Search in the Labor Market and the Real Business Cycle

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by Max Glischinski

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Introduction

- Neoclassical real business cycle model fails to capture many stylized labor market facts
  - frictionless market
  - no persistency in output
  - wages equaling the marginal product of labour
  - correlation of employment and productivity

- Solution: Model with frictions and transacting costs (search-model)
  - quantitative implications in a general equilibrium setting
Kydland and Prescott (1982), and Long and Plosser (1983)

Diamond (1982), Mortensen (1982), and Pissarides (1988)
Physical Environment

- Continuum of homogenous workers (more precise households)
  - assumed large - perfectly insure themselves
- Continuum of homogenous firms
- Perfect goods and capital markets
- Labour market inherits search frictions
  - driven by market-tightness: $\theta$
  - matches evolve according to matching function: $M$
- firms incurring vacancy posting cost: $a$
- workers search with certain search intensity: $c(S)$
- exogenous separation at rate: $\psi$
Social planner faces objective function
\[
E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) = U(C_t) - G(N_t), \ 0 < \beta < 1
\]
with \( U(C_t) = \log(C_t), \ G(N_t) = N^{1-1/\nu}/(1-1/\nu) \)

and production technology
\[
C_t + I_t + e^{\mu t} c(S_t)(1-N_t) + e^{\mu t} a V_t \leq e^{(1-\alpha)(\mu + z_t)} K_t^\alpha N_t^{1-\alpha}
\]
with \( c(S_t) = c_0 S_t^\eta, \ c_0 > 0, \ \eta > 1, \ S_t \geq 0, \ \alpha, \mu > 0 \)

subject to laws of motion
\[
K_{t+1} = (1-\delta) K_t + I_t
\]
\[
N_{t+1} = (1-\psi) N_t + M_t \ 0 < \delta, \ \psi < 1
\]
with \( M_t = V_t^{1-\lambda} [S_t (1-N_t)]^\lambda \)

thus endogenous probabilities for job-finding and hiring:
\[
p_t = M_t / S_t (1-N_t) = (S_t^{-1} \theta_t)^{1-\lambda}
\]
\[
q_t = M_t / V_t = (S_t \theta_t^{-1})^\lambda \text{ where } \theta = V_t / (1-N_t)
\]
Social planner chooses \( \{ C_t, K_{t+1}, N_{t+1}, S_t, V_t \}_{t=0}^{\infty} \) to maximize the objective function

\[
W \left( \{ z_t, \tilde{K}_t, N_t \} \right) = \max \left\{ U \left( \tilde{C}_t, N_t \right) + \beta E \left( W' | W \right) \right\}
\]

subject to

\[
\tilde{C}_t = \tilde{Y}_t - \tilde{I}_t - c (S_t) (1 - N_t) - a V_t
\]

\[
\tilde{K}_{t+1} = (1 - \delta) \tilde{K}_t + \tilde{I}_t
\]

\[
N_{t+1} = (1 - \psi) N_t + M_t
\]

\[
z_{t+1} = \rho z_t + \epsilon_{t+1}
\]
\[ U_{\tilde{c}_t} = \beta E \left\{ U_{\tilde{c}_{t+1}} \left[ f_{\tilde{K}_{t+1}} + (1 - \delta') \right] \mid \{ z_t, \tilde{K}_t, N_t \} \right\} \]

\[ \frac{aU_{\tilde{c}_t}}{MV_t} = \beta E \left\{ U_{\tilde{c}_{t+1}} \left[ f_{N_{t+1}} + c(S_{t+1}) \right] - G_{N_{t+1}} \right. \\
+ \left. \frac{aU_{\tilde{c}_{t+1}}}{MV_{t+1}} \left[ (1 - \psi) + M_{N_{t-1}} \right] \mid \{ z_t, \tilde{K}_t, N_t \} \right\} \]

\[ \frac{U_{\tilde{c}_t}c_{s_t}(1-N_t)}{Ms_t} = \beta E \left\{ U_{\tilde{c}_{t+1}} \left[ f_{N_{t+1}} + c(S_{t+1}) \right] - G_{N_{t+1}} \right. \\
+ \left. \frac{U_{\tilde{c}_{t+1}}c_{s_{t+1}}(1-N_{t+1})}{Ms_{t+1}} \left[ (1 - \psi) + M_{N_{t-1}} \right] \mid \{ z_t, \tilde{K}_t, N_t \} \right\} \]
Model Calibration

**Matched parameters:**

- Output elasticity wrt K \( \alpha \) 0.360
- Discount rate \( \beta \) 0.990
- Depreciation rate \( \delta \) 0.022
- Convexity of \( c(S_t) \) \( \eta \) 1.000
- Negative of Frisch elasticity \( \nu \) -1.250
- Elasticity of job matches wrt S \( \lambda \) 0.400
- Separation rate \( \psi \) 0.070
- Growth rate \( \mu \) 0.004
- Advertising cost \( a \) 0.050
- Search cost level \( c_0 \) 0.005
- Autocorr shock \( \rho \) 0.950
- St.dev shock \( \sigma_\varepsilon \) 0.007

**Calibrational exercise:**

Comparing artificial (100 times model simulation) and US time series (59:1-88:2, CITIBASE) outcome
## Results

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_C/\sigma_Y$</td>
<td>0.40 (0.032)</td>
<td>0.30 (0.025)</td>
<td>0.31 (0.025)</td>
<td>$\sigma_P/\sigma_Y$</td>
<td>0.68 (0.043)</td>
<td>0.70 (0.026)</td>
<td>0.74 (0.022)</td>
</tr>
<tr>
<td>$\sigma_T/\sigma_Y$</td>
<td>2.39 (0.058)</td>
<td>2.97 (0.114)</td>
<td>2.91 (0.115)</td>
<td>$\sigma_W/\sigma_Y$</td>
<td>0.37 (0.038)</td>
<td>0.31 (0.017)</td>
<td>0.34 (0.021)</td>
</tr>
<tr>
<td>$\sigma_K/\sigma_Y$</td>
<td>0.22 (0.026)</td>
<td>0.25 (0.038)</td>
<td>0.25 (0.030)</td>
<td>$\sigma_u/\sigma_Y$</td>
<td>6.11 (0.403)</td>
<td>5.85 (0.300)</td>
<td>4.63 (0.186)</td>
</tr>
<tr>
<td>$\sigma_E/\sigma_Y$</td>
<td>0.54 (0.038)</td>
<td>0.42 (0.008)</td>
<td>0.36 (0.007)</td>
<td>$\sigma_Y/\sigma_Y$</td>
<td>7.31 (0.345)</td>
<td>4.68 (0.660)</td>
<td>6.38 (0.600)</td>
</tr>
<tr>
<td>$\sigma_{LS}/\sigma_Y$</td>
<td>0.53 (0.045)</td>
<td>0.49 (0.041)</td>
<td>0.47 (0.037)</td>
<td>$\rho(V, u)$</td>
<td>-0.95 (0.009)</td>
<td>0.32 (0.063)</td>
<td>-0.15 (0.109)</td>
</tr>
<tr>
<td>$\sigma_Y$</td>
<td>1.87 (0.045)</td>
<td>1.12 (0.002)</td>
<td>1.07 (0.001)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Results

<table>
<thead>
<tr>
<th>Statistic</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>US data</td>
<td>$\rho(E_t, P_{t-1})$</td>
<td>-0.325</td>
<td>-0.151</td>
<td>0.092</td>
<td>0.345</td>
<td>0.577</td>
<td>0.687</td>
</tr>
<tr>
<td></td>
<td>(0.097)</td>
<td>(0.118)</td>
<td>(0.111)</td>
<td>(0.104)</td>
<td>(0.095)</td>
<td>(0.102)</td>
<td>(0.114)</td>
</tr>
<tr>
<td>Model</td>
<td>$\rho(E_t, P_{t-1})$</td>
<td>0.155</td>
<td>0.281</td>
<td>0.432</td>
<td>0.596</td>
<td>0.964</td>
<td>0.593</td>
</tr>
<tr>
<td>variable S</td>
<td>(0.137)</td>
<td>(0.130)</td>
<td>(0.118)</td>
<td>(0.103)</td>
<td>(0.011)</td>
<td>(0.077)</td>
<td>(0.090)</td>
</tr>
</tbody>
</table>

### II. Output and labor’s share of income

| US data   | $\rho(Y_t, LS_{t-1})$ | -0.005 | -0.216 | -0.481 | -0.739 | -0.782 | -0.728 | -0.610 |
|           | (0.112) | (0.108) | (0.089) | (0.060) | (0.067) | (0.093) | (0.110) |
| Model     | $\rho(Y_t, LS_{t-1})$ | -0.337 | -0.514 | -0.736 | -0.768 | -0.231 | -0.095 | -0.020 |
| variable S | (0.110) | (0.099) | (0.064) | (0.029) | (0.117) | (0.105) | (0.105) |

### III. Vacancies and unemployment

| US data   | $\rho(V_t, u_{t-1})$ | -0.535 | -0.769 | -0.928 | -0.954 | -0.824 | -0.607 | -0.357 |
|           | (0.097) | (0.072) | (0.042) | (0.009) | (0.054) | (0.097) | (0.120) |
| Model     | $\rho(V_t, u_{t-1})$ | 0.200 | 0.224 | 0.263 | 0.322 | -0.476 | -0.365 | 0.197 |
| variable S | (0.090) | (0.080) | (0.061) | (0.063) | (0.038) | (0.067) | (0.090) |
| Model     | $\rho(V_t, u_{t-1})$ | 0.094 | 0.035 | -0.045 | -0.153 | -0.824 | -0.590 | -0.400 |
| fixed S   | (0.085) | (0.083) | (0.099) | (0.110) | (0.013) | (0.079) | (0.100) |
## Results

### I. Unemployment

<table>
<thead>
<tr>
<th>Statistic</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>US data</td>
<td>$\rho(u_t, u_{t-1})$</td>
<td>1.000</td>
<td>0.899</td>
<td>0.687</td>
</tr>
<tr>
<td>59:1–88:2</td>
<td>(0)</td>
<td>(0.041)</td>
<td>(0.078)</td>
<td>(0.107)</td>
</tr>
<tr>
<td>Model</td>
<td>$\rho(u_t, u_{t-1})$</td>
<td>1.000</td>
<td>0.682</td>
<td>0.424</td>
</tr>
<tr>
<td>fixed S</td>
<td>(0)</td>
<td>(0.073)</td>
<td>(0.110)</td>
<td>(0.123)</td>
</tr>
</tbody>
</table>

### II. Output

<table>
<thead>
<tr>
<th>Statistic</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>US data</td>
<td>$\rho(Y_t, Y_{t-1})$</td>
<td>1.000</td>
<td>0.874</td>
<td>0.681</td>
</tr>
<tr>
<td>59:1–88:2</td>
<td>(0)</td>
<td>(0.049)</td>
<td>(0.078)</td>
<td>(0.093)</td>
</tr>
<tr>
<td>Model</td>
<td>$\rho(Y_t, Y_{t-1})$</td>
<td>1.000</td>
<td>0.781</td>
<td>0.500</td>
</tr>
<tr>
<td>fixed S</td>
<td>(0)</td>
<td>(0.051)</td>
<td>(0.102)</td>
<td>(0.124)</td>
</tr>
</tbody>
</table>

Max Glischinski  
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Incorporating labour market frictions into the neoclassical growth model inherits numerous improvements:

- productivity more volatile than wages
- productivity leads employment over the cycle
- countercyclical behaviour of labour share
- unemployment and vacancies highly volatile
- history dependence in wages (persistence ion un/employment and aggregate output)