Understanding the evolution of the US wage distribution: a theoretical analysis

Fatih Guvenen\textsuperscript{1} Burhanettin Kuruscu\textsuperscript{2}

\textsuperscript{1}University of Minnesota

\textsuperscript{2}University of Toronto

\textit{Presented by Francisco Javier Rodríguez}

\textit{(Universidad Carlos III de Madrid)}
Overview

1. Introduction
   - Motivation and research question

2. The model
   - Main features
   - Details

3. Theoretical Analysis
   - Steady State
   - Skill biased technological change (SBTC)

4. Quantitative exploration

5. Conclusions
Changes in the US wage distribution between 1970 and 2000:


2. Changes in overall, between-group and within-group inequality.

3. Relatively small rise in consumption inequality versus income inequality.

Can a human capital model à la Becker (1964)/Ben-Porath (1967) account for these empirical facts?
Main characteristics

- Overlapping generations model.
- Individuals begin life with a fixed endowment of raw labour and can accumulate human capital.
- Raw labour and human capital earn separate wages determined competitively in the market.
- Investment in human capital takes place on the job until certain threshold.
- Skills are general (not firm specific).
- Markets are complete and individuals are able to borrow from the rest of the world at the international interest rate.
- Individuals differ in their ability to accumulate human capital.
- Skill biased technological change as driving force for changes.
Individuals

Dynamic problem

\[
\max_{\{i_{js}\}_{s=1}^S} \left[ \sum_{s=1}^S \left( \frac{1}{1 + r} \right)^{s-1} \left( P_L \ell + P_H h_{js} \right) (1 - i_{js}) \right]
\]
subject to
\[
\begin{align*}
    h_{j, s+1} &= h_{js} + Q_{js} \\
    Q_{js} &= \tilde{A}_j \left( (\theta_L \ell + \theta_H h_{js}) i_{js} \right)^\alpha \\
    h_{j0} &= 0
\end{align*}
\]
Production

Aggregate factors

\[
L^\text{net} = \sum_{s=1}^{S} \mu(s) \int_j \ell (1 - i_{js}) \, dj \quad \text{and} \quad H^\text{net} = \sum_{s=1}^{S} \mu(s) \int_j h_{js} (1 - i_{js}) \, dj
\]

Production function

\[
Y = Z (\theta_L L^\text{net} + \theta_H H^\text{net})
\]

Prices

\[
P_H = \frac{\partial Y}{\partial H^\text{net}} = Z \theta_H \quad \text{and} \quad P_L = \frac{\partial Y}{\partial L^\text{net}} = Z \theta_L
\]
Rewriting the individual’s problem

Write \( C_j(Q_{js}) \equiv (\theta_L \ell + \theta_H h_{js}) i_{js} = \left( \frac{Q_{js}}{A_j} \right)^{\frac{1}{\alpha}} \), then

New individual’s problem

\[
\max_{\{Q_{js}\}_{s=1}^S} \left[ \sum_{s=1}^S \left( \frac{1}{1 + r} \right)^{s-1} (\theta_L \ell + \theta_H h_{js} - C_j(Q_{js})) \right]
\]

subject to

\[
h_{j,s+1} = h_{js} + Q_{js}
\]

\[
h_{j0} = 0
\]

Optimality condition

\[
C_j'(Q_{js}) = \left\{ \frac{\theta_H (t + 1)}{1 + r} + \frac{\theta_H (t + 2)}{(1 + r)^2} + \ldots + \frac{\theta_H (t + S - s - 1)}{(1 + r)^{S-s-1}} \right\}
\]
Fanning of wage profiles

Figure: Effect of the price of human capital on life-cycle wage profiles
Slight change in the framework for analytical tractability: *perpetual youth* version of overlapping generations model. Now, individuals don’t die at a given date, but face a probability of death of \((1 - \delta)\) in every period. Normalize population size to 1 and assume that every period a cohort of measure \((1 - \delta)\) is born. This way \(s\) is not a state variable anymore. Moreover, assume 
\[ r = \frac{1}{(\delta \beta)} - 1 \]
so that individuals will choose a constant consumption path over their life cycle.
Optimality conditions

New capital

\[ Q_j = \tilde{A}_j^{\frac{1}{1-\alpha}} \left( \frac{\alpha \delta \beta}{1 - \beta \delta} \theta_H \right)^{\frac{\alpha}{1-\alpha}} \]

Human capital

\[ h_{js} = Q (s - 1) \]

Investment

\[ i_{js} = \frac{C (Q_j)}{\theta_L \ell + \theta_H h_{js}} = \frac{\alpha \delta \beta}{1 - \delta \beta} \left( \frac{\theta_L \ell}{\theta_H Q_j} + (s - 1) \right)^{-1} \]
Averages

Average investment

\[ \bar{Q} \equiv \sum_{s=1}^{\infty} \mu(s) \int Q_j \, dj = \left( \frac{\alpha \delta \beta}{1 - \delta \beta} \theta_H \right)^{\frac{\alpha}{1 - \alpha}} \mathbb{E} \left[ \tilde{A}_j^{1 - \frac{\alpha}{\delta}} \right] \]

Average cost of investment

\[ C(\bar{Q}) \equiv \sum_{s=1}^{\infty} \mu(s) \int C(Q_j) \, dj = \frac{\alpha \delta \beta}{1 - \delta \beta} \theta_H \bar{Q} \]

Average human capital

\[ H(\bar{Q}) \equiv \sum_{s=1}^{\infty} \mu(s) \int h_{js} \, dj = \sum_{s=1}^{\infty} \mu(s)(s - 1) \times \int h_{js} \, dj = \frac{\delta}{1 - \delta} \bar{Q} \]
More averages

**Average wage**

\[
\bar{w} \equiv \sum_{s=1}^{\infty} \mu(s) \int_{j} w_{js} d\mu = \left[ \theta_L \ell + \theta_H H(\bar{Q}) \right] - C(\bar{Q})
\]

\[
= \theta_L \ell + \left( \frac{\delta}{1 - \delta} - \frac{\alpha \delta \beta}{1 - \delta \beta} \right) \theta_H \bar{Q}
\]

**Average consumption**

\[
\bar{c} = \left[ \theta_L \ell + \frac{\delta \beta}{1 - \delta \beta} \theta_H \bar{Q} \right] - C(\bar{Q}) = \theta_L \ell + \left( (1 - \alpha) \frac{\delta \beta}{1 - \delta \beta} \right) \theta_H \bar{Q}
\]
Skill biased technological change

We now explore the short and long-run effects of a one time permanent increase in the price of human capital from $\theta_H$ to $\theta'_H$ on

- Average wages and productivity
- College premium
- Overall inequality
Slowdown in labour productivity and average wages

\[ \bar{w}_I = [\theta_L \ell + \theta_H H (\bar{Q})] - C (\bar{Q}) \]
\[ \bar{w}_{SR} = [\theta_L \ell + \theta'_H H (\bar{Q})] - C (\bar{Q}') \]
\[ \bar{w}_{LR} = [\theta_L \ell + \theta'_H H (\bar{Q}')] - C (\bar{Q}') \]

**Condition 1**

\[ \frac{\alpha \delta \beta}{(1 - \delta \beta)(1 - \alpha)} > \frac{\delta}{1 - \delta} \]

**Proposition**

*If condition 1 holds, in response to SBTC, the average wage*

1. *increases in the long run* (\( \bar{w}_{LR} > \bar{w}_I \))
2. *falls in the short run* (\( \bar{w}_{SR} < \bar{w}_I \))
Between-group wage inequality (College premium)

\[
\omega^*_I = \frac{\theta_L \ell + \theta_H H (\bar{Q}_c) - C (\bar{Q}_c)}{\theta_L \ell + \theta_H H (\bar{Q}_n) - C (\bar{Q}_n)}
\]

\[
\omega^*_{SR} = \frac{\theta_L \ell + \theta'_H H (\bar{Q}_c) - C (\bar{Q}'_c)}{\theta_L \ell + \theta'_H H (\bar{Q}_n) - C (\bar{Q}'_n)}
\]

\[
\omega^*_LR = \frac{\theta_L \ell + \theta'_H H (\bar{Q}'_c) - C (\bar{Q}'_c)}{\theta_L \ell + \theta'_H H (\bar{Q}'_n) - C (\bar{Q}'_n)}
\]

**Proposition**

*If condition 1 holds, in response to SBTC, the college premium*

1. *increases in the long run* \((\omega^*_LR > \omega^*_I)\)
2. *falls in the short run* \((\omega^*_{SR} < \omega^*_I)\)
College premium within age groups

\[ \omega_i^*(s) = \frac{\theta_L \ell + \theta_H \bar{Q}_c (s - 1) - C (\bar{Q}_c)}{\theta_L \ell + \theta_H \bar{Q}_n (s - 1) - C (\bar{Q}_n)} \]

**Proposition**

Define

\[ s = 1 + \frac{\alpha \delta \beta}{1 - \delta \beta} \quad \text{and} \quad \bar{s} = 1 + \frac{\alpha \delta \beta}{(1 - \delta \beta)(1 - \alpha)} \]

In response to SBTC, the college premium

1. increases in the long run \((\omega_{LR}^*(s) > \omega_i^*(s))\) if and only if \(s > \bar{s}\)
2. falls in the short run \((\omega_{SR}^*(s) < \omega_i^*(s))\) if and only if \(s < \bar{s}\)
Within group, overall and consumption inequality

**Corollary**

\[
\frac{\omega_{LR}(\Omega|s)}{\omega_{I}(\Omega|s)} \text{ is increasing in } \Omega \text{ when } s > s
\]

**Proposition**

In response to SBTC, wage inequality

1. increases in the long run \((CV_{LR}(w) > CV_{I}(w))\)
2. increases in the short run \((CV_{SR}(w) > CV_{I}(w))\)

**Proposition**

If \(\beta = 1\), in response to SBTC, wage inequality rises more than consumption inequality in the long run, that is

\[
CV_{LR}(w)^2 - CV_{I}(w)^2 > CV_{LR}(c)^2 - CV_{I}(c)^2
\]
Quantitative exercise

Simulate the response of the model to a one-time change in the growth rate of the skill price.

<table>
<thead>
<tr>
<th>Year</th>
<th>Log 90–50 Data</th>
<th>Model</th>
<th>Log 50–10 Data</th>
<th>Model</th>
<th>Log education premium Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>1975</td>
<td>1.1</td>
<td>3.8</td>
<td>1.6</td>
<td>2.3</td>
<td>−2.9</td>
<td>−1.6</td>
</tr>
<tr>
<td>1980</td>
<td>3.1</td>
<td>9.1</td>
<td>0.3</td>
<td>−0.1</td>
<td>−6.0</td>
<td>−1.9</td>
</tr>
<tr>
<td>1985</td>
<td>7.1</td>
<td>16.3</td>
<td>6.0</td>
<td>2.0</td>
<td>2.3</td>
<td>1.6</td>
</tr>
<tr>
<td>1990</td>
<td>9.0</td>
<td>25.1</td>
<td>10.2</td>
<td>4.6</td>
<td>8.4</td>
<td>9.0</td>
</tr>
<tr>
<td>1995</td>
<td>15.2</td>
<td>33.2</td>
<td>8.7</td>
<td>7.4</td>
<td>12.8</td>
<td>14.7</td>
</tr>
<tr>
<td>2000</td>
<td>18.7</td>
<td>36.9</td>
<td>9.7</td>
<td>9.2</td>
<td>18.1</td>
<td>18.0</td>
</tr>
</tbody>
</table>

Notes: For a given year $t$ shown in column 1, the reported statistics are calculated by averaging the values for year $t-1$, $t$, and $t+1$ to smooth out noise in the data. All statistics have been normalized to 0.0 in 1970, so the figures for year 2000 also represent the cumulative change from 1970 to 2000.
<table>
<thead>
<tr>
<th>Year</th>
<th>Residual variance (×100)</th>
<th>College grads</th>
<th>High-school grads</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>σ²(log(W)) × 100 within:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
<td>Data</td>
</tr>
<tr>
<td>1970</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>1975</td>
<td>0.5</td>
<td>0.6</td>
<td>0.0</td>
</tr>
<tr>
<td>1980</td>
<td>1.9</td>
<td>1.0</td>
<td>2.3</td>
</tr>
<tr>
<td>1985</td>
<td>4.6</td>
<td>1.8</td>
<td>4.0</td>
</tr>
<tr>
<td>1990</td>
<td>6.0</td>
<td>2.8</td>
<td>6.3</td>
</tr>
<tr>
<td>1995</td>
<td>7.7</td>
<td>3.6</td>
<td>9.3</td>
</tr>
<tr>
<td>2000</td>
<td>8.9</td>
<td>3.9</td>
<td>10.4</td>
</tr>
</tbody>
</table>
Conclusions

- Key element: interaction between SBTC and heterogeneity in ability to accumulate human capital.
- The model is consistent with the great changes in the wage distribution since the 1970's.
- Incomplete markets could improve the model’s performance by reducing cross-sectional wage inequality and increasing consumption inequality.
The End