Scheinkman & Weiss (1986): Borrowing Constraints and Aggregate Economic Activity

Carlos III Macro Reading Group

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To build a simple theoretical model to study the effects of borrowing constraints (under incomplete markets) on aggregate economic activity.

Mechanism: Wealth distribution changes endogenously ⇒ Incentives to work/save vary over time
The model

Two types $i = 1, 2$ of infinitely-lived agents:

- Choose consumption $c^i_t \geq 0$ and labor $l^i_t \geq 0$.
- Preferences:
  \[
  U^i = E_0 \int_0^\infty e^{-\beta t} [u(c^i_t) - l^i_t] \, dt.
  \]
- Labor productivity depends on state $s \in \{1, 2\}$ (follows a Poisson process with switching rate $\lambda$):
  1. $s = 1$: Agent 1’s productivity is one, agent 2’s is zero.
  2. $s = 2$: Agent 2’s productivity is one, agent 1’s is zero.

$\Rightarrow$ No aggregate, only idiosyncratic risk!

There is one unit of a good called “money”:

- initial endowments $y^1_0 + y^2_0 = 1$
- in fixed supply 1
- yields dividend flow $\delta \geq 0$ ($\delta = 0$: fiat money, $\delta > 0$: Lucas tree).
Benchmark: Complete markets/efficient allocation

Set initial money endowments equal: \( y_0^1 = y_0^2 = \frac{1}{2} \). Then:

- Setting is symmetric \( \Rightarrow \) Pareto planner puts equal weights on both agents.
- At each \( t \) in state \( s = i \), planner sets \( l^j \neq i = 0 \) and chooses \( \{c_t^1, c_t^2, l_t^i\} \) to

\[
\max_{c_t^1, c_t^2, l_t^i} \{u(c_t^1) + u(c_t^2) - l_t^i\}
\]

s.t. \( c_t^1 + c_t^2 \leq l_t^i + \delta \).

- First-order condition:

\[
1 \quad \text{marginal disutility of labor} \quad = \quad u'(c_t^1) \quad = \quad u'(c_t^2)
\]

Result:

1. No consumption risk: \( c_t^i = \bar{c} \) for \( i = 1, 2 \), for all \( t \).
2. Only productive agent works (a constant amount): \( l_t^i = l(s = i)\bar{I} \).
Incomplete markets

Can only trade money against the consumption good (at each $t$):

- Let $\omega \in \Omega$ denote a history (or “event”).
- $q(t, \omega)$: Price of money/capital (in terms of the good).
- $y^i_t$: Agent $i$’s holdings of money.
- Borrowing constraint: $y^i_t \geq 0$.
- Budget constraint:

\[
\dot{y}^i = \frac{l^i(t, \omega) - c^i(t, \omega)}{q(t, \omega)} \quad \text{if productive: } s(t, \omega) = i
\]
\[
\dot{y}^i = -\frac{c^i(t, \omega)}{q(t, \omega)} \quad \text{if unproductive: } s(t, \omega) \neq i
\]

$\Rightarrow$ State:

1. $s$: productivity
2. $z$: Asset position of agent of type 1 (agent 2: $1 - z$)
Agent 1’s problem

Given prices $q(t, \omega)$, agent 1 chooses consumption and labor:

$$\max_{c^1(t,\omega), l^1(t,\omega)} E_0 \int_0^\infty e^{-\beta t} \left[ u(c_t^1) - l_t^1 \right] dt$$

s.t.  $\dot{y}^1 = \frac{l_s(t,\omega) = 1 l^1(t,\omega) - c^1(t,\omega)}{q(t,\omega)}$ (budget constraint)

$y^1(t, \omega) \geq 0$ (no borrowing)

Note (trick!):
If agent works, i.e. $l^1(t, \omega) > 0$, then $u'(c^1(t, \omega)) = 1$ and so $c^1(t, \omega) = \bar{c}$. 
State of the economy:

1. $s$: productivity
2. $z$: asset position of agent of type 1 (agent 2: $1 - z$)

Definition

An *equilibrium* is a stochastic process $z(t, \omega) \in [0, 1]$ and a (symmetric) pricing function $q(z, s)$ such that

1. $y^1 = z$ solves agent 1’s problem given $q(z, s)$
2. $y^2 = 1 - z$ solves agent 2’s problem given $q(z, s)$

Note:

- Symmetry means: $q(z, 1) = q(1 - z, 2)$.
- Market clearing is built in: $y^1 + y^2 = z + 1 - z = 1$. 
Solving for equilibrium

**Strategy:** Guess candidate (type of) equilibrium, then show it is indeed an equilibrium. Guess:

1. Productive agent saves: \( l^i(z, i) > c^i(z, i) \)
2. Unproductive agent must dissave: \( 0 < c^j(z, i) \) when \( z \in (0, 1) \).

Work with Hamiltonians (instead of Bellman equations). Define:

\[
p(z, i) : \text{Marginal (or shadow) value of money for agent 1.}
\]

Then:

- If the agent is not constrained: Indifferent between

\[
\begin{align*}
\underbrace{p(z, i)} & = \underbrace{q(z, i)u'(c^i(z, i))} \\
\text{keep marginal unit of money} & \quad \text{sell marginal unit and consume}
\end{align*}
\]

Note: If 1 is productive: Then \( 1 = u'(c^1) \), so \( p(z, 1) = q(z, 1) \).

- The co-state equation (like Euler equation) is:

\[
\begin{align*}
\underbrace{p_z(z, i) \dot{z}} & = \beta \underbrace{p(z, i)} + \lambda [p(z, i) - p(z, j)] \\
\text{change in shadow value over } dt
\end{align*}
\]
Phase diagram

Always stay in region III
Results

- When the unproductive agent gets close to zero assets, the unproductive agent’s valuation of the asset goes to infinity (Inada condition!). Zero assets are never reached.
- “The value of the asset and both the consumption of the non-productive agent and aggregate consumption increase with the fraction of the asset held by the non-productive agent.”
  **Intuition:** The poorer the unproductive agent, the more desperate to keep some assets. Asset price rises and consumption/output fall.
- There exists a unique ergodic distribution for \((z, s)\).
Numerical solution
This model:

1. Shows that wealth distribution can have effect on aggregate variables.
2. Has business cycles that are not efficient/socially optimal
   - Social planner chooses constant output...
   - ...but changes in wealth distribution lead to fluctuations under incomplete markets.
   - Potential welfare gains from re-distribution towards unproductive agent!
Further points the authors make

- This setting shows that fluctuations in employment are compatible with constant real wages.
- Can explain the failure of models of asset returns which work only on aggregate consumption (i.e. a representative agent).
- Model can generate non-linear responses, e.g. to increase in money supply:
  1. When output is low, monetary expansion raises output.
  2. When output is high, monetary expansion decreases output.