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Duarte and Restuccia (2009) –> At the industry level, rich compared to poor countries have much higher productivity levels in the production of agricultural goods and services but no so much in manufacturing.
Motivation

Relative Prices

- Herrendorf and Valentinyi (forthcoming) → At the level of final expenditure, poorest countries are particularly inefficient at producing agriculture, investment and consumption goods, while much less inefficient at producing services.

- Duarte and Restuccia (2009) → At the industry level, rich compared to poor countries have much higher productivity levels in the production of agricultural goods and services but no so much in manufacturing.

- Analysis taking into account input-output pattern is needed.
Intermediate goods play an essential role in development accounting.
The production of goods uses relatively larger values of intermediate goods than the production of services.
Goods intermediates are relatively more prevalent in poorer countries.
Goods intermediates are relatively more prevalent in poorer countries.

Do intermediate inputs help to explain some of the salient relative and aggregate productivity differences across countries?

**Figure 4.** Shares of intermediates from same own industry.
Firms operate in a competitive environment

Two specializations: final or intermediate goods, $j \in \{f, m\}$

Two industries: goods or services, $i \in \{g, s\}$

$$y_{ji} = A_{ji} \left( \gamma_{gi} x_{gji} + \gamma_{si} x_{sji} \right)^{\frac{\sigma_i \rho_i}{\rho_i - 1}}$$
Firms operate in a competitive environment

Two specializations: final or intermediate goods, \( j \in \{ f, m \} \)

Two industries: goods or services, \( i \in \{ g, s \} \)

\[
y_{ji} = A_{ji} \left( \gamma_{gi} \frac{\rho_{i}}{\rho_{i}-1} x_{gji} + \gamma_{si} \frac{\rho_{i}}{\rho_{i}-1} x_{sji} \right)^{\frac{\sigma_{i}\rho_{i}}{\rho_{i}-1}} l_{ji}^{1-\sigma_{i}}
\]

Firm’s maximization of profits

\[
\max_{x_{gji} \geq 0, x_{sji} \geq 0, l_{ji} \geq 0} \left( p_{ji} y_{ji} - p_{mg} x_{gji} - p_{ms} x_{sji} - \omega l_{ji} \right)
\]
A representative household solves

\[
\max_{c_g \geq 0, c_s \geq 0} u(c_g, c_s) = \max_{c_g \geq 0, c_s \geq 0} \left( \frac{1}{\rho} \omega_g c_g^{\rho-1} + \frac{1}{\rho} \omega_s c_s^{\rho-1} \right)^{\frac{\rho}{\rho-1}}
\]

subject to

\[
p_{fg} c_g + p_{fs} c_s \leq \omega (l_{fg} + l_{fs} + l_{mg} + l_{ms})
\]

\[
l_{fg} + l_{fs} + l_{mg} + l_{ms} = 1
\]
A representative household solves

\[
\max_{c_g \geq 0, c_s \geq 0} u(c_g, c_s) = \max_{c_g \geq 0, c_s \geq 0} \left( \frac{1}{\rho} \left( \omega_g c_g^{\rho-1} + \omega_s c_s^{\rho-1} \right) \right)^{\frac{\rho}{\rho-1}}
\]

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\]

\[
l_{fg} + l_{fs} + l_{mg} + l_{ms} = 1
\]

Market clearing implies

\[
c_i = y_{fi}, i \in \{g, s\}
\]

\[
x_{ifg} + x_{ifs} + x_{img} + x_{ims} = y_{mi}, i \in \{g, s\}
\]
Theoretical Implications

- Two possible scenarios

**Definition**

**Industry-neutral growth**: Percentage changes in efficiency across industries are identical conditional on the specialization, i.e.
\[
\frac{dA_f}{A_f} \equiv \frac{dA_{fg}}{A_{fg}} = \frac{dA_{fs}}{A_{fs}} \quad \text{and} \quad \frac{dA_m}{A_m} \equiv \frac{dA_{mg}}{A_{mg}} = \frac{dA_{ms}}{A_{ms}}.
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Theoretical Implications

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\]

**Definition**

**Specialization-neutral growth:** Percentage changes in efficiency across specializations are identical conditional on the industry, i.e.

\[
\frac{dA_g}{A_g} \equiv \frac{dA_{fg}}{A_{fg}} = \frac{dA_{mg}}{A_{mg}} \quad \text{and} \quad \frac{dA_s}{A_s} \equiv \frac{dA_{fs}}{A_{fs}} = \frac{dA_{ms}}{A_{ms}}.
\]
Price ratio between specializations

\[ \frac{p_{mi}}{p_{fi}} = \frac{A_{fi}}{A_{mi}}, \forall i \in \{g, s\} \]
Theoretical Implications

Prices

- Price ratio between specializations

\[
\frac{p_{mi}}{p_{fi}} = \frac{A_{fi}}{A_{mi}}, \forall i \in \{g, s\}
\]

- Final good price ratio

\[
\frac{p_{fs}}{p_{fg}} = \frac{(1 - \sigma_g)}{(1 - \sigma_s)} \left( \frac{\sigma_g}{1 - \sigma_g} \right) \frac{A_{fg} A_{mg}}{A_{fs} A_{ms}} \left( \gamma_{ss} + \gamma_{gs} \left( \frac{A_{fs}}{A_{fg}} \frac{A_{mg}}{A_{ms}} \frac{p_{fs}}{p_{fg}} \right)^{\rho_s - 1} \right) \frac{\sigma_g}{(1 - \sigma_g)(1 - \rho_g)}
\]

\[
\frac{p_{fg}}{p_{fs}} = \frac{(1 - \sigma_s)}{(1 - \sigma_g)} \left( \frac{\sigma_s}{1 - \sigma_s} \right) \frac{A_{fs} A_{ms}}{A_{fg} A_{mg}} \left( \gamma_{gg} + \gamma_{sg} \left( \frac{A_{fs}}{A_{fg}} \frac{A_{mg}}{A_{ms}} \frac{p_{fs}}{p_{fg}} \right)^{1 - \rho_g} \right) \frac{\sigma_s}{(1 - \sigma_s)(1 - \rho_s)}
\]
Proposition 1: Assume the economy becomes more efficient across the board in the sense that $dA_{fg}, dA_{fs}, dA_{mg}, dA_{ms} > 0$:

(i) Under industry-neutral technical change the relative price of final services to final goods $p_{fs}/p_{fg}$ is increasing (decreasing) if and only if $\sigma_g > (\sigma_s)$;

(ii) under specialization-neutral technical change $p_{fs}/p_{fg}$ is increasing (decreasing) if and only if $\frac{dA_g}{dA_s} > (\frac{1-\sigma_g}{1-\sigma_s})$. 
Theoretical Implications
Intermediate Good Intensity

- Define the composite intermediate input demand

\[
m_{ji} \equiv \left( \gamma_{gi}^{\rho_i} x_{gji} + \gamma_{si}^{\rho_i} x_{sji} \right) \\frac{\sigma_i \rho_i}{\rho_i - 1}
\]

\[
\tilde{p}_{ji} m_{ji} = p_{mg} x_{gji} + p_{ms} x_{sji} = \sigma_i, \forall j \in \{f, m\}, i \in \{s, g\}
\]

- Relative share of the industries’ intermediates

\[
\frac{p_{mg} x_{gjg}}{p_{mg} x_{gjg} + p_{ms} x_{sjg}} = \frac{\gamma_{gg}}{\gamma_{gg} + \gamma_{sg} \left( \frac{p_{ms}}{p_{mg}} \right)^{1-\rho_g}} \equiv \Gamma_{gg} \in (0, 1), \forall j \in \{f, m\}
\]

\[
\frac{p_{ms} x_{sj} s}{p_{mg} x_{gjs} + p_{ms} x_{sjs}} = \frac{\gamma_{ss}}{\gamma_{ss} + \gamma_{gs} \left( \frac{p_{ms}}{p_{mg}} \right)^{\rho_s - 1}} \equiv \Gamma_{ss} \in (0, 1), \forall j \in \{f, m\}
\]
**Proposition 2:** Assume the economy becomes more efficient across the board in the sense that \( dA_{fg}, dA_{fs}, dA_{mg}, dA_{ms} > 0 \):

(i) Under industry-neutral thechnical change the real intermediate input intensity \( m_{mg} / y_{mg} \) is decreasing (increasing) if and only if \( \sigma_g > (<) \sigma_s \), \( m_{ms} / y_{ms} \) is increasing (decreasing) if and only if \( \sigma_g > (<) \sigma_s \), \( m_{fg} / y_{fg} \) is increasing (decreasing) if and only if

\[
\frac{(1-\sigma_g)(1-\sigma_s) + \sigma_s(1-\sigma_g)(1-\Gamma_{ss}) + \sigma_s(1-\sigma_g)(1-\Gamma_{gg})}{(1-\sigma_g)(1-\sigma_s) + \sigma_s(1-\sigma_g)(1-\Gamma_{ss}) + \sigma_s(1-\sigma_g)(1-\Gamma_{gg})} \frac{dA_m/A_m}{dA_f/A_f} > (<)1,
\]

and \( m_{fs} / y_{fs} \) is increasing (decreasing) if and only if

\[
\frac{(1-\sigma_g)(1-\sigma_s) + \sigma_s(1-\sigma_g)(1-\Gamma_{gg}) + \sigma_s(1-\sigma_g)(1-\Gamma_{ss})}{(1-\sigma_g)(1-\sigma_s) + \sigma_s(1-\sigma_g)(1-\Gamma_{gg}) + \sigma_s(1-\sigma_g)(1-\Gamma_{ss})} \frac{dA_m/A_m}{dA_f/A_f} > (<)1;
\]

(ii) Under specialization-neutral technical change \( m_{fg} / y_{fg} \) and \( m_{mg} / y_{mg} \) are increasing (decreasing) and \( m_{fs} / y_{fs} \) and \( m_{ms} / y_{ms} \) are decreasing (increasing) if and only if

\[
\frac{1-\sigma_g}{1-\sigma_s} \frac{dA_s/A_s}{dA_g/A_g} > (<)1.
\]
Value-added in each specialized industry $j_i$ is defined as

$$VA_{ji} \equiv p_{ji}y_{ji} - p_{mg}x_{gji} - p_{ms}x_{sji} = (1 - \sigma_i)p_{ji}y_{ji}$$

GDP (per unit of labor)

$$GDP \equiv \sum_{j,i} VA_{ji}$$

Price deflator

$$P \equiv \left( \omega_g p_g^{1-\rho} + \omega_s p_s^{1-\rho} \right)^{\frac{1}{1-\rho}}$$
Proposition 3: Assume the economy becomes more efficient across the board in the sense that $dA_{fg}, dA_{fs}, dA_{mg}, dA_{ms} > 0$:

(i) Under industry-neutral technical change a percent increase in intermediate good production efficiency $A_m$ increases real theoretical GDP by a factor of $\frac{\sigma_g (1-\sigma_s)(1-\Omega_s)+\sigma_s (1-\sigma_g)\Omega_s+\sigma_g \sigma_s (2-\Gamma_{gg}-\Gamma_{ss})}{(1-\sigma_g)(1-\sigma_s)+\sigma_g (1-\sigma_s)(1-\Gamma_{gg})+\sigma_s (1-\sigma_g)(1-\Gamma_{ss})}$ of a percent increase in final good production efficiency $A_f$ where

$$\Omega_s \equiv \frac{p_{fs}c_s}{p_{fg}c_g+p_{fg}c_s} = \frac{\omega_s \left( \frac{p_{fs}}{p_{fg}} \right)^{1-\rho}}{\omega_g+\omega_s \left( \frac{p_{fs}}{p_{fg}} \right)^{1-\rho}} \in (0,1);$$

(ii) under specialization-neutral technical change a percent increase in goods production efficiency $A_g$ increases real theoretical GDP by a factor of $\frac{(1-\sigma_s)(1-\Omega_s)+\sigma_s (1-\Gamma_{ss})}{(1-\sigma_g)\Omega_s+\sigma_g (1-\Gamma_{gg})}$ of a percent increase in services production efficiency $A_s$.  

Jan Grobovsek, 2011 ()
## Accounting and Counterfactuals

### Calibration

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_g$</td>
<td>0.570</td>
<td>$\sum_k \left( \frac{p_{mg}(x_{gfg}+x_{smg})+p_{ms}(x_{sfg}+x_{gms})}{p_{fg}y_{fg}+p_{mg}y_{mg}} \right)^k / K$</td>
</tr>
<tr>
<td>$\sigma_s$</td>
<td>0.357</td>
<td>$\sum_k \left( \frac{p_{mg}(x_{gfs}+x_{gms})+p_{ms}(x_{sf}+x_{gms})}{p_{fs}y_{fs}+p_{ms}y_{ms}} \right)^k / K$</td>
</tr>
<tr>
<td>$\gamma_{gg}, \rho_g$</td>
<td>0.677, 0.178</td>
<td>$\left( \frac{p_{mg}(x_{gfg}+x_{gms})}{p_{ms}(x_{sf}+x_{gms})} \right)^k, \left( \frac{p_{ms}}{p_{mg}} \right)^k$</td>
</tr>
<tr>
<td>$\gamma_{ss}, \rho_s$</td>
<td>0.572, 0.223</td>
<td>$\left( \frac{p_{ms}(x_{sf}+x_{sms})}{p_{mg}(x_{gfg}+x_{gms})} \right)^k, \left( \frac{p_{ms}}{p_{mg}} \right)^k$</td>
</tr>
<tr>
<td>$\omega_g, \rho$</td>
<td>0.437, 0.749</td>
<td>$\left( \frac{p_{fgc_g}}{p_{fs}c_s} \right)^k, \left( \frac{p_{fs}}{p_{fg}} \right)^k$</td>
</tr>
<tr>
<td>$A^k_{fg}, A^k_{fs}, A^k_{mg}, A^k_{ms}$</td>
<td>-</td>
<td>$(p_{fs}/p_{gs})^k, (p_{mg}/p_{fg})^k, (p_{ms}/p_{fs})^k, y_{fg}^k, y_{fs}^k, y_{mg}^k, y_{ms}^k, l_g^k, l_s^k$</td>
</tr>
</tbody>
</table>

Table 1: Benchmark calibration
Accounting and Counterfactuals

Calibration

![Graphs showing comparisons between model and data for various economic indicators.](Image)
Accounting and Counterfactuals

Results

Efficiency levels across countries and industries

- $A_{cg}$ (US=1)
- $A_{cs}$ (US=1)
- $A_{mg}$ (US=1)
- $A_{ms}$ (US=1)

Data aggregate productivity, int'l prices (EUKlerns, US=1)
### Counterfactuals

<table>
<thead>
<tr>
<th>benchmark</th>
<th>$A_{mg}^k = A_{fg}^k, A_{ms}^k = A_{fs}^k$</th>
<th>$\sigma_g = \sigma_s = 0.5$</th>
<th>$\gamma_{gg} = \gamma_{ss} = 0.5, \rho_g = \rho_s \rightarrow 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{fg}^P/A_{fg}^R$</td>
<td>0.830</td>
<td>0.748</td>
<td>0.795</td>
</tr>
<tr>
<td>$A_{fs}^P/A_{fs}^R$</td>
<td>0.855</td>
<td>1.015</td>
<td>0.866</td>
</tr>
<tr>
<td>$A_{mg}^P/A_{mg}^R$</td>
<td>0.456</td>
<td>0.410</td>
<td>0.436</td>
</tr>
<tr>
<td>$A_{ms}^P/A_{ms}^R$</td>
<td>0.573</td>
<td>0.678</td>
<td>0.580</td>
</tr>
</tbody>
</table>

**Table 2:** Alternative calibrations, average efficiency levels of poorest to richest quintile
### Table 3: Scenarios of convergence to US efficiency levels

<table>
<thead>
<tr>
<th>Scenario</th>
<th>(GDP/l)$^P$</th>
<th>(p$<em>{fs}$/p$</em>{fg}$)$^P$</th>
<th>(GDP/l)$^R$</th>
<th>(p$<em>{fs}$/p$</em>{fg}$)$^R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>0.401</td>
<td>0.778</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Benchmark calibration</td>
<td>0.467</td>
<td>0.683</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A^k_{fg} = A^US_{fg}$, $A^k_{mg} = A^US_{mg}$</td>
<td>0.728</td>
<td>1.119</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A^k_{fs} = A^US_{fs}$, $A^k_{ms} = A^US_{ms}$</td>
<td>0.589</td>
<td>0.600</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A^k_{fg} = A^US_{fg}$, $A^k_{fs} = A^US_{fs}$</td>
<td>0.542</td>
<td>0.695</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A^k_{mg} = A^US_{mg}$, $A^k_{ms} = A^US_{ms}$</td>
<td>0.843</td>
<td>0.972</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Conclusions

- The main driving factor behind aggregate and sectoral relative productivity differences across countries is the efficiency of intermediate good production.
Conclusions

- The main driving factor behind aggregate and sectoral relative productivity differences across countries is the efficiency of intermediate good production.
- The technical structure of the input-output relationship is such that relatively minor inefficiencies in intermediate good production are magnified strongly.