Taxing Capital? Not a Bad Idea After All!

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Zero capital tax

- **Capital tax should be zero:**
  - **Seminal contributions:** Chamley (1986) and Judd (1985)
  - **Robust:** Larry E. Jones, Rodolfo E. Manuelli, and Peter E. Rossi (1997), Andrew Atkeson, V. V. Chari, and Patrick Kehoe (1999), and Chari and Kehoe (1999)
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- **Non-zero if:**
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- **This paper:**
  - Model with these two elements plus heterogeneity (via fixed effect on productivity).
  - Quantitatively characterize optimal capital and (progressive) labor tax.
Setting

Agents

Demographics:
- $J$ generations.
- Population grows at rate $n$.
- Conditional survival probability $\psi_j$.
- Exogenous retirement at age $j_r$.
- Accidental bequests $Tr_t$. 
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- **Heterogeneity** $(a, \eta, i, j)$
  - $a$: savings on risk free asset.
  - $j$: average productivity by age $\epsilon_j$.
  - $i$: Permanent differences in productivity $\alpha_i$ w.p. $p_i$.
  - $\eta$: Idiosyncratic risk: independent across agents Markov chain.
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- **Pre-tax labor income:** $y_p_t = \alpha_i \epsilon_j \eta w_t l(i, j, \eta)$
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- **Pre-tax labor income:** $y\rho_t = \alpha_i \epsilon_j \eta w_t l(i, j, \eta)$

- **Measure:** $\Phi_t(a, \eta, i, j)$

- **Preferences:**
  
  $E_{\Psi, \eta} \left[ \sum_{j=1}^{J} \beta^{j-1} u(c_j, 1 - l_j) \right]$
Labor tax $T(y_t)$ is a function of taxable labor income:

$$y_t = \begin{cases} 
    y p_t - \text{ess}_t & \text{if } j < j_r \\
    0 & \text{if } j \geq j_r 
\end{cases}$$

$$y p_t = \alpha_i \epsilon_j \eta_w t(l(i, j, \eta))$$  
$$\text{ess}_t = 0.5\tau_{ss,t} t \min\{y p_t, \bar{y}\}$$

Constant marginal capital tax rate $\tau_{K,t}$ over capital income $r_t(a + Tr_t)$.
Setting

Government

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  \end{cases}$$

  $$y_p t = \alpha_i \epsilon_j \eta w_t l(i,j,\eta)$$

  $$ess_t = 0.5 \tau_{ss,t} t \min\{y_p t, \bar{y}\}$$

- Constant marginal **capital tax** rate $\tau_{K,t}$ over capital income $r_t(a + Tr_t)$.

- Exogenous government **spending** $\{G_t\}_{t=1}^\infty$. 
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0 & \text{if } j \geq j_r 
\end{cases}, \quad yp_t = \alpha_i \epsilon_j \eta w_t l(i, j, \eta), \quad \text{ess}_t = 0.5 \tau_{ss,t} \min\{yp_t, \bar{y}\}$$

- Constant marginal **capital tax** rate $\tau_{K,t}$ over capital income $r_t(a + Tr_t)$.

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Constant marginal capital tax rate $\tau_{K,t}$ over capital income $r_t(a + Tr_t)$.

Exogenous government spending $\{G_t\}_{t=1}^{\infty}$.

Exogenous consumption tax $\tau_{C,t}$.

Payroll tax Each worker pays $\tau_{ss,t} \min\{yp_t, \bar{y}\}$. Retires get $SS_t$ as social security income.

Social security separated from spending budget.
Technology

\[ C_t + K_{t+1} - (1 - \delta)K_t + G_t \leq ZK_T^\alpha N_T^{1-\alpha} \]
Setting

Everything else

- **Technology**

\[ C_t + K_{t+1} - (1 - \delta)K_t + G_t \leq ZK_T^\alpha N_T^{1-\alpha} \]

- **Market structure**
  - One period risk free asset. No insurance for idiosyncratic shocks and mortality.
  - Borrowing constraint.
Technology

\[ C_t + K_{t+1} - (1 - \delta)K_t + G_t \leq ZK_T^\alpha N_T^{1-\alpha} \]

Market structure
- One period risk free asset. No insurance for idiosyncratic shocks and mortality.
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Competitive equilibrium

Stationary equilibrium A CE in which per capita variables, functions, prices and policies are constant and aggregate variables grow at rate \( n \).
Setting

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- **Technology**
  \[ C_t + K_{t+1} - (1 - \delta)K_t + G_t \leq ZK_T^\alpha N_T^{1-\alpha} \]

- **Market structure**
  - One period risk free asset. No insurance for idiosyncratic shocks and mortality.
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- **Competitive equilibrium**
- **Stationary equilibrium** A CE in which per capita variables, functions, prices and policies are constant and aggregate variables grow at rate \( n \).

- **Social Welfare** Government maximize the ex-ante lifetime utility of an agent born in the stationary equilibrium, by choosing labor tax \( T(y) \) and capital tax \( \tau_k \).
Functional forms

- Preferences:
  \[
  u(c, 1 - l) = \frac{(c^\gamma(1 - l)^{1-\gamma})^{1-\sigma}}{1 - \sigma}
  \]  

- Alternative preferences:
  \[
  u(c, 1 - l) = \frac{c^{1-\sigma_1}}{1 - \sigma_1} + \frac{(1 - l)^{1-\sigma_2}}{1 - \sigma_2}
  \]  

- Progressive tax:
  \[
  T^{GS}(y; \kappa_0, \kappa_1, \kappa_2) = \kappa_0(y - (y^{-\kappa_1} + \kappa_2)^{-1/\kappa_1})
  \]  

(proposed by Gouveia and Strauss 1994) \(\kappa_0\) controls the level, \(\kappa_1\) the progressivity and \(\kappa_2\) moves to ensure budget balance.
### Table 1—Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Demographics</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Retirement age $j_r$</td>
<td>46 (65)</td>
<td>Compulsory retirement (assumed)</td>
</tr>
<tr>
<td>Maximum age $J$</td>
<td>81 (100)</td>
<td>Certain death (assumed)</td>
</tr>
<tr>
<td>Survival probability $\psi_j$</td>
<td>Bell and Miller (2002)</td>
<td>Data</td>
</tr>
<tr>
<td>Population growth $n$</td>
<td>0.011</td>
<td>Data</td>
</tr>
<tr>
<td><strong>Preferences</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discount factor $\beta$</td>
<td>1.001</td>
<td>$K/Y = 2.7$</td>
</tr>
<tr>
<td>Risk aversion $\sigma$</td>
<td>4.0</td>
<td>$IES = 0.5$</td>
</tr>
<tr>
<td>Consumption share $\gamma$</td>
<td>0.377</td>
<td>Average hours $= \frac{1}{3}$</td>
</tr>
<tr>
<td><strong>Labor productivity process</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variance types $\sigma_\alpha^2$</td>
<td>0.14</td>
<td>$\text{var}(y_{22})$</td>
</tr>
<tr>
<td>Persistence $\rho$</td>
<td>0.98</td>
<td>Linear increase in $\text{var}(y_j)$</td>
</tr>
<tr>
<td>Variance shock $\sigma_\eta^2$</td>
<td>0.0289</td>
<td>$\text{var}(y_{60})$</td>
</tr>
<tr>
<td><strong>Technology</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capital share $\alpha$</td>
<td>0.36</td>
<td>Data</td>
</tr>
<tr>
<td>Depreciation rate $\delta$</td>
<td>0.0833</td>
<td>$I/Y = 0.255$</td>
</tr>
<tr>
<td>Scale parameter $Z$</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td><strong>Government policy</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption tax $\tau_c$</td>
<td>0.05</td>
<td>Mendoza, Razin, and Tesar (1994)</td>
</tr>
<tr>
<td>Marginal tax $\kappa_0$</td>
<td>0.258</td>
<td>Gouveia and Strauss (1994)</td>
</tr>
<tr>
<td>Tax progressivity $\kappa_1$</td>
<td>0.768</td>
<td>Gouveia and Strauss (1994)</td>
</tr>
<tr>
<td>Payroll tax $\tau_{ss}$</td>
<td>0.124</td>
<td>Data</td>
</tr>
</tbody>
</table>
Results

- Optimal taxes:
  - $\tau_k = 0.36$.
  - $\kappa_0 = 0.23$, $\kappa_1 = 0.7$: flat tax with 24% marginal rate and $7200$ deduction (with average income of $42\,000$).
Results

- Optimal taxes:
  - \( \tau_k = 0.36 \).
  - \( \kappa_0 = 0.23, \kappa_1 = 0.7 \): flat tax with 24% marginal rate and $7200 deduction (with average income of $42\,000).

- Comparison with benchmark (US tax with estimated \( \kappa' \)'s for the sum of capital an labor income)

<table>
<thead>
<tr>
<th>Table 2—Changes in Aggregate Variables in the Optimal Tax System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
</tr>
<tr>
<td>-----------------------------------------</td>
</tr>
<tr>
<td>Average hours worked</td>
</tr>
<tr>
<td>Total labor supply ( N )</td>
</tr>
<tr>
<td>Capital stock ( K )</td>
</tr>
<tr>
<td>Output ( Y )</td>
</tr>
<tr>
<td>Aggregate consumption ( C )</td>
</tr>
<tr>
<td>( CEV )</td>
</tr>
</tbody>
</table>

- CEV: increase in consumption at all ages, all states, keeping labor supply unchanged that generates the welfare change.
Results

- **Decomposition:**

<table>
<thead>
<tr>
<th>Total change (in percent)</th>
<th>1.33</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1.29</td>
</tr>
<tr>
<td>Level</td>
<td>-1.63</td>
</tr>
<tr>
<td>Distribution</td>
<td>2.97</td>
</tr>
<tr>
<td>Leisure</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>0.04</td>
</tr>
<tr>
<td>Level</td>
<td>0.41</td>
</tr>
<tr>
<td>Distribution</td>
<td>-0.37</td>
</tr>
</tbody>
</table>

- Drivers are a consumption redistribution across types and an increase in leisure.
Results
Life cycle profiles

![Graph 1: Assets](image1.png)

- **Assets**
  - Type 1 and Type 2
  - Benchmark and Optimal

![Graph 2: Labor supply](image2.png)

- **Labor supply**
  - Type 1 and Type 2
  - Benchmark and Optimal

![Graph 3: Consumption](image3.png)

- **Consumption**
  - Type 1 and Type 2
  - Benchmark and Optimal

![Graph 4: Total income taxes paid](image4.png)

- **Total income taxes paid**
  - Type 1 and Type 2
  - Benchmark and Optimal
Without endogenous labor, there is no case for capital tax.

With progressive $\tau^l$, we want to subsidize capital.

Adding BC and shocks does not provide a rationale for capital taxes when a progressive labor tax is available (contrary to Imrohoroglu(98)).
Results

Effect of life-cycle profile

Table 4—Summary of Quantitative Results

<table>
<thead>
<tr>
<th>Model</th>
<th>End. lab.</th>
<th>BC</th>
<th>Type</th>
<th>Idio.</th>
<th>Life cycle</th>
<th>$\beta$</th>
<th>$r$</th>
<th>$\tau_k$</th>
<th>$\tau_l$</th>
<th>Prog.</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>0.983</td>
<td>4.5</td>
<td>10</td>
<td>19</td>
<td>No</td>
</tr>
<tr>
<td>M2</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>1.001</td>
<td>3.2</td>
<td>-24</td>
<td>100</td>
<td>Yes</td>
</tr>
<tr>
<td>M3</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>1.001</td>
<td>4.3</td>
<td>-34</td>
<td>100</td>
<td>Yes</td>
</tr>
<tr>
<td>M4</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>0.979</td>
<td>4.7</td>
<td>20</td>
<td>17</td>
<td>No</td>
</tr>
<tr>
<td>M5</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>1.009</td>
<td>5.6</td>
<td>34</td>
<td>14</td>
<td>No</td>
</tr>
<tr>
<td>M6</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>1.009</td>
<td>5.2</td>
<td>32</td>
<td>18</td>
<td>Yes</td>
</tr>
<tr>
<td>M7</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>1.005</td>
<td>5.6</td>
<td>35</td>
<td>23</td>
<td>Yes</td>
</tr>
<tr>
<td>Bench</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>1.001</td>
<td>5.6</td>
<td>36</td>
<td>23</td>
<td>Yes</td>
</tr>
</tbody>
</table>

- Life cycle (mortality risk, wage profile, social security) affects significantly the size of $\tau_k$. Erosa, Gervais (2002)
- It is optimal to condition the tax code on age. If it is not possible, taxing capital allows for less labor taxes when old.
Fixed effects and idiosyncratic shocks matter for progressivity of labor tax, but not for capital tax.
Borrowing constraints do not seem to matter much.
- Progressive labor tax allows to tax the rich more and
- Presence of idiosyncratic shock force young to make precautionary savings early so few individuals are constrained.
Sensitivity analysis

- Different preferences: different elasticity of labor supply. Qualitatively the same.
- Progressivity of capital tax: not used.
- Government debt: gov. can lower taxes

### Table 6—Government Debt and the Optimal Tax System

<table>
<thead>
<tr>
<th>Debt/GDP ratio</th>
<th>$\tau_k$</th>
<th>$\tau_l$</th>
<th>$d_l$</th>
<th>$r$</th>
<th>$w$</th>
<th>$K$</th>
<th>Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.00</td>
<td>24</td>
<td>20</td>
<td>$13,400$</td>
<td>3.43</td>
<td>1.113</td>
<td>1.364</td>
<td>1.019</td>
</tr>
<tr>
<td>-0.20</td>
<td>39</td>
<td>20</td>
<td>$10,800$</td>
<td>5.29</td>
<td>1.025</td>
<td>1.073</td>
<td>1.004</td>
</tr>
<tr>
<td>0.00 (benchmark)</td>
<td>43</td>
<td>20</td>
<td>$10,200$</td>
<td>5.59</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>0.20</td>
<td>43</td>
<td>20</td>
<td>$9,500$</td>
<td>6.37</td>
<td>0.982</td>
<td>0.938</td>
<td>0.985</td>
</tr>
<tr>
<td>1.00</td>
<td>55</td>
<td>21</td>
<td>$4,000$</td>
<td>9.33</td>
<td>0.885</td>
<td>0.703</td>
<td>0.984</td>
</tr>
</tbody>
</table>
Conclusions

- Quantitatively characterize optimal capital and progressive labor taxes in a life-cycle economy with incomplete markets.
- The key drivers of non-zero capital tax are: 1. endogenous labor supply and 2. life-cycle elements.
- Altruism between generations?
DEFINITION 1: Given a sequence of government expenditures \( \{G_t\}_{t=1}^{\infty} \) and consumption tax rates \( \{\tau_{c,t}\}_{t=1}^{\infty} \) and initial conditions \( K_1 \) and \( \Phi_1 \), a competitive equilibrium is a sequence of functions for the household, \( \{v_t, c_t, a'_t, l_t\}_{t=1}^{\infty} \), production plans for the firm, \( \{N_t, K_t\}_{t=1}^{\infty} \), government labor income tax functions \( \{T_t: \mathbb{R}_+ \to \mathbb{R}_+\}_{t=1}^{\infty} \), capital income taxes, \( \{\tau_{K,t}\}_{t=1}^{\infty} \), social security taxes, \( \{\tau_{ss,t}\}_{t=1}^{\infty} \) and benefits, \( \{SS_t\}_{t=1}^{\infty} \), prices \( \{w_t, r_t\}_{t=1}^{\infty} \), transfers \( \{Tr_t\}_{t=1}^{\infty} \), and measures \( \{\Phi_t\}_{t=1}^{\infty} \), with \( \Phi_t \in \mathcal{M} \) such that:

(i) Given prices, policies, transfers, and initial conditions, for each \( t \), \( v_t \) solves the functional equation (with \( c_t, a'_t \), and \( l_t \) as associated policy functions):

\[
\begin{align*}
    v_t(a, \eta, i, j) &= \max_{c, a', i, l} \{ u(c, 1 - l) + \beta \psi_j \int v_{t+1}(a', \eta', i, j + 1)Q(\eta, d\eta') \} \\
    \text{subject to} \\
    (1 + \tau_{c,t})c + a' &= w_t e_j \alpha_i \eta l - \tau_{ss,t} \min \{w_t e_j \alpha_i \eta l, \bar{y}\} + (1 + r_t(1 - \tau_{K,t}))(a + Tr_t) - T_t[y_t].
\end{align*}
\]

\[\text{for } j < j_r,\]

\[
(1 + \tau_{c,t})c + a' = SS_t + (1 + r_t(1 - \tau_{K,t}))(a + Tr_t),
\]

\[\text{for } j \geq j_r,\]

\[
a' \geq 0, \quad c \geq 0, \quad 0 \leq l \leq 1.
\]
(ii) Prices $w_t$ and $r_t$ satisfy

\[
    r_t = \alpha Z \left( \frac{N_t}{K_t} \right)^{1-\alpha} - \delta,
\]

\[
    w_t = (1 - \alpha) Z \left( \frac{K_t}{N_t} \right)^\alpha.
\]

(iii) The social security policies satisfy

\[
    \tau_{ss,t} \int \min \{w_t \alpha \varepsilon_j \eta_i, \bar{y}\} \Phi_t(da \times d\eta \times di \times dj) = SS_t \int \Phi_t(da \times d\eta \times di \times \{j_r, \ldots, J\}).
\]

(iv) Transfers are given by

\[
    Tr_{t+1} \int \Phi_{t+1}(da \times d\eta \times di \times dj) = \int (1 - \psi_j) a_t'(a, \eta, i, j) \Phi_t(da \times d\eta \times di \times dj).
\]
(v) **Government budget balance:**

\[
G_t = \int \tau_{K,t} r_i(a + Tr_i) \Phi_t(da \times d\eta \times di \times dj) + \int T_t[y_t] \Phi_t(da \times d\eta \times di \times dj) \\
+ \tau_{c,t} \int c_t(a, \eta, i, j) \Phi_t(da \times d\eta \times di \times dj).
\]

(vi) **Market clearing:**

\[
K_t = \int a \Phi_t(da \times d\eta \times di \times dj),
\]

\[
N_t = \int \varepsilon_{j' \alpha_i \eta_i} \Phi_t(da \times d\eta \times di \times dj),
\]

\[
\int c_t(a, \eta, i, j) \Phi_t(da \times d\eta \times di \times dj) + K_{t+1} + G_t = ZK_t^\alpha N_t^{1-\alpha} + (1 - \delta)K_t.
\]
(vii) Law of Motion:

\[
\Phi_{t+1} = H_t(\Phi_t),
\]

where the function \(H_t : \mathcal{M} \to \mathcal{M}\) can be written explicitly as follows. For all \(J\) such that \(1 \notin J\):

\[
\Phi_{t+1}(\mathcal{A} \times \mathcal{E} \times \mathcal{I} \times J) = \int P_t((a, \eta, i, j); \mathcal{A} \times \mathcal{E} \times \mathcal{I} \times J) \Phi_t(da \times d\eta \times di \times dj),
\]

where

\[
P_t((a, \eta, i, j); \mathcal{A} \times \mathcal{E} \times \mathcal{I} \times J) = \begin{cases} Q(\epsilon, \mathcal{E}) \psi_j & \text{if } a'_t(a, \eta, i, j) \in \mathcal{A}, \ i \in \mathcal{I}, \ j + 1 \in J \\ 0 & \text{else} \end{cases}.
\]

For \(J = \{1\},

\[
\Phi_{t+1}(\mathcal{A} \times \mathcal{E} \times \mathcal{I} \times \{1\}) = (1 + n')^{\frac{\sum_{i \in I} P_i}{}} \text{ if } 0 \in \mathcal{A}, \ \bar{\eta} \in \mathcal{E}
\]