A Matching Model with Endogenous Skill Requirements

James Albrecht and Susan Vroman
1. Introduction

- An increase in wage inequality in the United States and in the United Kingdom and in unemployment in continental Europe have taken place over the past two decades. (See Bound and Johnson (1992) and Katz and Murphy (1992) for wage inequality in the United States and Machin (1996) for the United Kingdom. OECD (1994) documents unemployment trends).

- Skill-biased technical change is a common explanation for both of these trends. Different arguments:

  - Technical change has shifted demand in favor of high-skill workers (Katz and Murphy (1992) and Krugman (1994)).
  
  - Increased trade with developing countries lies behind the relative demand shift (Woods (1994)).
  
  - An increase in the supply of skilled labor can lead to a change in technology or in the mix of job types (Acemoglu (1998) and Beaudry and Green (1999)). Thus, "supply creates its own demand".
This paper

- Construct a labor market matching model that incorporates both skill differences across workers and differences in skill requirements across jobs.

- Examine several explanations that have been given for the increase in wage inequality in the United States and in the United Kingdom and in unemployment in continental Europe that have taken place over the past two decades.
2. THE MODEL

- Continuous time.
- Measure 1 of risk neutral workers who live forever.
- Exogenous distribution of skills: a fraction \( \rho \) of the workers low-skill level \( s_1 \) and a fraction \( 1 - \rho \) has high-skill level \( s_2 \).
- Jobs are either vacant or filled.
- Exogenous job destruction at rate \( \delta \).
- Jobs described by its skill requirement: technology is such that a filled job produces output \( x(s, y) \), given by

\[
x(s, y) = \begin{cases} 
y & \text{if } s \geq y \\
0 & \text{if } s < y
\end{cases}
\]
• When a vacant job is created, its skill requirement is chosen to maximize the value of the vacancy. Two skill requirements $y_1 = s_1$ and $y_2 = s_2$.

• Labor market tightness: $\theta = (v/u)$

• Matching technology (CRS): $m(u,v) = m(1,v/u) = m(\theta)u$

• Arrival rate of jobs for a worker: $m(\theta)$.

• Low-skill workers meet vacancies at the same rate as high-skill workers do, but they do not qualify for the high-skill vacancies.

• Fraction of vacancies that require low-skill levels $\phi \Rightarrow$ effective arrival rate for low-skill workers is $\phi m(\theta)$

• Arrival rate of workers for a vacant job: $m(\theta)/\theta$

• Fraction of unemployed low-skilled workers $\gamma \Rightarrow$ effective arrival rate to high-skill vacancies $(1-\gamma)((m(\theta))/\theta)$. 
Value Functions

- Employed worker of type $s$ on a job requiring skill $y$ (conditional on $s \geq y$):

$$rN(s, y) = w(s, y) + \delta[U(s) - N(s, y)]$$

- Low-skill unemployed:

$$rU(s_1) = b + m(\theta)\phi[N(s_1, s_1) - U(s_1)]$$

- High-skill unemployed:

$$rU(s_2) = b + m(\theta)[\phi \max[ N(s_2, s_1) - U(s_2), 0] + (1 - \phi)[N(s_2, s_2) - U(s_2)]]$$
• Firm of having a job with skill requirement $y$ filled by a worker of type $s$ (conditional on $s \geq y$):

$$rJ(s, y) = y - w(s, y) - c + \delta[V(y) - J(s, y)]$$

• Low-skill vacancy:

$$rV(s_1) = -c + \frac{m(\theta)}{\theta} \{\gamma[J(s_1, s_1) - V(s_1)] + (1 - \gamma) \max[J(s_2, s_1) - V(s_1), 0]\}$$

• High-skill vacancy

$$rV(s_2) = -c + \frac{m(\theta)}{\theta} (1 - \gamma)[J(s_2, s_2) - V(s_2)]$$
2.2. Match Formation and Wages

• Matches consummated between unemployed workers and vacancies when the joint surplus realized by the match is nonnegative:

\[ N(s, y) + J(s, y) \geq U(s) + V(y) \]

Substituting (conditional on \( s \geq y \))

\[ y - c \geq rU(s) \]

• Wage, \( w(s, y) \), is given by the Nash bargaining condition:

\[
(1 - \beta)[N(s,y) - U(s)] = \beta[J(s,y) - V(y)]
\]

Thus (conditional on \( y - c > rU(s) \))

\[ w(s,y) = \beta(y - c) + (1 - \beta)rU(s) \]
• At most three wages are paid in equilibrium

\[ w(s_1, s_1) = \beta(s_1 - c) + (1 - \beta)rU(s_1) \]
\[ w(s_2, s_1) = \beta(s_1 - c) + (1 - \beta)rU(s_2) \]
\[ w(s_2, s_2) = \beta(s_2 - c) + (1 - \beta)rU(s_2) \]

• Free entry of vacancies: \( V(s_1) = V(s_2) = 0. \)

Which implies

\[ J(s, y) = \frac{(1 - \beta)[y - c - rU(s)]}{r + \delta} \]
3. EQUILIBRIUM

• A steady-state equilibrium is a collection of four variables \( \{\theta, \phi, \gamma, u\} \) that satisfy:
  
  – All matches mutually advantageous relative to the alternative of continuing unmatched are formed.
  
  – Firm vacancy creation satisfies zero-value conditions.
  
  – The appropriate steady-state labor market flow conditions hold: the flow of low-skill workers into and out of unemployment be equal, and likewise for high-skill workers.

• Equilibrium depends on the parameters of the model. Two cases:
  
  – Equilibrium with cross-skill matching: it is beneficial for high-skill workers to match with low-skill vacancies. If parameters are consistent with cross-skill matching, there is a unique equilibrium of this type.
  
  – Equilibrium with ex post segmentation: it is not worthwhile for high-skill workers to take low-skill jobs. If parameters are consistent with ex post segmentation, there is a unique equilibrium of this type.
3.1. Cross-Skill Matching

• Worthwhile for high-skill workers to match with low-skill vacancies: \( s_1 - c \geq rU(s_2) \)

• Steady-state conditions:
  
  – Flow of low-skill workers out of unemployment equals the flow of low-skill workers back into unemployment:
    \[ \phi m(\theta) \gamma u = \delta (\rho - \gamma u) \]

  – Flow of high-skill workers out of unemployment equals the flow of high-skill workers back into unemployment:
    \[ m(\theta)(1 - \gamma)u = \delta (1 - \rho - (1 - \gamma)u) \]

• Solving equations above:
  
  \[ \phi = \frac{(1 - \gamma) \rho m(\theta) + (\rho - \gamma)\delta}{m(\theta)\gamma(1 - \rho)} \]

  \[ u = \frac{\delta (1 - \rho)}{(1 - \gamma)(\delta + m(\theta))} \]
• Substitute for the two unemployment values:

\[
\begin{align*}
   rU(s_1) &= \frac{b(r + \delta) + m(\theta)\phi \beta (s_1 - c)}{r + \delta + m(\theta)\phi \beta} \\
   rU(s_2) &= \frac{b(r + \delta) + \beta m(\theta)\phi s_1 + (1 - \phi)s_2 - c}{r + \delta + \beta m(\theta)}
\end{align*}
\]

• Zero-value conditions: for convenience, use equivalent conditions \( V(s_1) = V(s_2) = 0 \) and \( V(s_2) = 0 \). This leads to:

\[
(s_1 - c - b)(r + \delta) = (1 - \gamma)[(s_2 - c - b)(r + \delta) + \beta m(\theta)\phi(s_2 - s_1)]
\]

\[
c = \frac{m(\theta)}{\theta} \frac{(1 - \gamma)(1 - \beta)(s_1 - c - b)}{r + \delta + \beta m(\theta)}
\]

• Notice that \( \partial \theta/\partial s_2 = 0 \) and \( \partial \theta/\partial \rho = 0 \).

• Corner solution: only low-skill vacancies offered (can happen if \( \rho \) is sufficiently large and/or \( s_2 - s_1 \) is sufficiently small \( \Rightarrow \phi = 1 \)). The value of opening a high-skill vacancy must be negative

\[
s_1 - c - b > (1 - \rho) \left[ s_2 - c - b + \frac{\beta m(\theta^*) (s_2 - s_1)}{r + \delta} \right]
\]
3.2. Ex Post Segmentation

• High-skill workers match only with high-skill vacancies: \( s_1 - c < rU(s_2) \)

• Steady-state conditions:
  
  – Flow of low-skill workers out of unemployment equals the flow of low-skill workers back into unemployment
    \[
    \phi m(\theta) \gamma u = \delta (\rho - \gamma u)
    \]
  
  – Flow of high-skill workers out of unemployment equals the flow of high-skill workers back into unemployment:
    \[
    (1 - \phi)m(\theta)(1 - \gamma)u = \delta (1 - \rho - (1 - \gamma)u)
    \]

• Solving equations above:
  
  \[
  \phi = \frac{(1 - \gamma) \rho m(\theta) + (\rho - \gamma) \delta}{m(\theta)(\gamma + \rho - 2\gamma \rho)}
  \]
  \[
  u = \frac{\delta (\gamma + \rho - 2\gamma \rho)}{\gamma (1 - \gamma)(2 \delta + m(\theta))}
  \]
• Unemployment values:

\[ rU(s_1) = \frac{b(r + \delta) + m(\theta)\beta (s_1 - c)}{r + \delta + m(\theta)\beta} \]

\[ rU(s_2) = \frac{b(r + \delta) + \beta m(\theta)(1 - \phi)(s_2 - c)}{r + \delta + \beta m(\theta)(1 - \phi)} \]

• Zero-value conditions:

\[ c = \frac{m(\theta)}{\theta} (1 - \gamma) \left[ \frac{(1 - \beta)(s_1 - c - b)}{r + \delta + \beta m(\theta)\phi} \right] \]

\[ c = \frac{m(\theta)}{\theta} (1 - \gamma) \left[ \frac{(1 - \beta)(s_2 - c - b)}{r + \delta + \beta m(\theta)(1 - \phi)} \right] \]
3.3. Multiple equilibria

- Multiple equilibria can arise because the value of unemployment for an individual high-skill worker depends on the choices made by other high-skill workers:
  
  - If high-skill workers match only with high-skill vacancies ⇒ employers create more high-skill vacancies ⇒ the value of unemployment for the individual high-skill worker increases ⇒ ex post segmentation becomes more likely.
  
  - If high-skill workers accept any type of job ⇒ employers shift the vacancy mix in favor of low-skill jobs ⇒ cross-skill matching is obtained for a wider range of parameter values.

- Multiple equilibria do not occur for all possible parameter configurations:
  
  - For some parameter values (if \( p \) is large and/or \( s_2 - s_1 \) is small), it is worthwhile for an individual high-skill worker to accept a low-skill job, even if all other high-skill workers reject them ⇒ unique equilibrium, and it is one with cross-skill matching.
  
  - For other parameter values (if \( p \) is small and/or \( s_2 - s_1 \) is large), it is worthwhile for an individual high-skill worker to reject low-skill job offers, even if all other high-skill workers were to accept them ⇒ unique equilibrium, and it entails ex post segmentation.
  
  - For an intermediate range of parameter values, there are two pure-strategy equilibrium possibilities, one with cross-skill matching and one with ex post segmentation.
4. AN EXAMPLE

• Examine the effects of:

  – Increasing the productivity of high-skill jobs (skill-biased technical change).
  – Decreasing the productivity of low-skill jobs ("cheap import competition").
  – Changing the underlying worker skill distribution (shifts in the relative supplies of different worker skill types).

• Baseline parameter values chosen with three criteria:

  – Parameter values themselves should be reasonable.
  – Values of the endogenous variables that follow from these parameter values should also be reasonable.
  – Baseline parameters such that plausible variations illustrate the various equilibrium and comparative statics possibilities identified.
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### Table 2

Comparative statics for $s_1$ in cross-skill matching equilibrium—solution with $m(\theta) = 2\theta^{1/2}$ (Assumptions: $s_2 = 1.2$, $p = 2/3$, $b = 0.1$, $\beta = 0.5$, $\delta = 0.2$, $c = 0.3$, $r = 0.05$)

<table>
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<tr>
<th>$s_1$</th>
<th>$\theta$</th>
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<th>$\gamma$</th>
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### Table 3

Comparative statics for $p$ in cross-skill matching equilibrium—solution with $m(\theta) = 2\theta^{1/2}$ (Assumptions: $s_1 = 1$, $s_2 = 1.2$, $b = 0.1$, $\beta = 0.5$, $\delta = 0.2$, $c = 0.3$, $r = 0.05$)

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<th>$u$</th>
<th>$\gamma$</th>
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5. CONCLUSION

• Model highlights the role of skill in the labor market.

• The definition of skill used makes it possible to examine how low- and high-skill jobs interact, in addition to the usual interaction between low- and high-skill workers.

• Comparative statics properties:
  
  – A parameter shift (Skill-biased technical change):
    
    • increases wage dispersion within and between skill groups. This is consistent with recent experience in the United States (real wages rise for high-skill workers and fall for the less-skilled workers).
    
    • Increase in unemployment This is consistent with European developments, but has not been seen in the United States.

  – Reducing the value of low-skill production results in an increase in unemployment, but the wage effects are inconsistent with U.S. experience in that the real wages of both high- and low-skill workers fall.