Default Risk and Income Fluctuations in Emerging Economies

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Introduction
Emerging economies have more volatile business cycle, which may be related to access to international credit market.
  ▶ Volatile and countercyclical interest rate
  ▶ Countercyclical default risk
▶ This paper studies the relation between default events, interest rates and output and provides potential mechanism generating the co-movements.
  ▶ Further, quantitative analysis and predictions for the default experience of Argentina.
Model

- Small open economy with perfect information and endogenous default risk
- Dynamic with discreet infinite time
- Agents
  - Identical risk averse households
  - Benevolent Government
  - Competitive and risk neutral lenders (later, risk averse pricing)
- Stochastic income in terms of tradeable good
  - Markov process
  - I.I.D shocks
- Government borrows in international credit market to smooth consumption of households.
- Lump sum transfers by government to the households
Model

- Incomplete asset market
  - Government borrow/lend through a state invariant bond
- 1-period debt contract (bond) between government and lenders, which are not enforceable
- Penalty for default
  - Exclusion from the market (re-entry and autarky)
  - Output loss
- Outside option for lenders is international r.o.i
- Bond price is endogenous s.t. it reflects the likelihood of default event
- The model studies the relation between default events, interest rates, and output
Households

- $E_0 \sum_0^\infty \beta^t u(c_t)$
- $c$ is consumption, $\beta$ is discount factor, and $u(.)$ is increasing and strictly concave
- Stochastic income stream $y$, follows Markov process with compact support, $f(y', y)$

Government

- Chooses one period discount bond $B'$ at price $q(B', y)$ and to default or repay its debt.
- $B' \geq 0$ means government saves $q(B', y)B'$ units of period t goods to receive $B'$ units of goods the next period.
- Resource constraint for small open economy under
  - repayment: $c = y + B - q(B', y)B'$
  - default: $c = y^{def} = h(y) \leq y$
- $B' \geq Z$, No Ponzi scheme
Lenders

\[ \phi = qB' - \frac{1-\delta}{1+r}B' \]

- \( \delta \) is the probability of default,
- \( 0 \leq \delta \leq 1 \)
- \( r \), constant international rate of interest
- Perfectly informed about the endowment process
- Chooses one period discount bond \( B' \) at price \( q(B', y) \)
- \( 1 + r^c = \frac{1}{q} \), country’s rate of interest and spread, \( r^c - r \)
Model in equations

- Aggregate state $s=(B,y)$
- Government’s policy functions
  - $V^o(s) = \max_{c,d} \{ V^c(s), V^d(y) \}$
  - $V^d(y) = u(y^{\text{def}}) + \beta \int [\theta V^o(0, y') + (1 - \theta) V^d(y')] f(y', y) dy'$
  - $V^c(s) = \max_{B'} \left\{ u(y - q(B', y)B' + B) + \beta \int V^o(s') f(y', y) dy' \right\}$
- Default policy: $D(B) = \{ y : V^d(y) > V^c(s) \}$ (repayment set: $A(B)$)
- Bond price satisfy
  - $q(B', y) = \frac{1 - \delta(B', y)}{1 + r}$
Timing

- Government starts with some initial assets
- It observes the income shock
- It chooses whether to repay its debt obligations or default
- If it decides to repay, then taking as given the bond price, it chooses the amount to lend/borrow subject to the resource constraint.
- Creditors taking price as given choose the amount to lend/borrow
- In case of default, the country is penalised.
- Consumption takes place.
Equilibrium

- Recursive sequential equilibrium
- Stochastic general equilibrium model with endogenous default risk.
  - Volatile and countercyclical interest rate
  - Countercyclical default risk
- The model studies the relation between default events, interest rates, and output
The recursive equilibrium for this economy is defined as a set of policy functions for (i) consumption $c(s)$; (ii) governments asset holdings $B'(s)$, repayment sets $A(B)$, and default sets $D(B)$; and (iii) the price function for bonds $q(B', y)$ such that:

- Taking as given the government policies, households consumption $c(s)$ satisfies the resource constraint.
- Taking as given the bond price function $q(B', y)$, the governments policy functions $B'(s)$, repayment sets $A(B)$, and default sets $D(B)$ satisfy the government optimization problem.
- Bonds prices $q(B', y)$ reflect the governments default probabilities and are consistent with creditors expected zero profits.
Results

Note, $\delta(B', y) = \int_{D(B')} f(y', y) dy'$

- Default sets are shrinkings in assets.
  - For all $B^1 \leq B^2$, if default is optimal for $B^2$ then $D(B^2) \subseteq D(B^1)$

Under I.I.D shocks ($\theta = 0, h(y) = y$)

- If, for some $B$, the default set is non-empty, then there are no contracts available $\{q(B'), B'\}$ such that the economy can experience capital inflows, $B - q(B')B' > 0$.
- Default incentives are stronger the lower the endowment. For all $y_1 \leq y_2$, if $y_2 \in D(B)$ then $y_1 \in D(B)$. 
Results

\[ B \]

\[ B^* \]

\[ B' \]

\[ q(B')B' \]

Risky borrowing region

\[ \text{slope} = \frac{1}{1 + r} \]

**Figure 2. Total Resources Borrowed**
## Table 1—Business Cycle Statistics for Argentina

<table>
<thead>
<tr>
<th>Default episode</th>
<th>$x$: Q1–2002</th>
<th>std($x$)</th>
<th>corr($x, y$)</th>
<th>corr($x, r^c$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest rates spread</td>
<td>28.60</td>
<td>5.58</td>
<td>-0.88</td>
<td></td>
</tr>
<tr>
<td>Trade balance</td>
<td>9.90</td>
<td>1.75</td>
<td>-0.64</td>
<td>0.70</td>
</tr>
<tr>
<td>Consumption</td>
<td>-16.01</td>
<td>8.59</td>
<td>0.98</td>
<td>-0.89</td>
</tr>
<tr>
<td>Output</td>
<td>-14.21</td>
<td>7.78</td>
<td>-0.88</td>
<td></td>
</tr>
</tbody>
</table>

Sources: MECON, EMBI
## Table 2—Business Cycle Statistics for Other Defaulters

<table>
<thead>
<tr>
<th>Default episode</th>
<th>$x$: Q3–1999</th>
<th>std ($x$)</th>
<th>corr ($x, y$)</th>
<th>corr ($x, r^c$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ecuador</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interest rates spread</td>
<td>47.58</td>
<td>5.44</td>
<td>-0.63</td>
<td></td>
</tr>
<tr>
<td>Trade balance</td>
<td>10.96</td>
<td>4.47</td>
<td>-0.39</td>
<td>0.05</td>
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<tr>
<td>Consumption</td>
<td>-7.14</td>
<td>2.78</td>
<td>0.92</td>
<td>-0.53</td>
</tr>
<tr>
<td>Output</td>
<td>-6.46</td>
<td>2.53</td>
<td></td>
<td>-0.63</td>
</tr>
<tr>
<td>Default episode</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Russia</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interest rates spread</td>
<td>30.43</td>
<td>17.50</td>
<td>-0.70</td>
<td></td>
</tr>
<tr>
<td>Trade balance</td>
<td>12.40</td>
<td>5.40</td>
<td>-0.17</td>
<td>0.86</td>
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<tr>
<td>Consumption</td>
<td>-17.20</td>
<td>7.08</td>
<td>0.79</td>
<td>-0.80</td>
</tr>
<tr>
<td>Output</td>
<td>-12.60</td>
<td>11.80</td>
<td></td>
<td>-0.70</td>
</tr>
</tbody>
</table>

Sources: IFS, CBE, EMBI
Calibration

- Direct output cost:
  \[ h(y) = \begin{cases} \hat{y} & \text{if } y > \hat{y} \\ y & \text{if } y \leq \hat{y} \end{cases} \]

- Utility: \( u(c) = \frac{c^{1-\sigma}}{1-\sigma} \)

- Income: \( \log(y_t) = \rho \log(y_{t-1}) + \epsilon_t^y \)

<table>
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<tr>
<th>Table 3—Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk-free interest rate</td>
</tr>
<tr>
<td>Risk aversion</td>
</tr>
<tr>
<td>Stochastic structure</td>
</tr>
<tr>
<td>Calibration</td>
</tr>
<tr>
<td>Discount factor</td>
</tr>
<tr>
<td>Probability of reentry</td>
</tr>
<tr>
<td>Output costs</td>
</tr>
</tbody>
</table>
Quantitative Results

Bond price schedule

\[ q(B', y) \]

- \( y_{Low} \)
- \( y_{High} \)
Quantitative Results
Quantitative Results
Quantitative Results

Value function $v^o(B, y)$

- $y_{Low}$
- $y_{High}$

Graph showing the value function $v^o(B, y)$ with two lines representing $y_{Low}$ and $y_{High}$ as $B$ varies from $-0.25$ to $0.15$.
# Quantitative Results

<table>
<thead>
<tr>
<th></th>
<th>Default episodes</th>
<th>std(x)</th>
<th>corr((x, y))</th>
<th>corr((x, r^c))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest rates spread</td>
<td>24.32</td>
<td>6.36</td>
<td>-0.29</td>
<td></td>
</tr>
<tr>
<td>Trade balance</td>
<td>-0.01</td>
<td>1.50</td>
<td>-0.25</td>
<td>0.43</td>
</tr>
<tr>
<td>Consumption</td>
<td>-9.47</td>
<td>6.38</td>
<td>0.97</td>
<td>-0.36</td>
</tr>
<tr>
<td>Output</td>
<td>-9.60</td>
<td>5.81</td>
<td></td>
<td>-0.29</td>
</tr>
</tbody>
</table>

*Other statistics*

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th>Mean spread</th>
<th>Output deviation in default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean debt (percent output)</td>
<td>5.95</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Default probability</td>
<td>3.00</td>
<td></td>
<td></td>
<td>-8.13</td>
</tr>
</tbody>
</table>
Quantitative Results
Quantitative Results with risk averse pricing

Here, $q(b', y) = \int_{A(B')} m(y') f(y', y) dy'$ where $m$ is lenders’ stochastic discount factor and $m_{t+1} = \frac{1}{1+r} - \lambda \epsilon_{t+1}^y$

<table>
<thead>
<tr>
<th>Table 5—Business Cycle Statistics with Risk Averse Pricing Kernel</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<tr>
<td>------------------</td>
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<tr>
<td>Interest rates spread</td>
</tr>
<tr>
<td>Trade balance</td>
</tr>
<tr>
<td>Consumption</td>
</tr>
<tr>
<td>Output</td>
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Other statistics

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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5.95</td>
<td>10.40</td>
<td>3.00</td>
<td>-7.21</td>
</tr>
</tbody>
</table>

Calibration: $\beta = 0.882, \lambda = 24$
Conclusion

- Interest rates respond to output fluctuations through endogenous time-varying default probabilities.
- Study relationship between default and output in an environment of incomplete assets, and establishes that incomplete markets deliver default events in recessions.
- The model predicts the 2001 default of Argentina and matches well with data
  - volatility of interest rates,
  - high volatility of consumption relative to income,
  - negative correlation between output and interest rates, and
  - negative correlation between the trade balance and output.
- With risk averse pricing kernel, the model explains high average interest rate spread.