The Optimum Quantity Of Debt

S. Rao Aiyagari, Ellen R. McGrattan

Journal of Monetary Economics, 1998

May 21, 2014
Motivation (I)

- Important issues in fiscal policy: level, time path, and type of government debt

- 90’s: concerns regarding the high level of public debt in the US economy

**Goal of this paper:**
- Calculate the optimum quantity of risk-free public debt
- Compute the welfare costs of being at levels other than the optimum
Motivation (II)

Total US Government Debt (as a % of GDP)

Source: OECD
The Model (I)

- Augmented version of Aiyagari (1994a): Growth, government debt, distortionary income taxes and elastic labor supply

- **Representative-agent framework**: Continuum of infinitely lived agents of measure unity who receive the *same idiosyncratic shocks* to their labor productivities (No aggregate shocks)

- Saving behaviour: precautionary savings motives and borrowing constraints

- New feature: Government debt has a different role than in standard models
  - **Benefit side**: enhancement of liquidity (additional means of smoothing consumption and loosening borrowing constraints)
  - **Cost side**: implied taxes have adverse wealth distribution and incentive effects and government debt crowds out capital via higher interest rates
The Model (II)

Households

\[
\max \left\{ c_t, a_{t+1}, l_t \right\}_{t=0}^{\infty} \sum_{t=0}^{\infty} E \left[ \beta^t (c_t^{\eta} l_t^{1-\eta})^{1-\mu} / (1 - \mu) \right]
\]

s.t. \quad c_t + a_{t+1} \leq \bar{w}_t e_t (1 - l_t) + TR_t + (1 + \bar{r}) a_t

\[
0 \leq l_t \leq 1
\]

\[
c_t \geq 0
\]

\[
a_t \geq 0
\]

\[
\bar{w}_t = (1 - \tau_y) w_t = (1 - \tau_y) z_t F_2(K_t, z_t N)
\]

\[
\bar{r} = (1 - \tau_y) r = (1 - \tau_y) F_1(K_t, z_t N)
\]

\[a_0, e_0 \text{ given}\]
The Model (III)

**Firms**

Maximize profits subject to a Neoclassical Production Function:

\[ Y_t = F(K_t, z_t N_t) \]

**Government**

\[ G_t + TR_t + rB_t = B_{t+1} - B_t + \tau_y(w_t N + rA_t) \]

**Market Clearing Conditions**

\[ A_t = K_t + B_t \]

\[ N = Ee_t(1 - I_t) \]
The Model (IV)

Stationarized Version

\[
\max \left\{ \tilde{c}_t, \tilde{a}_{t+1}, l_t \right\}_{t=0}^{\infty} \quad E[Y_0^{\eta(1-\mu)} \sum_{t=0}^{\infty} \beta(1 + g)^{\eta(1-\mu)} \tilde{c}_t^{\eta} l_t^{1-\eta} (1-\mu)/(1-\mu) \mid \tilde{a}_0, e_0] \\
\tilde{c}_t + (1 + g)\tilde{a}_{t+1} \leq \bar{w}e_t(1 - l_t) + (1 + \bar{r})\tilde{a}_t + \chi \\
0 \leq l_t \leq 1 \\
\tilde{c}_t \geq 0 \\
\tilde{a}_t \geq 0 \\
\tilde{a}_0, e_0 \text{ given}
\]
Welfare

Welfare Criterion
\[ \Omega = \int \int V(a, e) dH(a, e) \]

- Utilitarian Social Welfare function
- Steady-state ex ante welfare (Welfare of a typical consumer before he realizes his initial assets and the productivity shock, which are assumed to be drawn from the steady-state joint distribution H)

Welfare effects

- ↑ debt ↑ r ⇒ ↑ a
- ↑ debt ↓ k ⇒ ↓ c
Parameterization

- Model period: one year
- Production function: Cobb-Douglas
- Stochastic process for $e_t$: first-order autoregressive process ($\log(e_t)$ is approximated with a first-order Markov process with seven states (Tauchen 1986))
- All parameter values based on post-second world war data for the US

\[ g = 0.0185, \quad \chi = 0.082, \quad \delta = 0.075, \quad \rho = 0.6, \quad \mu = 1.5, \]
\[ \gamma = 0.217, \quad \theta = 0.3, \quad \eta = 0.328, \quad \sigma = 0.3, \quad \beta = 0.991. \]
Results
Conclusion

- Optimum quantity of debt calculated for a model that is parameterized to mimic certain features of the US economy

- Optimum quantity of debt
  - high if debt is effective in smoothing out consumption over the lifetime of an individual
  - low if debt crowds out capital or if the incentive effects of higher distortionary taxes are important

- Optimum quantity of debt obtained equal to the average level in the post-war US economy

- However, welfare costs to being at levels other than the US level are small for a wide range of debt/GDP ratios

- Results consistent to some robustness checks
Thank you for your attention