3.2. Cournot Model

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3.2. Cournot Model

Assumptions:

- All firms produce an homogenous product
- The market price is therefore the result of the total supply (same price for all firms)
- Firms decide simultaneously how much to produce
- Quantity is the strategic variable. If OPEC was not a cartel, then oil extraction would be a good example of Cournot competition. Agricultural products? http://www.iser.osaka-u.ac.jp/library/dp/2010/DP0766.pdf ?
- The equilibrium concept used is Nash Equilibrium (Cournot-Nash)

3.2. Cournot Model

Graphically:

- Let's assume the duopoly case (n=2)
- MC=c
- Residual demand of firm 1:

  $RD_1(p,q_2)=D(p)-q_2$. The problem of the firm with residual demand RD is similar to the monopolist's.

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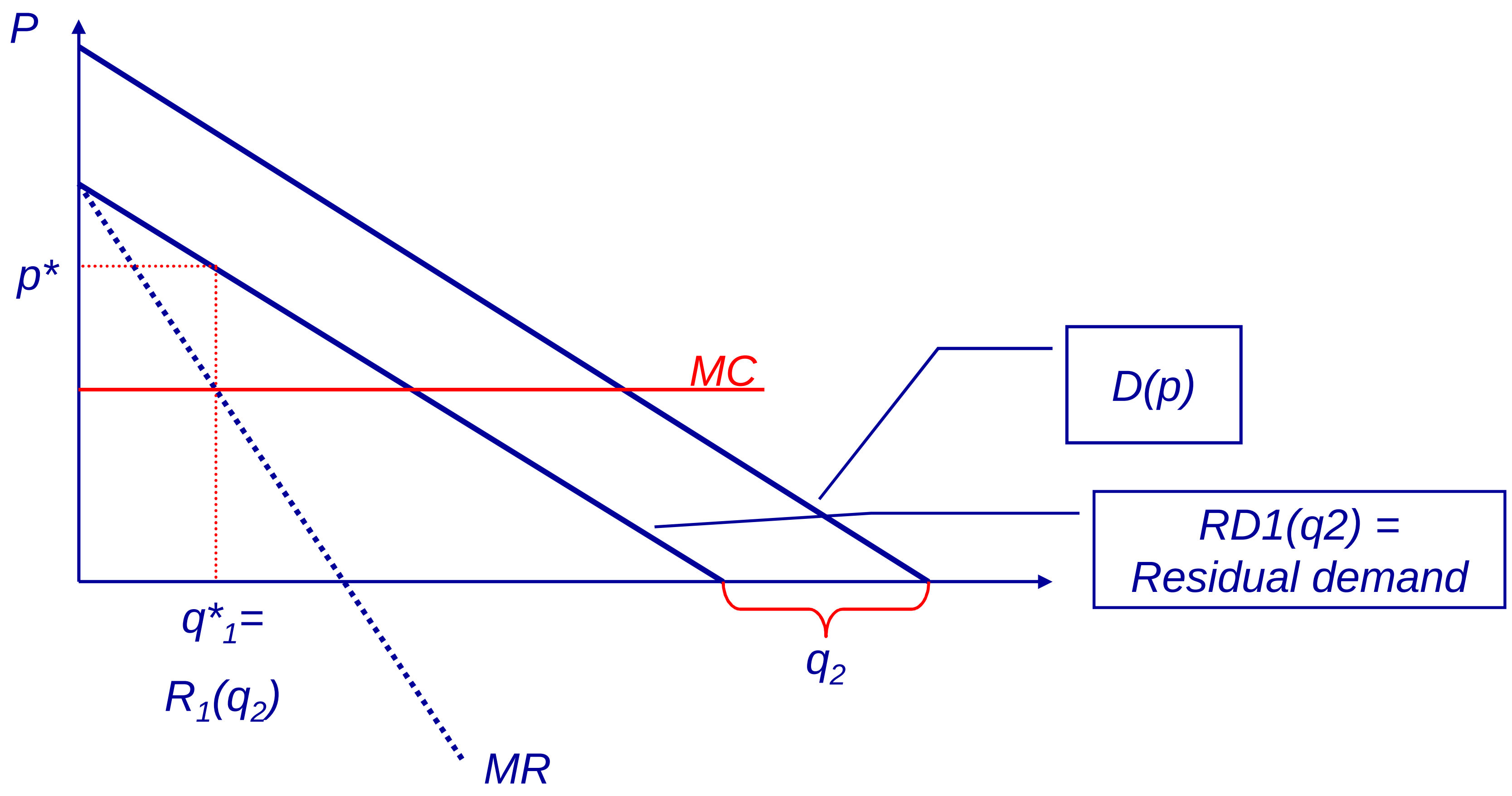
Graphically (cont.):



3.2. Cournot Model

Graphically (cont.):

$q^*_1(q_2)=R_1(q_2)$ is the optimal quantity as a function of q_2

Let's take 2 extreme cases q_2:

Case I: $\boxed{q_2=0} \Rightarrow RD_1(p,0)=D(p)$ whole demand

$\Downarrow$

$\boxed{q^*_1(0)=q^M}$

Firm 1 should produce the Monopolist's quantity

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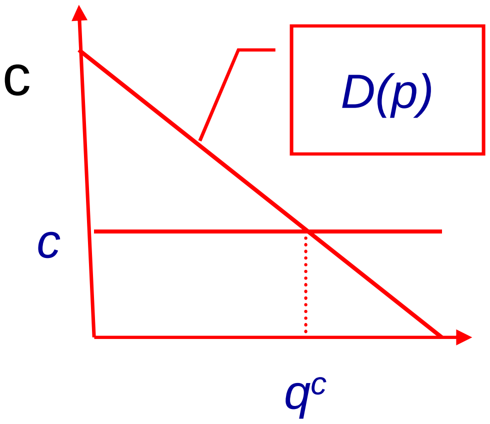
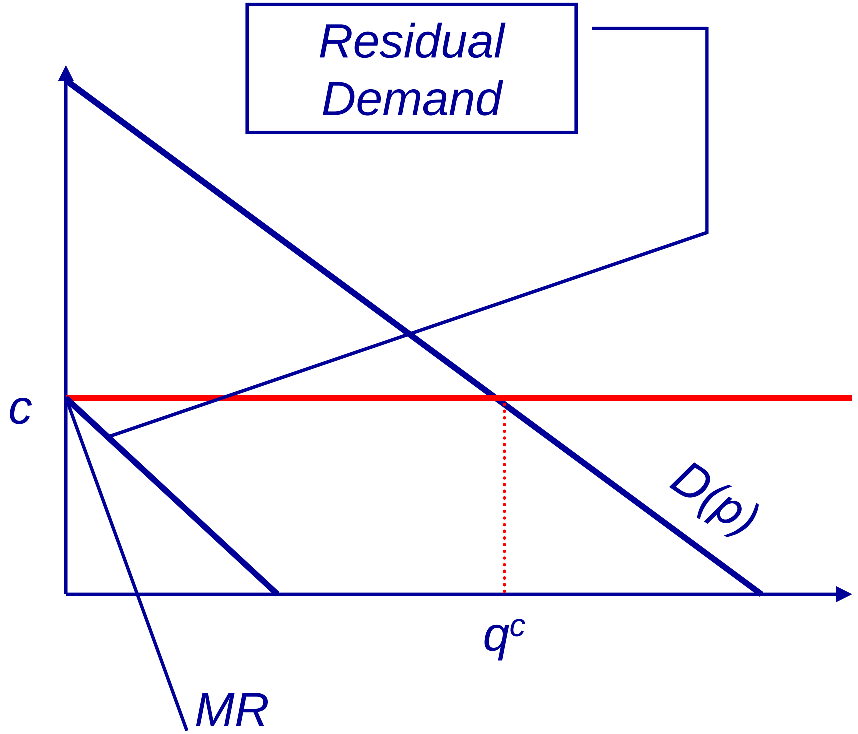
Case 2: $q_2=q^c \Rightarrow RD_1(p,q^c)=D(p)-q^c$





$MR<MC \Rightarrow q^*_1=0$

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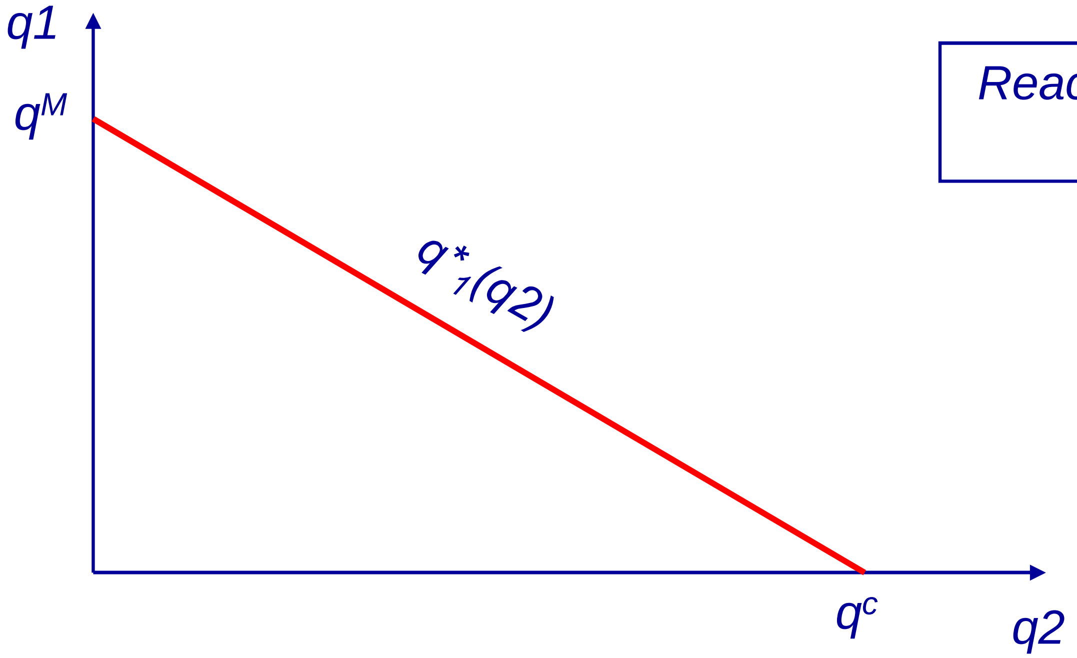
Note: If both demand and cost functions are linear, reaction function will be linear as well.



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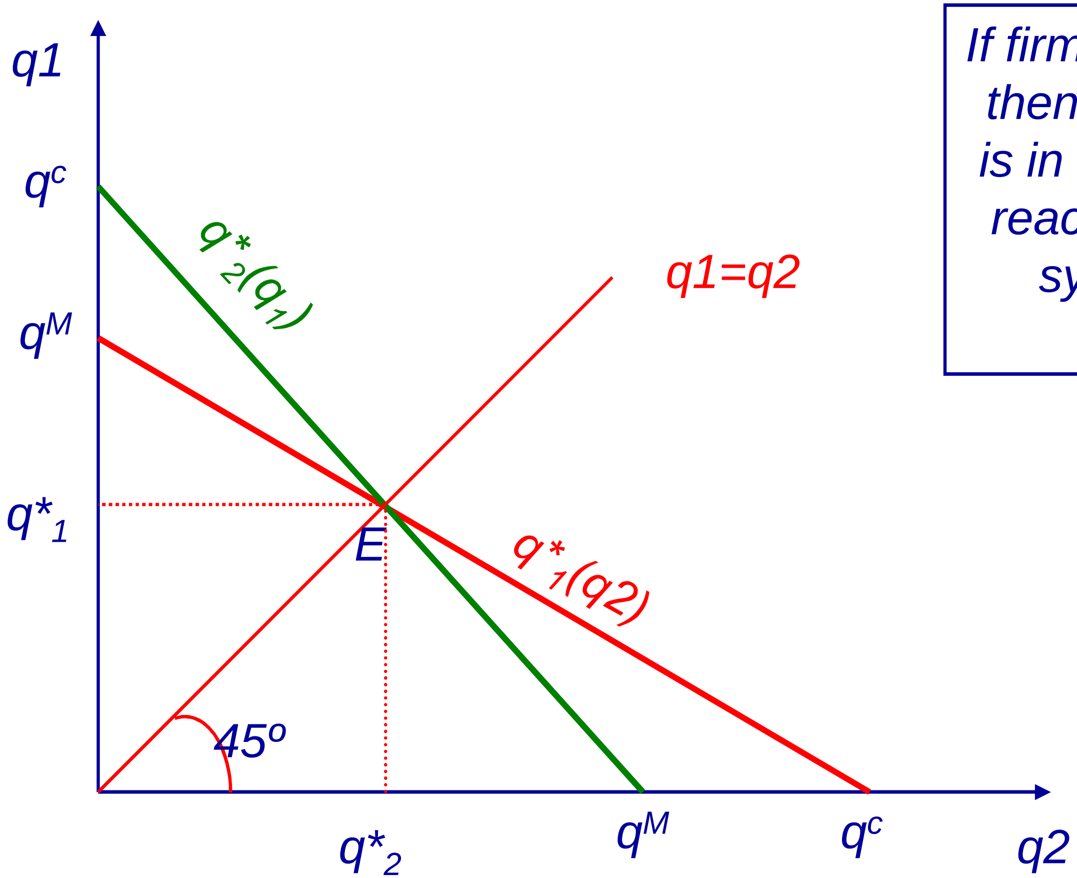

If firms are symmetric then the equilibrium is in the 45º line, the reaction curves are symmetric and $q^*_1=q^*_2$

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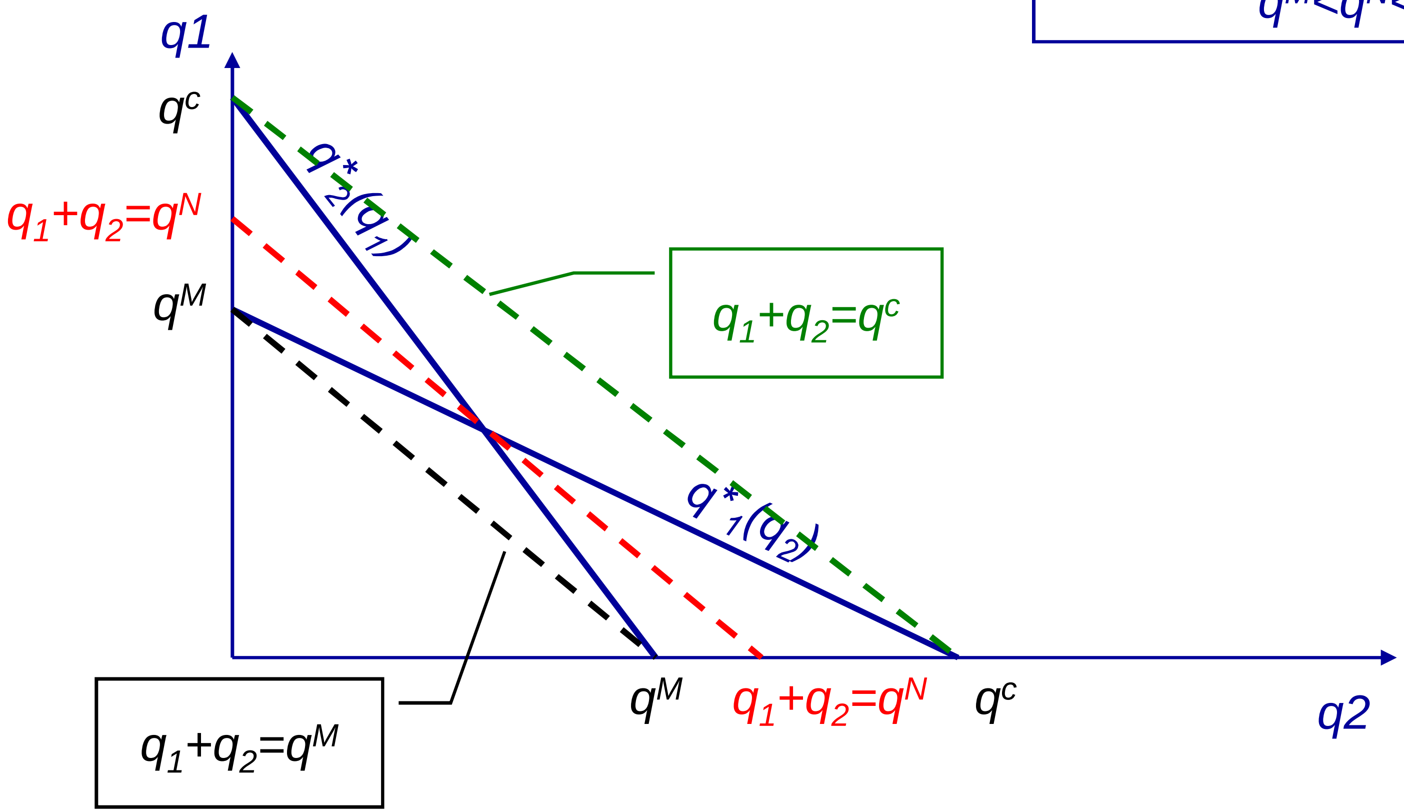
Comparison between Cournot, Monopoly and Perfect Competition

$q^M < q^N < q^c$



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Derivation of the Cournot Equilibrium for n=2

$P=a-bQ=a-b(q_1+q_2)$

$MC_1=MC_2=c$

For firm 1:

Takes the strategy of firm 2 as given, i.e. takes q_2 as a constant. Note the residual demand here

$$\underset{q_1}{Max}\,\Pi^1(q_1,q_2)=(p-c)q_1=(a-b(q_1+q_2)-c)q_1$$

$$\text{FOC: } \frac{\partial \Pi^1}{\partial q_1}=0 \Leftrightarrow a-bq_1-bq_2-c-bq_1=0$$

$$\Leftrightarrow 2bq_1=a-bq_2-c$$

$$\Leftrightarrow \boxed{q_1=\frac{a-c}{2b}-\frac{q_2}{2}}$$

Reaction function of firm 1: optimal quantity firm 1 should produce given q2. If q2 changes, q1 changes as well.

3.2. Cournot Model

We solve a similar problem for firm 2 and obtain a system of 2 equations and 2 variables.

$$\begin{cases} q_1 = \dfrac{a-c}{2b} - \dfrac{q_2}{2} \\ q_2 = \dfrac{a-c}{2b} - \dfrac{q_1}{2} \end{cases}$$

If firms are symmetric, then

$q_1^* = q_2^* = q^*$ i.e. we impose that the eq. quantity is in the 45º line

$$\Rightarrow q^* = \frac{a-c}{2b} - \frac{q^*}{2} \Leftrightarrow q^* = \boxed{\frac{a-c}{3b} = q_1^N} = q_2^N$$

Solution of the Symmetric equilibrium

3.2. Cournot Model

Solution of the Symmetric equilibrium

$$q_1^* = q_2^* = q^*$$

$$\Rightarrow q^* = \frac{a-c}{2b} - \frac{q^*}{2} \Leftrightarrow q^* = \frac{a-c}{3b} = q_1^N = q_2^N$$

Total quantity and the market price are:

$$Q^N = q_1^N + q_2^N = \frac{2}{3}\left(\frac{a-c}{b}\right)$$

$$p^N = a - bQ^N = a - \frac{2}{3}(a-c) = \frac{a+2c}{3}$$

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Comparing with Monopoly and Perfect Competition

$$\boxed{\underbrace{p^c}_{c} < \underbrace{p^N}_{\frac{a+2c}{3}} < \underbrace{p^M}_{\frac{a+c}{2}}}$$

Where we obtain that:

$$\boxed{\underbrace{\frac{\partial p^c}{\partial c}}_{=1} > \underbrace{\frac{\partial p^N}{\partial c}}_{=\frac{2}{3}} < \underbrace{\frac{\partial p^M}{\partial c}}_{=\frac{1}{2}}}$$

In perfect competition prices increase 1-to-1 with costs.

3.2. Cournot Model

In the Case of n≥2 firms:

$$\underset{q_1}{Max}\,\Pi_1\left(q_1,\ldots q_N\right)=\left(a-b(q_1+q_2+\ldots+q_N)-c\right)q_1$$

$$\text{FOC: } a-b(q_1+q_2+\ldots+q_N)-c-bq_1=0$$

$$\Leftrightarrow q_1=\frac{a-b(q_2+\ldots+q_N)-c}{2b}$$

If all firms are symmetric:

$$q_1=q_2=\ldots=q_N=q$$

$$q=\frac{a-b(n-1)q-c}{2b}\Leftrightarrow\left[1+\frac{1}{2}(n-1)\right]q=\frac{a-c}{2b}\Leftrightarrow\boxed{q^N=\frac{a-c}{(n+1)b}}$$

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Total quantity and the equilibrium price are:

$$\boxed{\begin{aligned} Q^N &= nq^N = \frac{n}{n+1}\frac{a-c}{b} \xrightarrow{n\to\infty} \frac{a-c}{b} = q^c \\ p^N &= a - bQ^N = a - b\frac{n}{n+1}\frac{a-c}{b} = \frac{a}{n+1} + \frac{n}{n+1}c \xrightarrow{n\to\infty} c \end{aligned}}$$

If the number of firms in the oligopoly converges to ∞, the Nash-Cournot equilibrium converges to perfect competition. The model is, therefore, robust since with n→ ∞ the conditions of the model coincide with those of the perfect competition.

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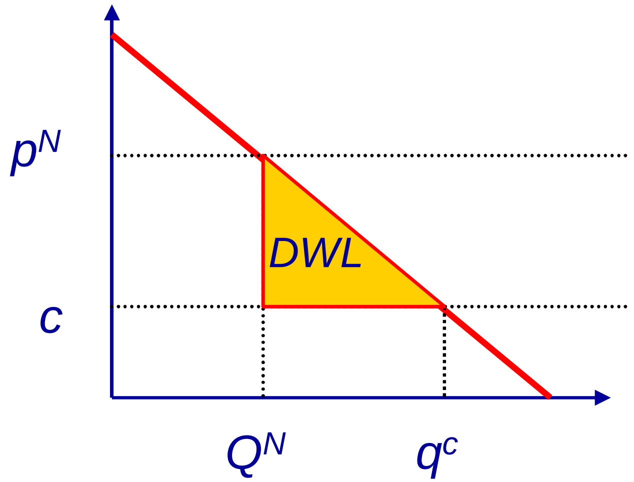
DWL in the Cournot model

= area where the willingness to pay is higher than MC



$$DWL = \frac{1}{2}\left(p^N - p^c\right)\left(Q^c - Q^N\right)$$

$$= \frac{1}{2}\left(\frac{1}{n+1}a + \frac{n}{n+1}c - c\right)\left(\frac{a-c}{b} - \frac{n}{n+1}\frac{a-c}{b}\right)$$

$$= \frac{1}{2b}\left(\frac{a-c}{n+1}\right)^2 \xrightarrow{n\to\infty} 0$$

When the number of firms converges to infinity, the DWL converges to zero, which is the same as in Perfect Competition. The DWL decreases faster than either price or quantity (rate of n^2)

3.2. Cournot Model

In the Asymmetric duopoly case with constant marginal costs.

linear demand $P(q_1+q_2)=a-b(q_1+q_2)$

$c_1 = \text{MC of firm 1}$

$c_2 = \text{MC of firm 2}$

The FOC (from where we derive the reaction functions):

$$\begin{cases} q_1P'(q_1+q_2)+P(q_1+q_2)-c_1=0 \\ q_2P'(q_1+q_2)+P(q_1+q_2)-c_2=0 \end{cases} \Leftrightarrow \begin{cases} -bq_1+a-b(q_1+q_2)-c_1=0 \\ -bq_2+a-b(q_1+q_2)-c_2=0 \end{cases}$$

$$\Leftrightarrow \begin{cases} q_1=\dfrac{a-bq_2-c_1}{2b} \\ q_2=\dfrac{a-bq_1-c_2}{2b} \end{cases}$$

Replace q_2 in the reaction function of firm 1 and solve for q_1

3.2. Cournot Model

In the Asymmetric duopoly case with constant marginal costs.

$$q_1 = \frac{a-c_1}{2b} - \frac{1}{2}\left(\frac{a-bq_1-c_2}{2b}\right) \Leftrightarrow \frac{3}{4}q_1 = \frac{a}{4b} + \frac{c_2}{4b} - \frac{c_1}{2b}$$

$$\Leftrightarrow q_1^* = \frac{a+c_2-2c_1}{3b}$$

Which we replace back in q_2:

$$q_2^* = \frac{a-bq_1^*-c_2}{2b} = \frac{a}{2b} - \frac{1}{2}\left(\frac{a+c_2-2c_1}{3b}\right) - \frac{c_2}{2b} = \frac{a-2c_2+c_1}{3b}$$

$$Q^* = q_1^* + q_2^* = \frac{a+c_2-2c_1}{3b} + \frac{a-2c_2+c_1}{3b} = \frac{2a-c_2-c_1}{3b}$$

$$p^* = a - b(q_1^* + q_2^*) = a - \frac{2a-c_2-c_1}{3} = \frac{a+c_2+c_1}{3}$$

3.2. Cournot Model

From the equilibrium quantities we may conclude that:

$$q_1^* = \frac{a + c_2 - 2c_1}{3b} \quad ; \quad q_2^* = \frac{a - 2c_2 + c_1}{3b}$$

If $\boxed{c_1 < c_2}$ (i.e. firm 1 is more efficient):

$$q_1^* - q_2^* = \frac{a}{3b} + \frac{c_2}{3b} - \frac{2c_1}{3b} - \frac{a}{3b} + \frac{2c_2}{3b} - \frac{c_1}{3b} = \frac{c_2 - c_1}{b} > 0$$

$$\Leftrightarrow q_1^* > q_2^*$$

In Cournot, the firm with the largest market share is the most efficient

3.2. Cournot Model

From the previous result, the more efficient firm is also the one with a larger price-Mcost margin:

$$L_1 = \underbrace{\frac{p - c_1}{p}}_{= \frac{s_1}{\varepsilon}} > \underbrace{\frac{p - c_2}{p}}_{= \frac{s_2}{\varepsilon}} = L_2$$

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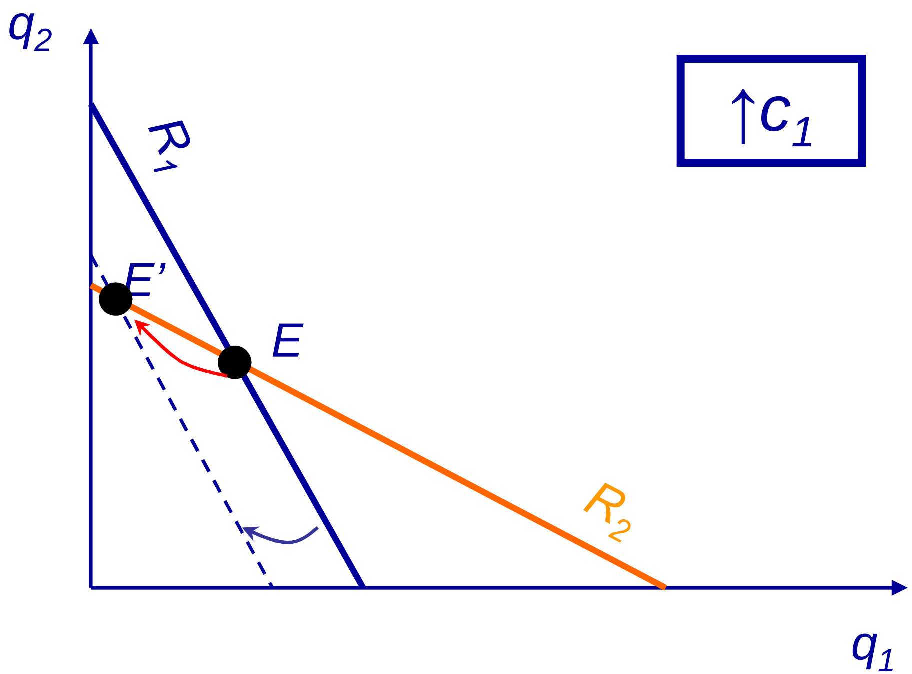
Comparative Statics:

The output of a firm ↓ when: ↑ own costs; ↓ costs of rival

$$q_i^* = \frac{a + c_j - 2c_i}{3b}$$



$\uparrow c_1$

Shifts the reaction curve of firm 1 to the left

$\uparrow q^*_2$ *and* $\downarrow q^*_1$

3.2. Cournot Model

Profits are:

$$\Pi^{1*} = \left(p^* - c_1\right) q_1^* = \left(a - b(q_1^* + q_2^*) - c_1\right) q_1^* =$$

$$= \left(a - b\left[\frac{2a - c_2 - c_1}{3b}\right] - c_1\right) \times \left(\frac{a + c_2 - 2c_1}{3b}\right) = \frac{\left(a + c_2 - 2c_1\right)^2}{9b}$$

Increase with rival's costs $\dfrac{\partial \Pi^1}{\partial c_2} > 0$

Decrease with own costs $\dfrac{\partial \Pi^1}{\partial c_1} < 0$

Symmetric to firm 2.

3.2. Cournot Model

More generally… for any demand and cost function. There is a negative externality between Cournot firms. Firms do not internalize the effect that an increase in the quantity they produce has on the other firms. That is when ↑q_i the firm lowers the price to every firm in the market (note that the good is homogenous). From the point of view of the industry (i.e. of max the total profit) there will be excessive production.

Externality: firms only take into account the effect of the price change in their own output. Then their output is higher than what would be optimal from the industry's point of view.

$$\underset{q_i}{Max}\,\Pi^i(q_i, q_j) = q_i P(Q) - C_i(q_i)$$

$$\text{CPO:} \frac{\partial \Pi_i}{\partial q_i} = 0 \Leftrightarrow \underbrace{q_i P'(Q)}_{\substack{\text{effect of the increase in quantity} \\ \text{on the inframarginal units}}} + \underbrace{P(Q) - C_i'(q_i)}_{\substack{\text{profitability of the} \\ \text{marginal unit}}} = 0$$

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If we define the Lerner index of the market as:

$$L \equiv \sum_i s_i L_i \text{ we obtain:}$$

$$\sum_i s_i L_i = \sum_i s_i \frac{s_i}{\varepsilon} = \frac{1}{\varepsilon} \sum_i s_i^2 = \frac{H}{\varepsilon}$$

Is the Herfindhal Concentration Index

3.2. Cournot Model

The positive relationship between profitability and the Herfindhal Concentration Index under Cournot:

Remember the FOC for each firm in that industry can be written as:

$$\frac{p-c_i}{p}=\frac{s_i}{\varepsilon}$$

The Industry-wide profits are then:

$$\Pi=\sum_{i=1}^{n}(p-c_i)q_i=\sum_{i=1}^{n}\frac{(p-c_i)}{p}\times pq_i=\sum_{i=1}^{n}\frac{s_i pq_i}{\varepsilon}=\sum_{i=1}^{n}\frac{s_i p}{\varepsilon}\times\frac{q_i}{Q}\times Q=$$

$$=\sum_{i=1}^{n}\frac{s_i^2 p}{\varepsilon}\times Q=\frac{pQ}{\varepsilon}\sum_{i=1}^{n}s_i^2=\frac{pQ}{\varepsilon}H=\kappa H$$

The concentration index is up to a constant an exact measure of industry profitability.

3.2. Cournot Model

Note: The Cournot model is often times criticized because in reality firms tend to choose prices not quantities. The answer to this criticism is that when the cournot model is modified to incorporate two periods, the first where firms choose capacity and the second where firms compete in prices. This two period model gives the same outcome as the simple Cournot model.