c-c_0)K_1= (1-(K_1+K_2))K_1$

And the model in capacities looks just like Stackelberg in quantities.

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Note:

We call accommodated entry if the incumbent prefers to let firm 2 enter than deter its entrance in the market:

$\bar{K}_1$ is the minimum level of capital that would deter firm 2 entrance:

K_1^S is the level of K1 that firm 1 would choose if it accomodates firm 2's entry

$$\begin{cases} \Pi^2(\bar{K}_1) \leq 0 \\ \Pi^1(\bar{K}_1) < \Pi^1(K_1^S) \end{cases}$$

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Nota:

We call deterred entry when the incumbent prefers not to let firm 2 in the market:

$$\Pi^1(\bar{K}_1) > \Pi^1(K_1^S)$$

We call blocked or blockaded entry if Fixed costs in case of entry are so high that the potential entrant does not enter even if the incumbent chooses the monopoly capacity, that is even if firm 1 acts as a monopolist without potential competition:

$$\Pi^2(K_1 = K_1^M) \leq 0$$

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$$\begin{cases} \Pi^1(K_1, K_2) = K_1(1 - K_1 - K_2) \\ \Pi^2(K_1, K_2) = K_2(1 - K_1 - K_2) \end{cases}$$

Note that these reduced form profit functions have the usual characteristics: 1) each firm suffers with the capital accumulation of the rival:

$$\frac{\partial \Pi^i}{\partial K_j} < 0$$

And capacities (like quantities) are strategic substitutes:

$$\frac{\partial^2 \Pi^i}{\partial K_i \partial K_j} < 0$$

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In the second stage, firm 2:

$$\underset{K_2}{Max} \Pi^2 = K_2(1 - K_1 - K_2)$$

$$\text{FOC: } \frac{\partial \Pi^2}{\partial K_2} = 0 \Leftrightarrow (1 - K_1 - K_2) - K_2 = 0 \Leftrightarrow K_2 = R_2(K_1) = \frac{1 - K_1}{2}$$

In the first stage, firm 1:

$$\underset{K_1}{Max} \, \Pi^1 = K_1(1 - K_1 - \underbrace{\frac{1 - K_1}{2}}_{= K_2})$$

$$\text{FOC: } 1 - K_1 - \frac{1 - K_1}{2} - \frac{K_1}{2} = 0$$

$$\Leftrightarrow K_1 = \frac{1}{2} \; ; K_2 = \frac{1}{4} \; ; \Pi^1 = \frac{1}{8} \; ; \Pi^2 = \frac{1}{16}$$

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Note that if firms selected capacities simultaneously, the equilibrium would be:

$$K_1 = K_2 = \frac{1}{3}\ ; \Pi^1 = \Pi^2 = \frac{1}{9}$$

Note: It is important that the capacity levels are sunk i.e. irreversible since expost in the Stackelberg-Spence-Dixit case, firm 1 is not in its reaction function. Ex-post, firm 1 would have liked to respond to K_2=1/4 with K_1=(1-1/4)/2=3/8<1/2. The fact that capacity is sunk is a commitment for firm 2 that after observing K2, firm 1 will not decrease its capacity level.

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Note that in this example, firm 1 cannot deter firm 2's entry, since:

$$K_2 = R_2(K_1) \leq 0 \Leftrightarrow \frac{1-K_1}{2} \leq 0 \Leftrightarrow K_1 \geq 1 \text{ in which case } \Pi^1 \leq 0$$

Firm 1 only limits the scale at which firm 2 enters, that is accommodates firm 2's entry

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If, on the contrary there were increasing returns to scale, for example fixed costs as the following:

$$\Pi^2(K_1,K_2)=\begin{cases} K_2(1-K_1-K_2)-F & \text{if } K_2>0 \\ 0 & \text{if } K_2=0 \end{cases}$$

Note that in the previous example $K_1=1/2$; $K_2=1/4$; $\Pi_2=1/16$ therefore if F<1/16, firm 2 would still have profits. Firm 1 can, however, now prevent the entry of firm 2 by overinvesting in capacity, and in doing so may attain higher profits.

If F>1/16, firm 1 deters entry of firm 2 just by choosing the same capacity $K_1=1/2$ (which by chance coincides with the monopoly capacity so entry would be blocked).

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So let's assume F<1/16. The capacity level that would deter entry by firm 2 is:

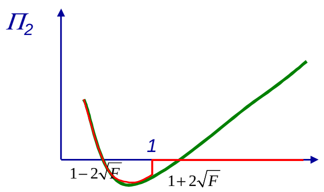
$$\Pi^2=0\Leftrightarrow K_2(1-\hat{K}_1-K_2)-F=0\Leftrightarrow \frac{1-\hat{K}_1}{2}\left(1-\hat{K}_1-\frac{1-\hat{K}_1}{2}\right)-F=0$$

$$\Leftrightarrow\left(\frac{1-\hat{K}_1}{2}\right)\left(\frac{1-\hat{K}_1}{2}\right)-F=0\Leftrightarrow \hat{K}_1^2-2\hat{K}_1+1-4F=0$$

$$\Leftrightarrow \hat{K}_1=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{2\pm\sqrt{4-4(1-4F)}}{2}=\frac{2\pm\sqrt{4-4+16F}}{2}=1\pm2\sqrt{F}$$

but for $\hat{K}_1>1\Rightarrow K_2=0\Rightarrow\Pi^2=0$

So the minimum capacity level $\hat{K}_1$ capable of deterring entry is $\hat{K}_1=1-2\sqrt{F}$



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for $\hat{K}_1$ and $K_2 = 0$:

$$\Pi^1(\hat{K}_1) = \left(1-2\sqrt{F}\right)\left(1-\left(1-2\sqrt{F}\right)\right) = 2\sqrt{F}\left(1-2\sqrt{F}\right)$$

Note that this function $\Pi^1(\hat{K}_1)$ attains a maximum at $F = \frac{1}{16}$

which is $\Pi^1(\hat{K}_1) = 2\sqrt{\frac{1}{16}}\left(1-2\sqrt{\frac{1}{16}}\right) = \frac{1}{4}$

since $F < \frac{1}{16} \Rightarrow \Pi^1(\hat{K}_1) < \frac{1}{4}$ but may be higher than the profit of accomodating entry $= \frac{1}{8}$

Proof:

$$\underset{F}{Max}\, 2\sqrt{F}\left(1-2\sqrt{F}\right) = 2\sqrt{F} - 4F = 2\left(\sqrt{F} - 2F\right)$$

FOC: $\frac{\partial}{\partial F} = 0 \Leftrightarrow \frac{1}{2}F^{-1/2} - 2 = 0 \Leftrightarrow \frac{1}{2} = 2\sqrt{F} \Leftrightarrow \sqrt{F} = \frac{1}{4} \Leftrightarrow F = \frac{1}{16}$

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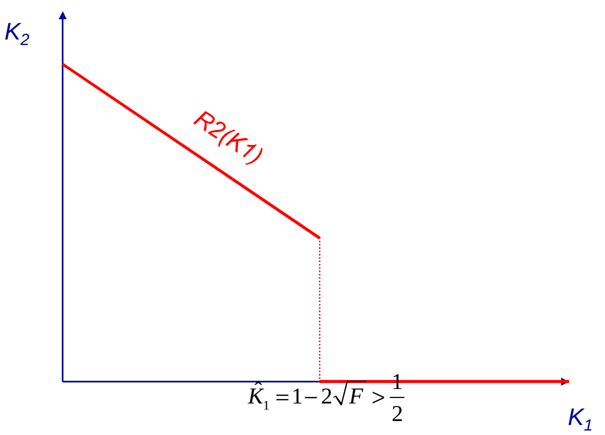
The level of K_1 that deters entry from firm 2 is:



$\hat{K}_1 = 1 - 2\sqrt{F} > \frac{1}{2}$

since $F < \frac{1}{16}$

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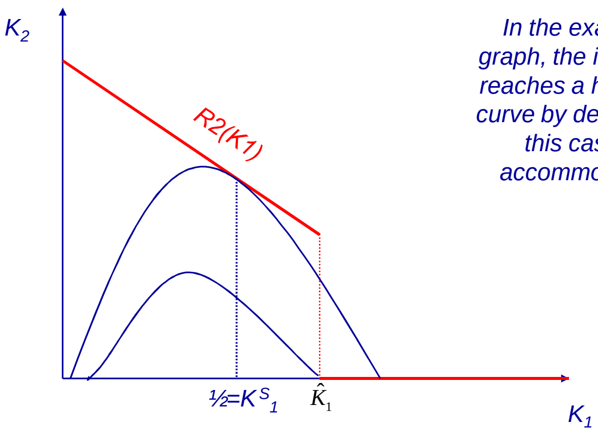
The level of K_1 that deters entry from firm 2 is:



In the example of the graph, the incumbent firm reaches a higher isoprofit curve by deterring entry in this case than by accommodating entry