people face significant lifespan risk in old age: 20 percent of 65-year olds will die before age of 75; 20 percent will live beyond age 90

life annuities: can provide insurance against this risk by converting a stock of wealth into lifelong income stream
Motivation

- People face significant lifespan risk in old age: 20 percent of 65-year-olds will die before age of 75; 20 percent will live beyond age 90.

- Life annuities: can provide insurance against this risk by converting a stock of wealth into lifelong income stream.

- Example: 2 periods, 50% survive into period 2, initial wealth = 90.

- No annuities: $c_1 = 60 / c_2 = 30$; with annuity: $c_1 = 60 / c_2 = 60$.

⇒ “annuity puzzle”
Motivation

- people face significant lifespan risk in old age: 20 percent of 65-year-olds will die before age of 75; 20 percent will live beyond age 90
- life annuities: can provide insurance against this risk by converting a stock of wealth into lifelong income stream
- example: 2 periods, 50% survive into period 2, initial wealth = 90
- no annuities: $c_1 = 60 / c_2 = 30$; with annuity: $c_1 = 60 / c_2 = 60$
- life-cycle models predict large welfare gains; but: virtually non-existent in retirees’ portfolios in most countries! ⇒ “annuity puzzle”
To what extent can bequest motives explain lack of annuitization?

- intuition: annuitized wealth is not bequeathable
- literature: beq motives can explain lack of full annuitization; but cannot explain why most people do not annuitize any of their wealth!
To what extent can bequest motives explain lack of annuitization?

- intuition: annuitized wealth is not bequeathable
- literature: beq motives can explain lack of full annuitization; but cannot explain why most people do not annuitize any of their wealth!
- main question: how strong do bequest motives have to be in order to eliminate purchases of life annuities?
- main result: *at available rates*, moderate bequest motive are enough to eliminate even partial annuitization – much weaker than those required if actuarially fair annuities are available
Lifespan Risk and Annuities

- individual with some initial wealth: optimal consumption/saving plan?
- saving decision (without bequest motive): future consumption vs. risk of incidental bequest \(\Rightarrow\) annuities relax this trade-off
- no annuities: \(c_1 = 60 / c_2 = 30\); with annuity: \(c_1 = 60 / c_2 = 60\)
Lifespan Risk and Annuities

- individual with some initial wealth: optimal consumption/saving plan?
- saving decision (without bequest motive): future consumption vs. risk of incidental bequest $\Rightarrow$ annuities relax this trade-off
- no annuities: $c_1 = 60 / c_2 = 30$; with annuity: $c_1 = 60 / c_2 = 60$
- with bequest motive: partial annuitization – if actuarially fair, set aside assets to bequeath and annuitize all future consumption
- US market: actuarially unfair annuities (10 – 15 percent loads)
The Model

- life-cycle model of retirement: from age 65 to death
- at age 65, once-and-for-all choice: how much wealth to annuitize?
- choose consumption path to maximize:

\[
EU = \sum_{t=65}^{T} \beta^{t-65} S_t u(c_t) + \sum_{t=66}^{T+1} p_t v \left( \frac{b_t}{(1 + r)^{t-65}} \right)
\]

where \( S_t \) is the probability of surviving to at least age \( t \), and \( p_t = S_t - S_{t-1} \) is the probability of dying between age \( t - 1 \) and age \( t \)
The Model

\[
\max \quad EU = \sum_{t=65}^{T} \beta^{t-65} S_t u(c_t) + \sum_{t=66}^{T+1} p_t v \left( \frac{b_t}{(1 + r)^{t-65}} \right)
\]

subject to, \quad \forall t \in \{66, 67, \ldots, T + 1\}

\[
b_t = (1 + r)^{t-65}(N - \Pi) + \sum_{s=1}^{t-65} (1 + r)^s \left[ (y_{pre} + y_{ann}) - c_{t-s} \right] \geq 0
\]

\[N: \text{initial (age 65) non-annuity wealth}; \quad \Pi \in [0, N]: \text{annuitized portion}\]

\[y_{pre}: \text{constant pre-existing income (Social Security, employer pensions, ...)}\]
The Model

\[
\text{max } EU = \sum_{t=65}^{T} \beta^{t-65} S_t u(c_t) + \sum_{t=66}^{T+1} p_t v \left( \frac{b_t}{(1 + r)^{t-65}} \right)
\]

subject to

\[
b_t = (1 + r)^{t-65} (N - \Pi) - \sum_{s=1}^{t-65} (1 + r)^s \left[ c_{t-s} - (y_{pre} + y_{ann}) \right] \geq 0
\]

and

\[
\Pi(y_{ann}, \lambda) = \sum_{t=65}^{T} \frac{S_t y_{ann}}{(1 + r)^{t-65}} / (1 - \lambda)
\]
Parameterization

- $\beta = \frac{1}{1+r} = \frac{1}{1.03}$

- utility from consumption is CRRA with $\sigma = 2$

- utility from bequests is

$$v(b) = a \cdot \left( \sum_{i=1}^{\infty} \beta^{i-1} \right) \left( y_{pre} + \frac{b}{\sum_{i=1}^{\infty} (1+r)^{-(i-1)}} \right)^{1-\sigma}$$

where $a \geq 0$ is the degree of altruism

- half of total wealth is already annuitized

- mortality probabilities from US Social Security Administr. life tables
Welfare Gains from Annuities

Fraction of wealth bequeathed with fair annuities, $b^*/N$
(for cases with bequest motives)

Altruism, $a$
(for cases with bequest motives)
To summarize:

- People without bequest motive willing to pay 25.3 percent of their non-annuity wealth for access to actuarially fair annuities.
- Annuities with 10 percent load: still willing to pay 17.1 percent.
- People with bequest motive gain much less ⇒ e.g. someone who would bequeath 1/5 of total wealth, willing to pay only 3.7 percent (zero load) and 0 percent (10% load).
- Beq motives signif. increase sensitivity of annuity purchases to loads.

Why do bequest motives reduce annuity gains so much?
Welfare Gains Decomposed

WTP/N

- **Trade bequests for consumption**
- **Smooth consumption**
- **Insure bequests**

Fraction of wealth bequeathed with fair annuities, \( b^*/N \)
(for cases with bequest motives)
Aggregate Demand for Annuities:

- simulate model with various bequest motives estimated in the literature
- 2006 wave of Health and Retirement Study (HRS): 65-69-year-old singles
- demand for annuity: function of age, wealth and income; 10-percent load

<table>
<thead>
<tr>
<th>Table 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulated and empirical annuity ownership rates among 65–69-year-old single retirees in the Health and Retirement Study.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bequest motive</th>
<th>Annuity ownership rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>69.1% 67.6% 61.0%</td>
</tr>
<tr>
<td>De Nardi et al. (2010)</td>
<td>60.6% 62.3% 56.4%</td>
</tr>
<tr>
<td>Hurd and Smith (2002)</td>
<td>24.7% 15.6% 11.7%</td>
</tr>
<tr>
<td>Kopczuk and Lupton (2007)</td>
<td>21.7% 13.6% 10.6%</td>
</tr>
<tr>
<td>De Nardi (2004)</td>
<td>19.0% 18.5% 15.4%</td>
</tr>
<tr>
<td>Lockwood (2010)</td>
<td>16.8% 16.8% 12.6%</td>
</tr>
<tr>
<td>Ameriks et al. (2009)</td>
<td>0.3% 0.0% 0.0%</td>
</tr>
</tbody>
</table>

Average ownership, middle four bequest motives:
- 20.5% 16.1% 12.6% 4.4%

Empirical ownership rate:
- 3.6%

Model
- Medical spending: No Yes Yes Yes
- Minimum annuity size: $0 $0 $20,000 $20,000
- Fixed cost: $0 $0 $2,000 $2,000

Note. The simulation uses annuities with a ten percent load, typical of the U.S. private market.
### Table 2
Bequest motives used to calculate the aggregate demand for annuities.

<table>
<thead>
<tr>
<th>Paper</th>
<th>Bequest motive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ameriks et al. (2009)</td>
<td>$v(b) = \frac{\omega}{1-\sigma} \left( \phi + \frac{b}{\omega} \right)^{1-\sigma}$</td>
</tr>
<tr>
<td>De Nardi (2004)</td>
<td>$v(b) = \phi_1 \left( 1 + \frac{b}{\phi_2} \right)^{1-\sigma}$</td>
</tr>
<tr>
<td>De Nardi et al. (2010)</td>
<td>$v(b) = \theta \left( \frac{k+b}{1-\sigma} \right)^{1-\sigma}$</td>
</tr>
<tr>
<td>Hurd and Smith (2002)$^a$</td>
<td>$v(b) = \theta b$</td>
</tr>
<tr>
<td>Kopczuk and Lupton (2007)</td>
<td>$v(b) = \theta b$</td>
</tr>
<tr>
<td>Lockwood (2010)</td>
<td>$v(b) = \left( \frac{m}{1-m} \right)^{\sigma} \left( \frac{m}{1-m} (c_0 + b)^{1-\sigma} \right)$</td>
</tr>
</tbody>
</table>
Table 3
Summary statistics of the sample of 65–69-year-old single retirees used to estimate the demand for annuities.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female</td>
<td>0.73</td>
</tr>
<tr>
<td>Age</td>
<td>67.1</td>
</tr>
<tr>
<td>Non-annuity wealth</td>
<td>$271,209</td>
</tr>
<tr>
<td>Income</td>
<td>$19,689</td>
</tr>
<tr>
<td>Any annuity</td>
<td>0.043</td>
</tr>
<tr>
<td>Life annuity</td>
<td>0.036</td>
</tr>
<tr>
<td>Have children</td>
<td>0.90</td>
</tr>
<tr>
<td>N</td>
<td>794</td>
</tr>
</tbody>
</table>