Tobin’s Marginal q & Average q: A Neoclassical Interpretation

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Two theories of Investment

▶ Neoclassical: Firms maximize present discounted value of profits subject to constraints of a production function and an adjustment cost related to capital investment (firm can control investment, the capital flow, but the capital stock is predetermined).

▶ Tobin’s q theory: The rate of investment is a function of (marginal) q, the ratio of the market value of additional investment goods to their replacement cost (here again, some adjustment cost is implicit, or else firm would inc/dec capital stock immediately, to set q=1).
In this paper, Hayashi shows that these two theories are just two sides of the same coin. He introduces a neoclassical model and shows that this embeds q-theory.

What really matters for a firm is the marginal q adjusted for tax rules concerning corporate tax rates, investment tax credits, and depreciation formulas. This is introduced as modified q.
Firms problem

Consider a firm acting to max the PV of future after-tax net receipts

$$\max V(0) = \int_0^\infty R(t) \exp[\int_0^t r(s)ds] dt$$

$$s.t. R(t) = [1 - u(t)]\pi(t) + u(t) \int_0^\infty D(x, t - x) p_I(t - x) l(t - x) dx$$

- depreciation tax deductions

- investment tax credits

$$\dot{K} = \psi(l, K; t) - \delta K(t)$$

$$\pi(t) = p(t) F(l, K; t) - w(t) N(t)$$
Introducing $q$

We can now define Tobin’s marginal $q$ as

$$q = \lambda / p_I$$

and average $q$ as

$$h = V / (p_I K)$$

one of the FOCs of the firms maximization problem is then given by

$$\frac{q}{1 - k - z} = \frac{1}{\psi_I}$$

This gives an optimal investment rule

$$I = \alpha(\tilde{q}, K; t)$$

where $\tilde{q} = \text{modified } q = \frac{q}{1 - k - z}$
Relation between Marginal and Average $q$

In empirical work, average $q$ is often used as an instrumental variable for marginal $q$, as the first can be observed while the second cannot.

Propostion 1: Suppose the firm is a price-taker in its output market, and that the production function and the installment function are both linearly homogeneous, then marginal $q$ is equal to average $q$ minus the tax credits from current capital stock (call this $a$).

Propostion 2: Suppose the firm is a price-maker in its output market, and that the production function and the installment function are both linearly homogeneous, then marginal $q$ is equal to average $q$ minus the tax credits from current capital stock minus some monopoly rent.

So under certain assumptions, there exists a strong relationship between the two.
We have seen that optimal investment is a function of modified $q$. We have also seen that modified $q$ can be written as

$$
\tilde{q} = \frac{h - a}{1 - k - z}
$$

Hayashi investigates the relation between average and modified $q$. Finds that modified $q$ is much more stable over time (post-war US data). This occurs because $q$ moves with the by inflation hump (via $a$) and by the upward trend in the rate of investment tax credits.

He also runs an OLS of $1/K$ on constant and modified $q$. Modified $q$ is highly significant with $R$ squared of 0.46 but the Durban-Watson statistic suggests there is a problem with positive serial correlation.
In summary, Hayashi demonstrates that Tobin’s q-theory is embedded within the neoclassical framework for the investment decisions of firms.

He also highlights that what really matters is modified q; taking account of investment tax credits and the tax treatment of depreciation.