Exercises for Industrial Organization
Master de Economía Industrial
2012-2013

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1 Concentration Measures

1. Imagine two industries A and B with concentration curves given by the following picture. Which is the right sentence?

(a) The Hirshman-Herfindahl index (HHI) is larger for A than for B
(b) The Hirshman-Herfindahl index (HHI) is lower for A than for B
(c) The Hirshman-Herfindahl index (HHI) is equal for A and B
(d) The picture does not allow us to conclude which of the industries has a larger Hirshman-Herfindahl index (HHI).

2. The following table characterizes the two industries, A and B where, for example, s1 denotes the market share of firm 1. Which of the following is the right answer:

(a) The concentration ratio C3 is the same for both industries but the Hirschman-Herfindahl index is higher for industry A.
(b) The concentration ratio C3 is higher for industry A and the Hirschman-Herfindahl index is higher for industry A.
(c) The concentration ratio C3 is higher for industry A but the Hirschman-Herfindahl index is lower for industry A.

(d) The concentration ratio C3 is the same for both industries A but the Hirschman-Herfindahl index is lower for industry A.

<table>
<thead>
<tr>
<th>Market shares</th>
<th>s1</th>
<th>s2</th>
<th>s3</th>
<th>s4</th>
<th>s5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industry A</td>
<td>60%</td>
<td>20%</td>
<td>10%</td>
<td>5%</td>
<td>5%</td>
</tr>
<tr>
<td>Industry B</td>
<td>30%</td>
<td>30%</td>
<td>30%</td>
<td>10%</td>
<td>-</td>
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</tbody>
</table>

3 **Elasticities [Exam December 2010]**: The elasticity of the demand for cigarettes is 0.5. The Government is considering raising the tax on tobacco by 25% (which would raise the price of cigarettes by 25%). The objective of the Government is to reduce the cigarettes’ demand by 25%. Will the Government succeed?

- a. No, the change in demand is less than 25%
- b. Yes, the change in demand is greater than 25%
- c. Yes, the change in demand is of exactly 25%
- d. No, demand will not change.

4 [From Luis Cabral] Suppose you only know the value of the market shares for the largest m firms in a given industry. While you do not possess sufficient information to compute the Herfindahl index, you can find a lower and an upper bound for its values. How?

5 [From December Exam 2011] Cournot: Suppose there are two firms selling Ageless a product that reduces the aches and pains associated with aging joints. Market research indicates that the elasticity of demand for Ageless is 1.25 (at all points on the demand curve). The marginal cost of Ageless is $3 for both firms. Suppose each firm serves 50% of the market and compete a la Cournot. What is the profit-maximizing uniform price per dose of Ageless?

- a. The optimal price is $5
- b. The optimal price is $3
- c. The optimal price is $15
- d. None of the above.

2 **Monopoly**

1. Suppose the monopolist has the following demand function \( P(q) = \max\{40 - 3q, 0\} \). The cost function is \( C(q) = q^2 \). Compute the monopoly price and quantity, the Lerner index and the inefficiency in this market.
2. Suppose that all consumers have unit demand. They buy either 0 or 1 unit of the good produced by the monopolist. All the consumers are identical and have valuation (i.e. maximum willingness to pay) for the good. Show that monopoly pricing would not create a welfare loss.

3. Monopoly [Exam December 2010]: You have finally gotten Food and Drug Administration (FDA) approval to sell your new patented wonder drug, which reduces the aches and pains associated with aging joints. You will market this drug under the brand name of Ageless and you will be a monopoly in the market. Market research indicates that the elasticity of demand for Ageless is 1.25 (at all points on the demand curve). The marginal cost of Ageless is $1. (a) What is the profit-maximizing uniform price per dose of Ageless?
   a. The optimal price is $1
   b. The optimal price is $5
   c. There is no information to compute the optimal price
   d. None of the above.

3 Price Discrimination:

1. T-shirts designed by a famous Italian designer, named “Marco”, are exclusively sold in only one store in Milan and in one store in Paris. Both of these stores belong to Marco. Marco comes from Italy and his t-shirts are more successful in Italy than in France. In particular:

   - In Milan he has 100 customers, each having the following demand: \( p_M = 40 - 2q \).
   - In Paris he has 100 customers, each having the following demand: \( p_P = 20 - 2q \).

   The marginal cost that he faces in both cities is 2. Assume throughout the exercise that consumers cannot travel between the two cities.

   1. (a) Marco has to decide on the price that it will appear on the price tag that he puts on his t-shirts. The same price tag will be put in his t-shirts that are available in both of his stores. What price will he choose?

     (b) Assume now that he can put different price tags on the t-shirts that are available in his two stores. Calculate the price that he will choose for the store in Milan and for the store in Paris.

     (c) Assume now that Marco has decided to sell his t-shirts only to customers that are members of his on-line community. Customers, both in Milan and in Paris, can go to his website and sign-up in order to become members. Whenever a customer signs up he/she has to pay a fixed membership fee \( A \), which is the same both for consumers in Milan and in Paris. Once
a customer becomes a member then he/she can visit Marco store, show his/her membership card and buy Marco’ t-shirts at a price $p$ per shirt. What is the value of $A$ and $p$ that Marco will charge if he wants to have customers both in Milan and in Paris? Which are his profits?

(d) What is the value of $A$ and $p$ that Marco will charge if he decides to keep only the store in Milan? Does he have incentives to keep only the store in Milan given that if he keeps both stores he will have to set the same $A$ and $p$ in both of them?

2 Suppose a firm faces two types of consumers. Consumers of type 1 and 2 have different demand functions. Their individual demand functions are:

For type 1:

$$p = 4 - 2q$$

For type 2:

$$p = 8 - 4q$$

Assume the marginal cost of production is constant and equal to zero. There are a total of 100 consumers where 60% are of type 1.

(a) Compute the optimal tariffs, quantities and profits if the monopoly firm could perfectly discriminate amongst consumers.

(b) Compute the aggregate demand of both markets. Compute the optimal uniform price if the Government would force the monopoly to charge an uniform price i.e. a tariff $T(q) = p \times q$.

(c) Assume now the monopoly cannot distinguish between consumers ex-ante. Assume the Government allows it to use two-part tariffs. Find the optimal two-part tariff that maximizes the monopolist’s profits.

3 Imagine a monopoly with 100 consumers. Consumers may be of two types according to the following demand functions:

$$p = 8 - 2q$$  \hspace{1cm} ((A))

$$p = 2 - q$$  \hspace{1cm} ((B))

The marginal cost of the monopolist is constant and equal to 1 Euro. There are no fixed costs. The percentage of consumers of each type is 50%.

1. (a) Compute the optimal tariffs for the monopolist if it is capable of perfectly discriminate between the two types of consumers.

(b) Suppose the Government decides to ban this type of discrimination and forces the monopolist to set a uniform price equal to both consumers. What is the optimal uniform price for the monopoly? (Note: The Government does not care if the price is low or high)
(c) Is the Government ban optimal from the social point of view? How much would the monopolist be willing to pay for the Government to allow the situation in a) i.e. perfect discrimination.

(d) Suppose the Government allows the monopolist to set a (SINGLE) two-part tariff. What would be the optimal two-part tariff that from the point of view of the monopolist?

(e) Suppose now the the Government subsidizes the monopolist production giving it $s = 2$ euros per unit. How would the market change?

4 [Non-linear Price discrimination] Suppose a monopolist faces two types of consumer: high valuation, low valuation. There are $N$ consumers 50% of each type. Costs are zero. He considers the possibility of price discriminating by offering a menu of two different goods at different prices. The two goods he may offer are $A$ and $B$. The table of valuations for these goods is:

<table>
<thead>
<tr>
<th></th>
<th>$A$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>high</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>low</td>
<td>13</td>
<td>9</td>
</tr>
</tbody>
</table>

(a) Compute the optimal pricing strategy.

(b) What would change if the matrix of valuation was:

<table>
<thead>
<tr>
<th></th>
<th>$A$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>high</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>low</td>
<td>13</td>
<td>11</td>
</tr>
</tbody>
</table>

i.e. the low valuation type values more the low quality good than the high valuation type.

5 Suppose in a given market there are two types of consumers depending on the value of the parameter $\theta$. $\theta$ takes two possible values $\{\theta_2, \theta_1\}$ where $\theta_2 > \theta_1$. The consumer’s utility function is given by: $U(q, T) = \theta_i V(q) - T$, where $q$ is the quantity consumed and $T$ is the total amount of money payed to consumer $q$. The function $V(q)$ is:

$$V(q) = \begin{cases} 
\frac{16 - (4 - 2q)^2}{4} & \text{si } q > 0 \\
0 & \text{si } q = 0 
\end{cases}$$

Assume marginal costs of production are zero. There are 100 consumers and the proportion of consumers with low demand type is 60%. Take $\theta_i \in \{0.5, 1\}$.

(a) Compute the demand functions of the two types of consumers if they pay a uniform price i.e. $T(q) = pq$.

(b) Compute the tariffs, quantities and profit if the monopolist can perfectly discriminate between consumers.

(c) Compute the aggregate demand function for the two markets. Compute the uniform price if the Government forces the monopolist to charge a uniform price i.e. $T(q) = pq$. 

(d) Suppose the monopolist is not able to distinguish between consumers ex-ante. If the Government allows the monopolist to charge two-part tariffs what two-part tariff would the monopolist set?

(e) Suppose the marginal cost rises to $\frac{3}{2}$, repeat d).

6 **Price Discrimination [Exam December 2010]**: First time subscribers to the Economist pay a lower rate than repeat subscribers.

a. This is an example of first degree price discrimination

b. This is an example of second degree price discrimination

c. This is an example of third degree price discrimination

d. None of the above

4 **Bundling:**

1. Consider the following demand functions $p_1 = 8 - 2q_1$ and $p_2 = 4 - \frac{1}{2}q_2$

a) Compute the gross consumer surplus of each consumer type as a function of quantity in the bundle.

b) Fill in the blanks where $\pi^i(q)$ is the profit of selling a bundle of size $q$ only to consumer of type $i$. Assume the marginal cost is constant and equal to 2 and there are no fixed costs. The last row $\max\{\pi^1(q), \pi^2(q), \pi^{1,2}(q)\}$ determines the maximum profit associated with each bundle of size $q$. What’s the best strategy?

2 Suppose a monopoly faces 100 consumers each with an inverted demand that is equal to $P = 15 - 5Q$. Marginal cost is constant and equals 5.

a) What is the optimal price and profit in the case of uniform pricing

b) Can the monopolist increase profits by selling the good in a bundle? If yes, what is the optimal size of the bundle. (Show all your computations).

5 **Monopoly Tying:**

1. Congratulations, you have been appointed the general manager of the Paradise Hotel, which is the only hotel on Paradise Island, located somewhere in the Pacific Ocean. This hotel also owns the only restaurant in town that serves breakfast. As the hotel manager, your first responsibility is to decide whether to include breakfast in the standard hotel rate or to charge extra for breakfast. The table below shows the willingness to pay of type 1 and type 2 hotel guests for hotel room (R) and
for breakfast(B), as well as the expected number of guests of each type and the hotel’s marginal
cost of providing each service.

<table>
<thead>
<tr>
<th>Guest Type</th>
<th>Hotel room (R)</th>
<th>Breakfast (B)</th>
<th>Expected number of guests</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type 1</td>
<td>100 euros</td>
<td>5 euros</td>
<td>200</td>
</tr>
<tr>
<td>Type 2</td>
<td>60 euros</td>
<td>10 euros</td>
<td>800</td>
</tr>
<tr>
<td>Marginal cost</td>
<td>40 euros</td>
<td>2 euros</td>
<td></td>
</tr>
</tbody>
</table>

Solve the following problems:

1. (a) Compute the hotel’s profit-maximizing room rate \( p_R \), breakfast price \( p_B \), and resulting profit
   \( \pi \), given that both services are sold separately (No Tying).
   
   b Suppose now that the hotel rents a room together with breakfast. Compute the package’s
   profit-maximizing price \( p_{RB} \) and the corresponding profit level \( \pi \). Conclude whether the hotel
   should tie the two services in a single package or sell them separately.

2. [From December 2011 Exam] [25 points] Tying: Consider a cable TV operator facing viewers and
   unit costs described by Table 1 below:

   1. (a) Assume there are no fixed costs.
   
      (b) What are the optimal prices \( p_C \), \( p_B \) and \( p_H \) and the resulting profit if each channel is sold
      separately (i.e. no tying).
   
      (c) Compute the profit-maximizing rate of \( p_{CBH} \) and the profit level in the case the operator
      offers a single package composed of all three channels (i.e. pure tying)
   
      (d) Can you find any alternative package that offers a higher profit than options a) and b). If so
      explain and describe.
6 Oligopoly

6.1 Cournot

1. In Cournot competition the equilibrium output of firm $i$ is a function:
   
   (a) increasing in firm $i$’s costs.
   (b) Decreasing in firm $i$’s costs and decreasing in the rival’s costs.
   (c) Decreasing in firm $i$’s costs and increasing in the rival’s costs.
   (d) Decreasing in the rival’s costs.

2. Suppose the aggregate demand of an homogenous product is given by $P = 64 - 2Q$. There are 2 firms producing the homogenous good, both with constant marginal cost equal to 4.

   (a) Compute the equilibrium price, quantities and profits (for each firm) in the case of Cournot competition.
   (b) Suppose these firms survive only one period in the market. How much would firm 1 be willing to pay for the ownership of firm 2 and convert into a monopoly?
   (c) And how much would a third firm (firm 3), which did not belong to this market, be willing to pay for firm 2?
   (d) If an auction would take place which firm (i.e. firm 1 or firm 3) would finally buy firm 2? And for what price? Explain.
   (e) Compute the Deadweight Loss of the Cournot Equilibrium with 2 firms as a percentage of the Deadweight Loss in Monopoly.

3 In a Cournot duopoly, the demand is given by $D(p) = \{500 - 50p, 0\}$ and the cost functions are respectively $C_i(q) = c_i q$ with $c_1 = 8$ and $c_2 = 6$. Firm 2 cannot produce more than 25 units of output. Compute the Cournot equilibrium.

4 In an industry there are three firms with cost functions given by: $C_i(q_i) = c_i q_i$ where $c_1 = c_2 = 2$ and $c_3 = 4$. The demand function is given by $D(p) = \max\{120 - p, 0\}$.

   (a) Compute the price, quantities and profits in a Cournot equilibrium.

b Repeat the computations assuming firms 1 and 2 merge. Who wins and who looses with respect to situation a).

c Suppose now that firm 3 is the owner of 25% of the stock of firm 1, such that its profit is now $\Pi_3 + 0.25\Pi_1$. However, firm 3 does not have the power to decide on the production of firm 1 (i.e. firm 1, therefore, maximizes $\Pi_1$). Compute the Cournot equilibrium. Who wins and who looses with respect to situation a)
5 Consider a market with 2 firms competing a la Cournot. Firm 1 has constant marginal costs equal to $c_1$ and firm 2 has constant marginal costs equal to $c_2$. Assume $c_1 < c_2$. The demand is given by $P = 1 - bQ$, where $Q = q_1 + q_2$.

(a) Compute the equilibrium values of $q_1$, $q_2$, $Q$, and $P$.
(b) For what values of $c_1$ and $c_2$ would a merger between the two firms be beneficial to society. Explain.

6 There are two firms in a market with linear demand $Q = 1 - p$. The firms’ marginal costs are constant and equal to $c_1$ and $c_2$ such that $c_1 + c_2 = 2c$ (where $c$ is a constant). Show that when firms become more asymmetric (larger difference between $c_i$ and $c$) the Cournot equilibrium implies a higher level of concentration. (Tirole ex. 5.7)

7 Suppose you have 3 identical firms in a market. Demand is given by $P = 1 - Q$ where $Q = q_1 + q_2 + q_3$. Marginal costs are zero. (Tirole 5.3)

(a) Compute the Cournot equilibrium
(b) Show that if two of the firms merge (i.e. the market becomes a monopoly) their total profit decreases. What happens to the profit of the unmerged firm? Explain.
(c) What happens if the three firms were to merge? Explain.
(d) If firms were selling differentiated goods and were competing in prices would it be beneficial to them if two of the firms merged? (explain the intuition taking into account that prices are strategic complements).

8 Show that industry-wide profits of a Cournot oligopoly are equal to $\Pi = \sum_{i=1}^{n} \Pi_i = \frac{PQ}{\varepsilon} H$ where $H$ is the Hirshman-Herfindahl index of concentration.

9 Show that under Cournot competition the "average Lerner Index" $\sum_i \alpha_i L_i$ is equal to the Hirshman-Herfindahl divided by the elasticity of demand.

### 6.2 Stackelberg

1. Which of the following is the correct statement if two asymmetric firms with constant marginal costs and a single homogenous product compete a la Stackelberg? (Hint: In order to solve this exercise you only need to reason, there is no need for computations)

(a) The leader’s profit may be lower than the profit it would obtain in the Cournot model.
(b) The leader’s profit is always lower than the profit it would obtain in the Cournot model.
(c) The leader’s profit is always larger than the profit it would obtain in the Cournot model.
(d) The follower’s profit is always the same as the profit it would obtain in the Cournot model.

2 May a firm which is leader in a Stackelberg model of competition has lower profits than if Cournot was played instead?

(a) Yes, if the costs of the leader are higher than the costs of the follower.
(b) No, never.
(c) Yes, if the costs of the follower are higher than those of the leader.
(d) Yes, always.

2. **Stackelberg:** In the Stackelberg model with two asymmetric firms with constant marginal costs and an homogenous product, which of the following sentences is the correct one? (Hint: there is no need to do any calculation)

a) The leader may have lower profit than the profit it would obtain in a Cournot game.
b) The leader will always have lower profits than in the Cournot game.
c) The leader always has a larger profit than the profit it would obtain in a Cournot game.
d) The follower always has the same profit as in the Cournot game.

### 6.3 Oligopoly - Mixed Topics

1. Suppose you have a duopoly with homogenous goods and equal constant marginal costs, $c = 2$, and an aggregated demand curve given by $P = 11 - Q$. Which of the following is true when firms compete a single period.

   (a) If firms compete in quantities (a la Cournot) the Deadweight Loss (DWL) is 0
   (b) If firms compete in quantities (a la Cournot) the Deadweight Loss (DWL) is always smaller than if firms compete in prices (a la Bertrand)
   (c) If firms compete in quantities (a la Cournot) the Deadweight Loss (DWL) is equal to 4.5
   (d) If firms compete in prices (a la Bertrand) the Deadweight Loss (DWL) is 4.5

2 Oligopolies producing homogenous goods, produce the efficient level of output?

(a) Yes, if they compete in **prices**
(b) Yes, if they compete in **quantities**
(c) No, porque because there a small number of firms in the market.
(d) Yes, whenever firms’ profit is zero.
7 Repeated Games

1. A solution to the Bertrand paradox is to make the realistic assumption that firms interact more than once. Accordingly, consider the problem of 2 firms competing in prices and choosing prices more than once. Which of the following statements is correct?

(a) If the number of periods equals $\infty$ and the firms are sufficiently patient, there exists equilibrium with price greater than the marginal cost of production.
(b) If the number of periods is finite and the firms do not value the future much, there exists equilibrium with price greater than the marginal cost of production.
(c) If the number of periods is finite and the firms are sufficiently patient, there exists equilibrium with price greater than the marginal cost of production.
(d) The existence of other equilibria, other than the one of the static game with price equal to marginal cost, only depends on the discount factor.

2. Consider three firms producing a homogeneous good. Firms have zero marginal cost and no fixed cost. They compete in prices à la Bertrand. The market demand is $D(p) = 1 - 1/5p$, where $p$ denotes price.

(a) Suppose that the firms face no capacity constraints and interact only once. What is the Nash equilibrium market price? What prices do the firms charge? How much does each firm sell? Explain.
(b) Given your answer to (a), compute profits for each firm, the consumers’ surplus, the DWL and the Lerner index.
(c) Suppose that firms interact infinitely often. Firms discount future profits using the discount factor $d \in (0, 1)$. Calculate the minimum discount factor for which the price $P = 2$ can be sustained as a subgame perfect Nash equilibrium.

3. Consider an industry with $N \geq 3$ firms producing an homogeneous good. Marginal costs are constant and equal to 5 for all firms and there are no fixed costs. Market demand is given by $D(p) = 5000 - 10p$, where $p$ denotes the market price. Firms compete in prices. Consumers buy from the firm selling at the lowest price. If the lowest price is offered by several firms, consumers randomize uniformly over these firms.

(a) Suppose the firms compete for only one period. Determine the Nash equilibrium prices $p_1^*, p_2^*, \ldots, p_N^*$ and the resulting equilibrium profits $\pi_1^*, \pi_2^*, \ldots, \pi_N^*$. How does your answer change when firm 1’s marginal cost increases to 6? Derive.
(b) Suppose the firms compete for an infinite number of periods. Market demand is $D(p) = 5000 - 10p$ in each period and firms discount profits with a discount factor $\delta = \frac{7}{9}$. Determine whether there exists a subgame perfect Nash equilibrium in trigger strategies. How does your
answer depend on the number of firms, \( N \)? Clearly derive. (Hint: Trigger strategies are: all firms start by choosing the agreed upon price. Then, if any firm deviates at any point in time, each firm prices at marginal cost thereafter.)

\section*{8 Product Differentiation}

1. Another solution to the Bertrand paradox is to allow for heterogeneity of the product. Consider a problem with 2 firms producing differentiated products and competing in prices. Of the statements below, choose the correct one.

(a) More substitutability between the products implies higher profits for firms.

(b) As substitutability between the products increases, the equilibrium prices approach the Cournot equilibrium.

(c) As substitutability between the products increases, we approach the equilibrium of the monopoly problem with independent demands.

(d) As substitutability increases, we approach the equilibrium price of the Bertrand model.

2. In class, we studied models of horizontal differentiation with location choice. For these models, the equilibrium price:

(a) Does not depend on the transportation costs if this is incurred by the consumer.

(b) Depends negatively on the transportation costs because firms are compensating the consumer for the additional cost.

(c) Depends positively on the transportation cost since higher transportation costs imply more differentiation between products.

(d) Depends negatively on the transportation costs since the consumer always buys from the lowest-pricing firm.

2. In Spain, the prices of most medicines are regulated. In Villapequeña, a small village with a single street where people’s houses are uniformly distributed along the street, there is enough demand to open two pharmacies. Given that pharmacies cannot compete in prices, location is the only variable in which they can compete. The mayor announced that the owners of the future pharmacies must deliver in a sealed envelope their desire location simultaneously. The owners of the pharmacies have chosen:

(a) The two pharmacies will locate in the centre

(b) The two pharmacies will locate in opposite extremes of the villages.

(c) Both owners decide not to open a pharmacy
There is no Nash Equilibrium.

4 Picture a triangular city. In this city there are 3 firms, A, B and C, producing a homogenous good at zero marginal cost. The firms are located at the vertices of the triangle as the figure illustrates:

The distance between neighboring vertices equals 1 and the consumers are distributed uniformly along each side of the triangle, with a total mass of population of size 1. All consumers have unit demand (that is, they are willing to buy one unit of the good) and they are willing to pay a maximum of $s$ euros for the product. The transportation costs are given by $td^2$, where $d$ denotes distance between consumer and firm.

1. (a) Derive the demand functions for each firm.
   (b) Compute the equilibrium prices and profits for each firm. (Hint: The equilibrium is symmetric).
   (c) Suppose now that the entire population (all consumers) is located exactly at firm C’s location. What is firm C’s reaction function? (Hint: Assume that A and B are choosing identical prices.) Prove that is an equilibrium (that is, $p_A = p_B = 0$ and $p_C = t$, with all consumers buying from C constitutes an equilibrium). Clearly explain.
   (d) Keep the assumption that the entire population (all consumers) are located exactly where C is, but assume that firm C disappears (only A and B remain in the market). What are the
new equilibrium prices? Explain your answer. (Hint: you do not need new derivations to answer this question, just reason your answer with your intuition.)

5 Motta and Garcia Fontes model the competition between pharmacies in the Spanish market using a model of horizontal differentiation similar to Hotelling’s model. Suppose now that there is a city with a single street. The citizens of that city are uniformly distributed along the street and buy up to 1 unit of the good sold in the pharmacies. The citizens’ willingness to pay for that good is $S > 0$. Citizens obtain zero utility if they do not buy the good. Transport costs are $t \times d$ where $d$ indicates distance to the pharmacy. To simplify, assume that there are no fixed costs and marginal costs are equal to zero in both pharmacies.

(a) Suppose there are only two pharmacies in the city, A y B, located in 0 and 1, respectively as shown in the figure below. Note that there are consumers to the left of 0 and to the right of 1. Derive the demand curves of A and B. (hint: the last consumer to the left of 0 to buy from pharmacy A is indifferent between buying or not buying, the same is true for the last consumer, to the right of 1, to buy from B).

(b) Suppose the Government fixes the prices to $p < S$ and that the market is covered. Compute the quantities demanded from A and B in the equilibrium.
(c) Suppose now the Government allows the use of discounts $dA$ and $dB$ such that $pA = p - dA$ and $pB = p - dB$, where $dA \geq 0$ and $dB \geq 0$. In which situations is optimal for the pharmacies to use discounts? And in those situations what are the level of the discounts in equilibrium?

6 Imagine a linear city, a segment $[0, 1]$, where all consumers are located at the point 0. Denote by $s > 0$ the consumers’ valuation for the good. Consumers must pay a transportation cost of $dt$ to travel a distance equal to $d$. Assume the marginal cost is zero.

(a) If only one firm enters this market, which location will it choose? and what price would it charge? Explain.

(b) If there are two firms in this market $A$ and $B$ where $A$ is located at zero and $B$ is located at 1. What is the Nash equilibrium if firms compete in prices? And what are the profits of the firms? Does the value of $s$ matters?

(c) If there is a cost of entry $F > 0$ and if $N \geq 1$ firms may enter the market, how many firms will enter? and what the market price will be? Explain.

7 Suppose that in the linear city there is only one restaurant, located at the center of the street with a length of 1km. Assume the restaurant’s costs are zero. Consumers are uniformly distributed on the street which is the interval $[0, 1]$, where at each point of the interval lives one consumer. Suppose the transportation cost of each consumer is $1$ for each unit of distance. The utility of a consumer who lives at $a$ units of distance from the restaurant is $U = S - a - p$ where $p$ is the price of a meal and $s$ is a constant. If the consumer does not eat at the restaurant the utility is $U = 0$. Answer the following questions:

(a) Suppose the parameter $S$ satisfies the following: $0 < S < 1$. Find the number of consumers eating at this restaurant. Compute the monopoly’s price and profits.

(b) Answer the previous question assuming that $S > 1$.

8 Imagine a road with two sellers A and B located at 0 and 1 respectively. The road’s length is one but there is a mountain right in the middle of the road (at 1/2) that impedes the passage of consumers. Suppose there are $N$ potential consumers. Half of them live at 0 and the other half at 1. As usual assume all consumers have a gross consumer surplus equal to $s$ and transportation costs is equal to one per unit of distance (i.e. $1 \times d$), where $d$ is distance to the chosen firm, and firms have zero marginal costs.

(a) Compute the equilibrium prices and compute the firms’ profits

(b) Now suppose a tunnel is constructed to connect the two sides of the road. Compute the new equilibrium prices and the firm’s profits.
9 Vertical Differentiation. Consider a model of vertical differentiation where two firms offer goods with qualities \( v_1 = 1 \) and \( v_2 = 2 \), both produced at zero cost. There is a continuum of consumers that value a good of quality \( v \) as: \( s + x_i v \) where \( x_i \) (the marginal valuation of quality) is uniformly distributed along \([1, 2]\). [Diego, Hoja de Ejercicios 4, #4]

(a) What are the equilibrium prices for \( v_1 = 1 \) and \( v_2 = 2 \)?

(b) If firms choose their quality levels simultaneously \((v_1, v_2) \in \{1, 2\}^2\), before competition in prices take place, what would be the levels of \((v_1, v_2)\)?