Recurrent Hyperinflations and Learning
Albert Marcet and Juan P. Nicolini (AER, 2003)

Manuel M. Mosquera T.
UC3M

November 24, 2015
Motivation of Literature

- Expectations play a key role in macroeconomics (e.g. agents are forward-looking, forecasts of future inflation and output have a key role in monetary policy)
- Theories of expectation: Static (1930s), Adaptative (1960s), RE(from 1970s),...Learning.
- Standard assumption in theory of economic policy is RE.
- RE is ”too much”. **Bounded Rationality:**
  1. One possible form: Agents as smart as good economists (model agents as ”econometricians”). **True model is not known.**
  2. Learning process: formulate and estimate the model. With new data, reformulate and reestimate. **May or may not converge to RE.**
Motivation of Paper: Explaining Hyperinflations

- **Main:** Develop a model that accounts for the main features of the recurrent hyperinflations of the 1980’s and to study the policy recommendations that arise from it.

- **A side:** address the main critics to the use of boundedly rational models to match empirical observations (deal with Lucas Critique!)
Facts: relationship between money and prices

- Quantitative Theory for the long-run.
- Simple story about hypers: high seignorage-high inflation rates
- Fails to explain time series of very high inflation countries.
- Evidence on Recurrent Hyperinflations
  1. Recurrence. Long periods of stable moderate inflation. Short periods extremely high
  2. Exchange rate rules (ERR) stop hyperinflations in short-run. Eventually new episodes
  3. During hyperinflations seignorage and inflation are not highly correlated
  4. Average inflation and seignorage are strongly correlated across countries. Hyperinflations occur only in countries where seignorage is high on average.
Facts: South American Countries during 1980’s

**Figure 1. Monthly Inflation Rate (in logs)**

- **Argentina**
- **Bolivia**
- **Brazil**
- **Peru**
The Model

Standard Hyperinflation Assumptions

- OLG monetary model of a small open economy (in the spirit of Sargent and Wallace 1987).
- Demand for real balances:
  \[
  \frac{M_t^d}{P_t} = \phi - \gamma \phi \frac{P_{t+1}^e}{P_t}
  \]
  if \(1 - \gamma \left( \frac{P_{t+1}^e}{P_t} \right) > 0\) and 0 o.w.
- Money supply is driven by the need of finance seignorage. Also, government is concern with level of inflation. When inflation gets out of control adoption of ERR.

If no ERR is in place at \(t\) (floating regime):

\[
M_t = M_{t-1} + d_t P_t
\]

where \(d_t\) is i.i.d.

Both equations plus a hypothesis for expectations formation determine equilibrium values \(\{M_t, P_t\}\).
If ERR is in place at \( t \) (crawling peg or fixed exchange rate). Object is to stabilize inflation:

\[
\frac{P_t^f}{P_{t-1}^f} \frac{e_t}{e_{t-1}} = \beta = \frac{P_t}{P_{t-1}}
\]

Government buys or sells foreign reserves as needed to meet a target exchange rate s.t. satisfies the above equation. Where \( \beta \) is targeted inflation and the last inequality follows from PPP assumption.

- Money supply is now given by:
  \[
  M_t = M_{t-1} + d_t P_t + e_t(R_t - R_{t-1})
  \]

Equilibrium: \( P_t \) by PPP and \( M_t \) by supply and demand, given expectations.

- To close the model. Government chooses exchange rate regimes to satisfy:

\[
\frac{P_t}{P_{t-1}} \leq \beta_u
\]
As long as seignorage is not too high:

- Two stationary RE equilibrium with constant expected inflation levels (high-inflation and low-inflation equilibria)
- A continuum of bubble equilibria that converge to high-inflation equilibrium (rational bubble as hyperinflation)
- Marcet and Sargent (1989): Under LSL low-inflation equilibrium is locally stable, but high-inflation equilibrium is always unstable. Hence, bubble equilibria cannot be learned by agents.
- RE equilibrium does not match facts.
Learning

General Setup

• An economic model satisfies:
  \[ x_t = g(x_{t-1}, x^e_{t+1}, \xi_t, \eta) \]
  
  \(x\) contains all variables of the model, \(\xi\) is an exogenous shock, \(\eta\) is a vector of parameters.

• Agents expectations are given by:
  \[ x^e_{t+1} = z(\beta_t(\mu), x_t) \]
  
  \(\beta_t(\mu)\) statistics inferred from past data, \(z\) is a forecast function.

• Statistics are generated by:
  \[ \beta_t(\mu) = f(\beta_{t-1}(\mu), x_t, \mu) \]
  
  \(f\) is a learning mechanism and \(\mu\) learning parameters

• Three equations determine equilibrium sequence for given parameters
Main critics to literature: (i) too many degrees of freedom, (ii) irrational expectations and (iii) expectations exogenous to the model.

Bounded Rationality means "upper bound" on rationality. AN propose lower bound on rationality:

1. **Asymptotic Rationality (AR).** Expectations given by \((z, f, \mu)\) satisfy AR in the model \((g, \eta)\) if for all \(\epsilon > 0\), \(\pi^{\epsilon, T} \to 1\) as \(T \to \infty\). Predictions errors made by agents almost as small as prediction errors made with true conditional expectation.

2. **Epsilon-Delta Rationality (EDR).** \(\pi^{\epsilon, T} \geq 1 - \delta\). Agents does not change learning-scheme.

3. **Internal Consistency (IC).** Predictions errors made by agents with learning parameters are as least as good predictions errors with alternative parameters.
Learning Equilibrium

- Learning mechanism given by:

\[ \beta_t = \beta_{t-1} + \frac{1}{\alpha_t} \left( \frac{P_{t-1}}{P_{t-2}} - \beta_{t-1} \right) \]

\( \frac{1}{\alpha_t} \) is the gain of a stochastic approximation algorithm. One assumption is:

\[ \alpha_t = \alpha_{t-1} + 1 \quad \alpha_0 = 1 \]

. In this case perceived inflation is just sample mean of past inflation (OLS)

\[ \beta_{t+1} = \frac{1}{t} \sum_{i=1}^{t} \frac{P_i}{P_{i-1}} \]

. Other possible assumption: \( \alpha_t = \hat{\alpha} > 1 \)

\[ \beta_{t+1} = \frac{1}{\hat{\alpha}} \sum_{i=0}^{t} (1 - \frac{1}{\hat{\alpha}})^i \frac{P_{t-i}}{P_{t-i-1}} \]

. "Tracking". Past discounted at a geometric rate. More importance to recent events.
Learning Equilibrium

- AN specify a learning mechanism that mix both alternatives. OLS in stable periods and Tracking in unstable periods.

\[ \alpha_t = \alpha_{t-1} + 1 \]

if \( \left| \frac{P_{t-1}}{P_{t-2}} - \beta_{t-1} \right| / \beta_{t-1} < v \) or \( \alpha_t = \hat{\alpha} \) o.w.

So,

\[ \frac{P_t}{P_{t-1}} = H(\beta_t, \beta_{t-1}, d_t) \]

where,

\[ H(\beta_t, \beta_{t-1}, d_t) = \frac{1 - \gamma \beta_{t-1}}{1 - \gamma \beta_t - d_t / \phi} \]

if \( 0 < \frac{1 - \gamma \beta_{t-1}}{1 - \gamma \beta_t - d_t / \phi} < \beta_u \) or \( 1 - \gamma \beta_{t-1} > 0 \). Otherwise:

\[ H(\beta_t, \beta_{t-1}, d_t) = \bar{\beta} \]
Actual inflation as a function of perceived inflation
Learning Equilibrium

- Equilibrium is an stochastic process \( \{P_t, M_t, \beta_t\} \) and \( \hat{\alpha} \) s.t.:
  - Given \( \hat{\alpha} \), \( \{P_t, M_t, \beta_t\} \) satisfy: equations for \( x_{t+1}^e \), \( f \), \( \alpha \) and prices.
  - \( \hat{\alpha} \) is IC

- Characterization of solution by simulation
  - Parameters: \( \phi = 0.37 \), \( \gamma = 0.4 \), \( \nu = 10\% \), \( \beta_u = 50 \) (means 5000\% of inflation), \( E(d_t) = 0.05 \).
  - Initial beliefs equal to low-inflation RE stationary equilibrium.
Learning Equilibrium

Figure 3. Simulation of the (log) Inflation
Learning Equilibrium

<table>
<thead>
<tr>
<th>Table 1—Probabilities of Hyperinflations Occurring in Ten Years for Different Deficit Means and Learning Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deficit mean = 4.5 percent</td>
</tr>
<tr>
<td>Alpha</td>
</tr>
<tr>
<td>0.5</td>
</tr>
<tr>
<td>0.4</td>
</tr>
<tr>
<td>0.3</td>
</tr>
<tr>
<td>0.2</td>
</tr>
<tr>
<td>0.1</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>Deficit mean = 4.7 percent</td>
</tr>
<tr>
<td>Alpha</td>
</tr>
<tr>
<td>0.4</td>
</tr>
<tr>
<td>0.3</td>
</tr>
<tr>
<td>0.2</td>
</tr>
<tr>
<td>0.1</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>Deficit mean = 4.9 percent</td>
</tr>
<tr>
<td>Alpha</td>
</tr>
<tr>
<td>0.2</td>
</tr>
<tr>
<td>0.1</td>
</tr>
<tr>
<td>0</td>
</tr>
</tbody>
</table>
Conclusion

• Consistent with main stylized facts
• Consistent with both policy recommendations: ”orthodox” and ”heterodox”
• Quasi-rational learning rule is more successful than RE, and can skip main critics of literature.
• When there is hyperinflation, learning forecast can do better than true conditional expectation.
• With adequate assumptions on lower bounds, irrationality can be productive.