3.3. Stackelberg Model

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1

3.3. Stackelberg Model

- 2-period model
- Same assumptions as the Cournot Model except that firms decide sequentially.
- In the first period the leader chooses its quantity. This decision is irreversible and cannot be changed in the second period.
- In the second period, the follower chooses its quantity after observing the quantity chosen by the leader (the quantity chosen by the follower must, therefore, be along its reaction function).

3.3. Stackelberg Model

Important Questions:

1. Is there any advantage in being the first to choose?
2. How does the Stackelberg equilibrium compare with the Cournot?

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Let's assume a linear demand P(Q)=a-bQ

$Mc_1=Mc_2=c$

In sequential games we first solve the problem in the second period and afterwards the problem in the 1st period.

<u>2nd period </u>(firm 2 chooses q_2 given what firm 1 has chosen in the 1st period q_1):

$$\boxed{\underset{q_2}{Max}\,\Pi^2 = \left(P(q_1+q_2)-c\right)q_2 = \left(a-b(q_1+q_2)-c\right)q_2}$$

given

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$$\underset{q_2}{Max}\,\Pi^2 = \left(P(q_1+q_2)-c\right)q_2 = \left(a-b(q_1+q_2)-c\right)q_2$$

$$\text{FOC: } \frac{\partial \Pi^2}{\partial q_2} = 0 \Leftrightarrow a-2bq_2-bq_1-c=0$$

$$\Leftrightarrow q_2^* = R_2(q_1) = \frac{a-bq_1-c}{2b} = \text{Cournot's reaction function}$$

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In the 1st period (firm 1 chooses q1 knowing that firm 2 will react to it in the 2nd period according to its reaction function $q_2=R_2(q_1)$):

$$\underset{q_1}{Max}\,\Pi^1 = \left(P(q_1+q_2)-c\right)q_1 = \left(a-b(q_1+R_2(q_1))-c\right)q_1$$

$$\text{FOC: } \frac{\partial \Pi^1}{\partial q_1} = 0 \Leftrightarrow a-2bq_1-bR_2(q_1)-bq_1R_2'(q_1)-c=0$$

$$\Leftrightarrow a-2bq_1-b\left[\frac{a-bq_1-c}{2b}\right]+bq_1\frac{1}{2}-c=0$$

$$\Leftrightarrow \left(\frac{a-c}{2}\right)-2bq_1+bq_1\frac{1}{2}+bq_1\frac{1}{2}=0$$

$$\Leftrightarrow \left(\frac{a-c}{2}\right)-bq_1=0 \Leftrightarrow \boxed{q_1^* = \frac{a-c}{2b}} = \frac{3}{2}q_1^N > q_1^N = \frac{a-c}{3b}$$

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Given q_1^* we solve for q_2

$$q_2^* = \frac{a-c}{2b} - \frac{1}{2}q_1^* = \frac{a-c}{2b} - \frac{1}{2}\left(\frac{a-c}{2b}\right) = \boxed{\left(\frac{a-c}{4b}\right)} = \frac{3}{4}q_2^N < q_2^N$$

Therefore $q_1^* > q_2^*$

$$q_1^* + q_2^* = \frac{a-c}{2b} + \frac{a-c}{4b} = \boxed{\frac{3(a-c)}{4b}} > \frac{2(a-c)}{3b} = Q^N$$

a>c

$$p^* = a - bQ^* = a - b\frac{3(a-c)}{4b} = \boxed{\frac{a+3c}{4}} > c$$

But $p^* < \dfrac{a+2c}{3} = p^N$

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The equilibrium profits of both firms:

$$\Pi^{1*} = \left(p^* - c\right)q_1^* = \left(\frac{a+3c}{4} - c\right)\left(\frac{a-c}{2b}\right) = \left(\frac{a-c}{4}\right)\left(\frac{a-c}{2b}\right) = \frac{(a-c)^2}{8b} > \Pi_1^N = \frac{(a-c)^2}{9b}$$

$$\Pi^{2*} = \left(p^* - c\right)q_2^* = \left(\frac{a+3c}{4} - c\right)\left(\frac{a-c}{4b}\right) = \left(\frac{a-c}{4}\right)\left(\frac{a-c}{4b}\right) = \frac{(a-c)^2}{16b} < \Pi_2^N = \frac{(a-c)^2}{9b}$$

Note: The profit of firm 1 must be at least as large as in Cournot because firm 1 could have always obtain the Cournot profits by choosing the Cournot quantity q_1^N, to which firm 2 would have replied with its Cournot quantity $q_2^N=R_2(q_1^N)$ since firm 2's reaction curve in Stackelberg is the same as in Cournot.

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Conclusion:

a) $q_1^* > q_2^*$ (the leader produces more)

b) $p^* > c$ (There will be a DWL)

c) $\Pi^{1*} > \Pi^{2*}$ (the leader has higher profits, there is an advantage of being the first to choose)

d) $Q^* > Q^N \Rightarrow p^* < p^N$

The leader has a higher profit for two reasons: 1) the leader knows that by increasing q1 the follower will reduce q2 (strategic substitutes). 2) the decision is irreversible (otherwise the leader would undo its choice and we would end up in Cournot again)

The sequential game (Stackelberg) leads to a more competitive equilibrium than the simultaneous move game (Cournot).

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Graphically: The isoprofit curves for firm 1 are derived as:

$$\Pi^1(q_1,q_2) = \left(a - b(q_1+q_2) - c\right)q_1$$

therefore:

$$\overline{\pi} = \left(a - b(q_1+q_2) - c\right)q_1 = aq_1 - bq_1^2 - bq_1q_2 - cq_1$$

$$\Leftrightarrow bq_1q_2 = \left(a-c\right)q_1 - bq_1^2 - \overline{\pi}$$

$$\Leftrightarrow q_2 = \frac{(a-c)}{b} - q_1 - \frac{\overline{\pi}}{q_1}$$

$$\frac{\partial q_2}{\partial q_1} = -1 + \frac{\overline{\pi}}{q_1^2}; \quad \frac{\partial^2 q_2}{\partial q_1^2} = -\frac{2\overline{\pi}}{q_1^3} < 0$$

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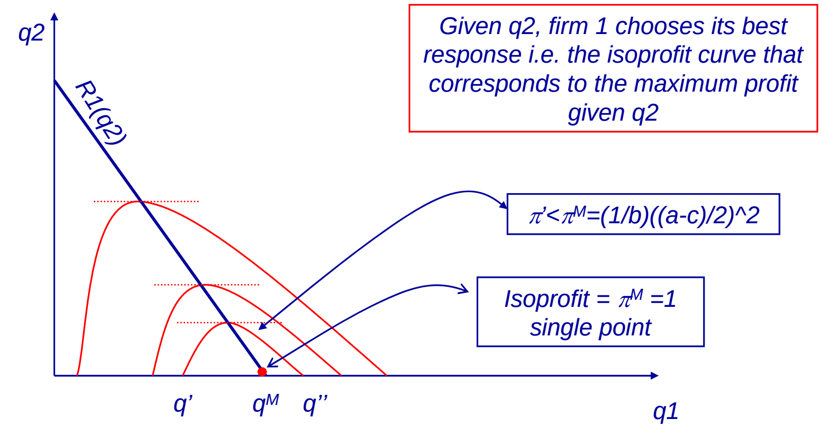
Graphically(cont):



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Graphically(cont):

The reaction function intercepts the isoprofit curves where the slope becomes zero (i.e. horizontal)

$$R_1(q_2) = \arg\max_{q_1} \Pi^1(q_1, q_2) \Leftrightarrow \Pi_1^1(R_1(q_2), q_2) = 0$$

Moreover we know that:

$$\Pi^1(q_1, q_2) = \overline{\pi} \Rightarrow \Pi_1^1 dq_1 + \Pi_2^1 dq_2 = 0 \Leftrightarrow \frac{dq_2}{dq_1} = -\frac{\Pi_1^1}{\Pi_2^1}$$

therefore at the best response $q_1 = R_1(q_2)$ the derivative is zero : $\left.\frac{dq_2}{dq_1}\right|_{q_1 = R_1(q_2)} = 0$

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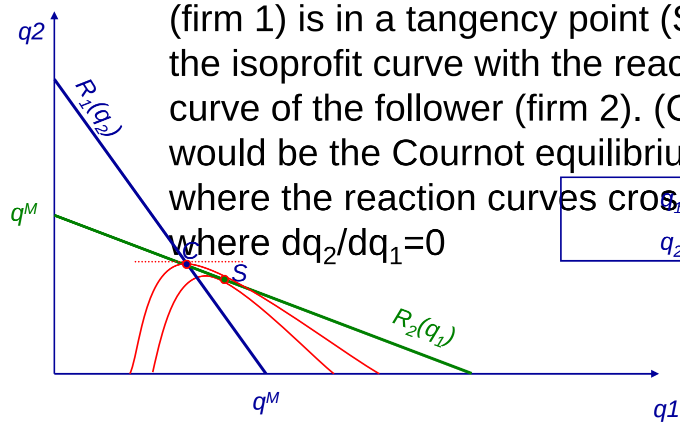
<u>Graphically(cont):</u> the optimum of the leader (firm 1) is in a tangency point (S) of the isoprofit curve with the reaction curve of the follower (firm 2). (C) would be the Cournot equilibrium, where the reaction curves cross and where $dq_2/dq_1=0$

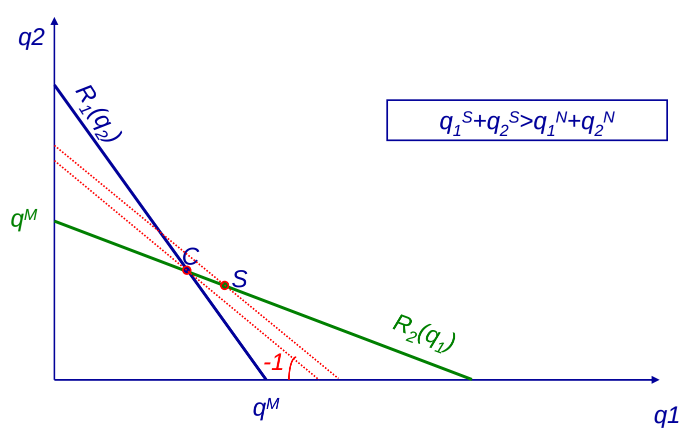


$q_1^S>q_1^N$

$q_2^S<q_2^N$

3.3. Stackelberg Model

<u>Graphically(cont):</u>



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<u>Differences between Cournot and Stackelberg:</u>

- In Cournot, firm 1 chooses its quantity given the quantity of firm 2
- In Stackelberg, firm 1 chooses its quantity given the reaction curve of firm 2

Nota: the assumption that the leader cannot revise its decision i.e. that q1 is irreversible is crucial here in the derivation of the Stackelberg equilibrium. The reason is that at the end of period 2, after firm 2 has decided on q2, firm 1 would like to change its decision and produce the best response to q1, $R_1(q_2)$. This flexibility, however, would hurt firm 1 since firm 2 would anticipate this reaction and the result could be no other but Cournot. This is the a paradox since firm1 is better off if we reduce its alternatives.

Is it plausible to think that q1 cannot be changed? This seems more plausible for the case of capacities than for the case of quantities.

3.3. Stackelberg Model

Note: When firms are symmetric, i.e. they have the same costs, then the Stackelberg solution is more efficient than Cournot (higher total quantity, lower price). This may not be the case for the asymmetric case. If the leader is the less efficient firm (higher costs) then it may well be the case that Cournot is more efficient than Stackelberg, since Stackelberg would be giving an advantage to the more inefficient firm.