Search in the Labor Market and the Real Business Cycle

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Introduction

- Neoclassical real business cycle model fails to capture many stylized labor market facts
    - frictionless market
    - no persistency in output
    - wages equaling the marginal product of labour
    - correlation of employment and productivity
- Solution: Model with frictions and transacting costs (search-model)
    - quantitative implications in a general equilibrium setting

Related Literature

- Kydland and Prescott (1982), and Long and Plosser (1983)
- Diamond (1982), Mortensen (1982), and Pissarides (1988)

Physical Environment

- Continuum of homogenous workers (more precise households)
  - assumed large - perfectly insure themselfes
- Continuum of homogenous firms
- Perfect goods and capital markets
- Labour market inherits search frictions
  - driven by market-tightness: θ
  - matches evolve accroding to matching function: M
- firms incurring vacancy posting cost: a
- workers search with certain search intensity: $c(S)$
- exogenous seperation at rate: ψ

Planners Problem

- Social planner faces objective function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) = U(C_t) - G(N_t), 0 < \beta < 1$$

with $U(C_t) = \log(C_t),\ G(N_t) = N^{1-1/\nu}/(1-1/\nu)$

- and production technology

$$C_t + I_t + e^{\mu_t} c(S_t)(1-N_t) + e^{\mu_t} a V_t \leq e^{(1-\alpha)(\mu_t+z_t)} K_t^{\alpha} N_t^{1-\alpha}$$

with $c(S_t) = c_0 S_t^{\eta}$, $c_0 > 0$, $\eta > 1$, $S_t \geq 0$, $\alpha, \mu > 0$

subject to laws of motion

$$\begin{aligned} K_{t+1} &= (1-\delta) K_t + I_t \\ N_{t+1} &= (1-\psi) N_t + M_t \quad 0 < \delta, \psi < 1 \end{aligned}$$

with $M_t = V_t^{1-\lambda} [S_t (1-N_t)]^{\lambda}$

thus endogenous probabilities for job-finding and hiring:

$$\begin{aligned} p_t &= M_t / S_t (1-N_t) = \left(S_t^{-1} \theta_t\right)^{1-\lambda} \\ q_t &= M_t / V_t = \left(S_t \theta_t^{-1}\right)^{\lambda} \text{ where } \theta = V_t/(1-N_t) \end{aligned}$$

Planners Problem

Social planner chooses$\{C_t, K_{t+1}, N_{t+1}, S_t, V_t\}_{t=0}^{\infty}$ to maximize the objective function

$$W\left(\left\{z_t, \tilde{K}_t, N_t\right\}\right) = \max_{\{\tilde{I}_t, V_t, S_t\}} \left\{U\left(\tilde{C}_t, N_t\right) + \beta E\left(W'|W\right)\right\}$$

subject to

$$\begin{aligned}
\tilde{C}_t &= \tilde{Y}_t - \tilde{I}_t - c\left(S_t\right)\left(1 - N_t\right) - aV_t \\
\tilde{K}_{t+1} &= \left(1 - \delta\right)\tilde{K}_t + \tilde{I}_t \\
N_{t+1} &= \left(1 - \psi\right)N_t + M_t \\
z_{t+1} &= \rho z_t + \varepsilon_{t+1}
\end{aligned}$$

$$U_{\tilde{c}_t} = \beta E\left\{U_{\tilde{C}_{t+1}}\left[f_{\tilde{K}_{t+1}} + (1-\delta')\right] | \left\{z_t, \tilde{K}_t, N_t\right\}\right\}$$

$$\frac{aU_{\tilde{c}_t}}{M_{V_t}} = \beta E\left\{U_{\tilde{C}_{t+1}}\left[f_{N_{t+1}} + c\left(S_{t+1}\right)\right] - G_{N_{t+1}} + \frac{aU_{\tilde{C}_{t+1}}}{M_{V_{t+1}}}\left[(1-\psi) + M_{N_{t-1}}\right] | \left\{z_t, \tilde{K}_t, N_t\right\}\right\}$$

$$\frac{U_{\tilde{c}_t} c_{S_t}(1-N_t)}{M_{S_t}} = \beta E\left\{U_{\tilde{C}_{t+1}}\left[f_{N_{t+1}} + c\left(S_{t+1}\right)\right] - G_{N_{t+1}} + \frac{U_{\tilde{C}_{t+1}} c_{S_{t+1}}(1-N_{t+1})}{M_{S_{t+1}}}\left[(1-\psi) + M_{N_{t-1}}\right] | \left\{z_t, \tilde{K}_t, N_t\right\}\right\}$$

Model Calibration

- Matched parameters:

Output elasticity wrt K	α	0.360
Discount rate	β	0.990
Depreciation rate	δ	0.022
Convexity of $c(S_t)$	η	1.000
Negative of Frisch elasticity	ν	-1.250
Elasticity of job matches wrt S	λ	0.400
Seperation rate	ψ	0.070
Growth rate	μ	0.004
Advertising cost	a	0.050
Search cost level	c_0	0.005
Autocorr shock	ρ	0.950
St.dev shock	σ_ε	0.007

- Calibrational exercise:

  Comparing artificial (100 times model simulation) and US time series (59:1-88:2, CITIBASE) outcome

Results

Statistic	US data 59:1–88:2	Model variable S	Model fixed S	Statistic	US data 59:1–88:2	Model variable S	Model fixed S
σ_C/σ_Y	0.40	0.30	0.31	σ_P/σ_Y	0.68	0.70	0.74
	(0.032)	(0.025)	(0.025)		(0.043)	(0.026)	(0.022)
σ_I/σ_Y	2.39	2.97	2.91	σ_W/σ_Y	0.37	0.31	0.34
	(0.058)	(0.114)	(0.115)		(0.038)	(0.017)	(0.021)
σ_K/σ_Y	0.22	0.25	0.25	σ_u/σ_Y	6.11	5.85	4.63
	(0.026)	(0.038)	(0.030)		(0.403)	(0.300)	(0.186)
σ_E/σ_Y	0.54	0.42	0.36	σ_V/σ_Y	7.31	4.68	6.38
	(0.038)	(0.008)	(0.007)		(0.345)	(0.660)	(0.600)
σ_{LS}/σ_Y	0.53	0.49	0.47	$\rho(V, u)$	−0.95	0.32	−0.15
	(0.045)	(0.041)	(0.037)		(0.009)	(0.063)	(0.109)
σ_Y	1.87	1.12	1.07				
	(0.045)	(0.002)	(0.001)				

Results

	Statistic	− 3	− 2	− 1	0	1	2	3
I. *Employment and average labor productivity*								
US data 59:1–88:2	$\rho(E_t, P_{t-\tau})$	− 0.325 (0.097)	− 0.151 (0.118)	0.092 (0.111)	0.345 (0.104)	0.577 (0.095)	0.687 (0.102)	0.730 (0.114)
Model variable S	$\rho(E_t, P_{t-\tau})$	0.155 (0.137)	0.281 (0.130)	0.432 (0.118)	0.596 (0.103)	0.964 (0.011)	0.593 (0.077)	0.305 (0.090)
II. *Output and labor's share of income*								
US data 59:1–88:2	$\rho(Y_t, LS_{t-\tau})$	− 0.005 (0.112)	− 0.216 (0.108)	− 0.481 (0.089)	− 0.739 (0.060)	− 0.782 (0.067)	− 0.728 (0.093)	− 0.610 (0.110)
Model variable S	$\rho(Y_t, LS_{t-\tau})$	− 0.337 (0.110)	− 0.514 (0.099)	− 0.736 (0.064)	− 0.768 (0.029)	− 0.231 (0.117)	− 0.095 (0.105)	− 0.020 (0.105)
III. *Vacancies and unemployment*								
US data 59:1–88:2	$\rho(V_t, u_{t-\tau})$	− 0.535 (0.097)	− 0.769 (0.072)	− 0.928 (0.042)	− 0.954 (0.009)	− 0.824 (0.054)	− 0.607 (0.097)	− 0.357 (0.120)
Model variable S	$\rho(V_t, u_{t-\tau})$	0.200 (0.090)	0.224 (0.080)	0.263 (0.061)	0.322 (0.063)	− 0.476 (0.038)	− 0.365 (0.067)	0.197 (0.090)
Model fixed S	$\rho(V_t, u_{t-\tau})$	0.094 (0.085)	0.035 (0.083)	− 0.045 (0.099)	− 0.153 (0.110)	− 0.824 (0.013)	− 0.590 (0.079)	− 0.400 (0.100)

Results

	Statistic	0	1	2	3
I. *Unemployment*					
US data 59:1–88:2	$\rho(u_t, u_{t-\tau})$	1.000 (0)	0.899 (0.041)	0.687 (0.078)	0.427 (0.107)
Model fixed S	$\rho(u_t, u_{t-\tau})$	1.000 (0)	0.682 (0.073)	0.424 (0.110)	0.226 (0.123)
II. *Output*					
US data 59:1–88:2	$\rho(Y_t, Y_{t-\tau})$	1.000 (0)	0.874 (0.049)	0.681 (0.078)	0.464 (0.093)
Model fixed S	$\rho(Y_t, Y_{t-\tau})$	1.000 (0)	0.781 (0.051)	0.500 (0.102)	0.278 (0.124)

Conclusion

- Incorporating labour market frictions into the neoclassical growth model inherits numerous improvements:
  - productivity more volatile than wages
  - productivity leads employment over the cycle
  - countercyclical behaviour of labour share
  - unemployment and vacancies highly volatile
  - history dependence in wages (persistence ion un/employment and aggregate output)